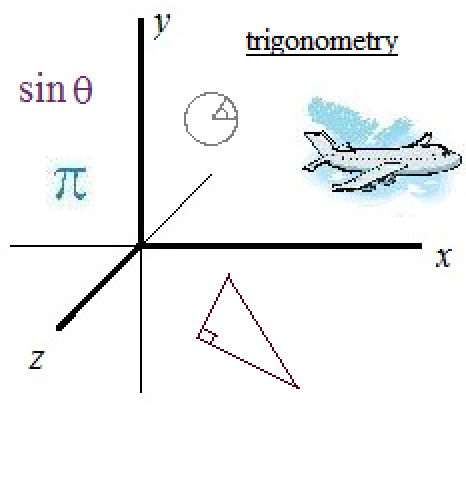
Trigonometry Identities IV: Review

Practice Questions (with Answers)

Includes simplifying, verifying, and solving trig equations.



Trig idenitites Review

Example: a) Using sum/difference formulas, find $\sin(15^\circ)$

b) Using half angle formulas, find $\sin(15^\circ)$

a) $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$

$= \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} = .2588$

b) $\sin(15^\circ) = \sin(\frac{30^\circ}{2})$

$= \sqrt{\frac{1 - \cos(30^\circ)}{2}}$ since this is quadrant I, the result is +

$= \sqrt{\frac{\frac{1}{2} - \frac{\sqrt{3}/2}{2}}{}} = \sqrt{\frac{2}{4} - \frac{\sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2} = .2588$

Example: a) Use angle sum/difference formula to find $\cos(\frac{5\pi}{6})$.

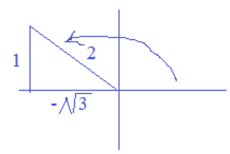
b) Use simple method to find $\cos(\frac{5\pi}{6})$.

a) $\cos(\frac{5\pi}{6}) = \cos(\pi - \frac{\pi}{6})$

$= \cos(\pi)\cos(\frac{\pi}{6}) + \sin(\pi)\sin(\frac{\pi}{6})$

$= (-1) \cdot \frac{\sqrt{3}}{2} + 0 \cdot \frac{1}{2} = \frac{-\sqrt{3}}{2}$

b) $\cos(\frac{5\pi}{6})$



$\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{-\sqrt{3}}{2}$

Example: find $\sin(\frac{\pi}{12})$ $\cos(\frac{\pi}{12})$ and $\tan(\frac{\pi}{12})$

converting to degrees: $\frac{\pi}{12} = 15$ degrees it's easier to see: 45 - 30

since we know exact values of π $\frac{\pi}{2}$ $\frac{\pi}{3}$ $\frac{\pi}{4}$ we'll convert to 12th's

$\frac{12\pi}{12}$ $\frac{6\pi}{12}$ $\frac{4\pi}{12}$ $\frac{3\pi}{12}$

$\frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$ $\frac{\pi}{3} - \frac{\pi}{4}$

$\sin(\frac{\pi}{12}) = \sin(\frac{\pi}{3})\cos(\frac{\pi}{4}) - \cos(\frac{\pi}{3})\sin(\frac{\pi}{4})$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6} - \sqrt{2}}{4} = .2588$

$\cos(\frac{\pi}{12}) = \cos(\frac{\pi}{3})\cos(\frac{\pi}{4}) + \sin(\frac{\pi}{3})\sin(\frac{\pi}{4})$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2} + \sqrt{6}}{4} = .9659$

$\tan(\frac{\pi}{12}) = \frac{\sin(\frac{\pi}{12})}{\cos(\frac{\pi}{12})}$

$\frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{2} + \sqrt{6}}{4}}$

$.2679 = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2} + \sqrt{6}}$

Warm-up:

Trigonometry Identity Review Test

A) Simplify: $\cos\theta + \sin\theta\tan\theta - \sec\theta$

$$\frac{\sin x + \cos x}{\tan x + 1}$$

B) Solve: $2\sin x + \csc x - 1 = 0$ where x is in the interval $[0, 2\pi)$

$$\sin x = \sqrt{3}\cos x$$

Trigonometry Identity Review Test

I. Simplify:

$\cos(-x)\tan(x)$

$\dfrac{\sin\theta}{\tan\theta}$

$\cos x + \tan x \sin x$

$\dfrac{(\sec y + 1)(\sec y - 1)\cos y}{\sin y}$

$\dfrac{\cot A + \tan A}{\sec^2 A}$

$\dfrac{1 + \cos 2x}{\cot x}$

Trigonometry Identity Review Test

II. Verify

$$\frac{1 + \sec\theta}{\sin\theta + \tan\theta} = \csc\theta$$

$$\cos^4 y - \sin^4 y = \cos 2y$$

$$\cot x - \tan x = \frac{2\cos^2 x - 1}{\sin x \cos x}$$

$$2\csc(2x) = \csc^2 x \tan x$$ hint: $\frac{\sin x}{\sin x} = 1$

$$2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 1$$

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$$ Hint: conjugates

III. Solve

Trigonometry Identity Review Test

$\csc^2 x - 2\csc(x) = 2 - 4\sin(x)$ where $0^\circ < x < 360^\circ$

Find ALL solutions: $\sin x \tan x + \tan x = 0$

$3\sin^2 x - 1 = \sin^2 x$ for $0 \leq x < 2\pi$

$\cos 4y(\cos y + 1) = 0$ for $0 \leq y < 2\pi$

$3\cos(x)\cot(x) + 7 = 5\csc(x)$ where $0^\circ < x < 360^\circ$

$2\sin\left(\frac{x}{3}\right) + 1 = 3$ Find ALL solutions

IV. Evaluate

Trigonometry Identity Review Test

Compare: $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ and $\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$

Find EXACT values (without a calculator)

$\tan(15^\circ)$

$\cos(75^\circ)$

$\sin(105^\circ)$ use 1/2 angles

$\sin(195^\circ)$ hint: 15° is a reference angle

V. More questions

Trigonometry Identity Review Test

$8 - 2\tan x - 5\sec^2 x = 0$

where $x \in (0^\circ, 360^\circ)$

$\sin(-x)\tan(-x) + \cos(-x) = ?$

$\operatorname{Tan}x + \operatorname{Sin}^2 x \operatorname{Sec}x = 0$ where $0 < x < 2\pi$

$4\tan^2 x - 1 = \tan^2 x$ (all solutions)

Find $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ if $\tan\theta = \frac{8}{15}$ in quad I

Trigonometry Identity Review Test

Verify: $4\cos^2 x + 3\sin^2 x = \cos^2 x + 3$

Verify: $\dfrac{1 + \sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1 - \cos\theta}$

Verify: $\dfrac{1 - 2\sin^2 x}{\sin x\cos x} = \cot x - \tan x$

If $\sin\theta = \dfrac{1}{3}$

and, the angle is in quadrant II

what is $\cos(\theta + \dfrac{\pi}{3})$?

$\sin(\theta - \dfrac{\pi}{6})$?

Find $\cos(2\text{Sin}^{-1}\left(\dfrac{3}{5}\right))$

Find $\tan(\arcsin\dfrac{x}{5})$

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

Preferable to ordinary computer cookies...

Essential part of a well-rounded, *academic diet.*

Try with (*t*), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

SOLUTIONS-→

Warm-up: SOLUTIONS

Trigonometry Identity Review Test

A) Simplify: $\cos\theta + \sin\theta\tan\theta - \sec\theta$

Change to sines and cosines (quotient property and reciprocal property)

$$\cos\theta + \sin\theta\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}$$

$$\cos\theta + \frac{\sin^2\theta}{\cos\theta} - \frac{1}{\cos\theta}$$

Add first and second terms...

$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} - \frac{1}{\cos\theta}$$

Pythagorean Identity

$$\frac{1}{\cos\theta} - \frac{1}{\cos\theta} = \boxed{0}$$

$$\frac{\sin x + \cos x}{\tan x + 1}$$

Quotient property for tangent

$$\frac{\sin x + \cos x}{\frac{\sin x}{\cos x} + 1}$$

$$\frac{\frac{\sin x + \cos x}{1}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}$$

$$\frac{\sin x + \cos x}{1} \cdot \frac{\cos x}{\sin x + \cos x} = \boxed{\cos x}$$

B) Solve: $2\sin x + \csc x - 1 = 0$ where x is in the interval $[0, 2\pi)$

Reciprocal identity $\quad 2\sin x + \frac{1}{\sin x} - 1 = 0$

multiply all terms by sinx $\quad 2\sin^2 x + 1 - \sin x = 0$

Factor $\quad (2\sin x + 1)(\sin x - 1) = 0$

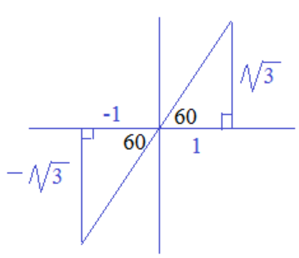
Solve $\quad \sin x = \frac{-1}{2} \qquad \sin x = 1$

$$\boxed{x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6} \quad x = \frac{\pi}{2}}$$

$\sin x = \sqrt{3}\cos x$

$$\frac{\sin x}{\cos x} = \sqrt{3}$$

$$\tan x = \sqrt{3}$$



$x = 60, 240, 420, \ldots$

or $\boxed{60^\circ + 180n}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \ldots$

or $\boxed{\frac{\pi}{3} + \pi k}$

n and k are any integer...

Trigonometry Identity Review Test

SOLUTIONS

I. Simplify:

$\cos(-x)\tan(x)$

$\cos(x)\cdot\tan(x)$ — even identity for cosine

$\cos(x)\cdot\dfrac{\sin(x)}{\cos(x)}$ — quotient identity for tangent

simplify

$\boxed{\sin(x)}$

$\dfrac{\sin\theta}{\tan\theta}$

$\dfrac{\sin\theta}{\dfrac{\sin\theta}{\cos\theta}}$ — quotient identity

$\dfrac{\sin\theta}{1}\cdot\dfrac{\cos\theta}{\sin\theta}$

$\boxed{\cos\theta}$

$\cos x + \tan x\sin x$

$\cos x + \dfrac{\sin x}{\cos x}\cdot\sin x$ — (tangent) quotient identity

$\cos x + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x}$ — Pythagorean identity

$\dfrac{1}{\cos x}$ — Reciprocal identity

$\boxed{\sec x}$

$\dfrac{(\sec y + 1)(\sec y - 1)\cos y}{\sin y}$

$\dfrac{(\sec^2 y + \sec y - \sec y - 1)(\cos y)}{\sin y}$

$(\sec^2 y - 1)\dfrac{(\cos y)}{(\sin y)}$

$\tan^2 y\ \dfrac{(\cos y)}{(\sin y)}$

$\tan^2 y\ (\cot y)$

$\tan^2 y\cdot\dfrac{1}{\tan y}$

$\boxed{\tan y}$

$\dfrac{\cot A + \tan A}{\sec^2 A}$

$\dfrac{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}}{\sec^2 A}$

$\dfrac{\dfrac{\cos^2 A}{\sin A\cos A} + \dfrac{\sin^2 A}{\cos A\sin A}}{\sec^2 A}$

$\dfrac{\dfrac{\cos^2 A + \sin^2 A}{\sin A\cos A}}{\sec^2 A}$

$\dfrac{\dfrac{1}{\sin A\cos A}}{\dfrac{1}{\cos^2 A}}$

$\dfrac{\cos^2 A}{\sin A\cos A}$ $\quad \dfrac{\cos A}{\sin A} \boxed{= \cot A}$

$\dfrac{1 + \cos 2x}{\cot x}$

$\dfrac{1 + 2\cos^2 x - 1}{\cot x}$

$\dfrac{2\cos^2 x}{\dfrac{\cos x}{\sin x}}$

$2\cos^2 x\cdot\dfrac{\sin x}{\cos x}$

$2\cos x\sin x \boxed{= \sin 2x}$

Identities:

$\text{Tan} = \dfrac{\text{Sin}}{\text{Cos}}$

$\text{Cot} = \dfrac{\text{Cos}}{\text{Sin}}$

Identities:

$\text{Cos}2x = \text{Cos}^2 x - \text{Sin}^2 x$

$\text{Cos}2x = 2\text{Cos}^2 x - 1$

$\sin 2x = 2\sin x\cos x$

II. Verify

SOLUTIONS

Trigonometry Identity Review Test

$$\frac{1+\sec\theta}{\sin\theta+\tan\theta} = \csc\theta$$

$$\frac{1+\frac{1}{\cos\theta}}{\sin\theta+\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}}$$

$$\frac{\cos\theta+1}{\sin\theta\cos\theta+\sin\theta} = \frac{\cos\theta+1}{\sin\theta(\cos\theta+1)}$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$2\csc(2x) = \csc^2 x\tan x \qquad \text{hint: } \frac{\sin x}{\sin x} = 1$$

$$\frac{2}{\sin(2x)}$$

$$\frac{2}{2\sin x\cos x}$$

$$\frac{1}{\sin x\cos x}\cdot\frac{\sin x}{\sin x}$$

$$\frac{1 \quad \sin x}{\sin^2 x\cdot\cos x}$$

$$\csc^2 x\cdot\tan x$$

$$2\sec^2 x - 2\sec^2 x\sin^2 x - \sin^2 x - \cos^2 x = 1$$

$$2\sec^2 x(1-\sin^2 x) - \sin^2 x - \cos^2 x = 1$$

$$2\sec^2 x(\cos^2 x) - \sin^2 x - \cos^2 x = 1$$

$$2 - \sin^2 x - \cos^2 x = 1$$

$$2 - 1(\sin^2 x + \cos^2 x) = 1$$

$$2 - 1(1) = 1$$

$$1 = 1$$

$$\cos^4 y - \sin^4 y = \cos 2y$$

$$(\cos^2 y - \sin^2 y)(\cos^2 y + \sin^2 y) = \cos 2y$$

$$(\cos^2 y - \sin^2 y)(1) = \cos 2y$$

$$\cos 2y = \cos 2y$$

$$\cot x - \tan x = \frac{2\cos^2 x - 1}{\sin x\cos x}$$

$$\frac{\cos^2 x + \cos^2 x - 1}{\sin x\cos x} \qquad \text{"split the 2"}$$

$$\frac{\cos^2 x}{\sin x\cos x} + \frac{\cos^2 x - 1}{\sin x\cos x} \qquad \text{separate fraction}$$

$$\frac{\cos x}{\sin x} + \frac{-(-\cos^2 x + 1)}{\sin x\cos x} \qquad \text{simplify and change negative}$$

$$\frac{\cos x}{\sin x} - \frac{(1 - \cos^2 x)}{\sin x\cos x}$$

$$\frac{\cos x}{\sin x} - \frac{\sin^2 x}{\sin x\cos x} \qquad \text{Pythagorean identity}$$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \qquad \text{quotient/tangent identity}$$

$$\cot x - \tan x$$

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{2\sin^2 x - 1}{1 + 2\sin x\cos x} \qquad \text{Hint: conjugates}$$

$$\frac{\sin x + \cos x}{\sin x + \cos x}\cdot\frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\frac{\sin^2 x - \sin x\cos x + \sin x\cos x - \cos^2 x}{\sin^2 x + 2\sin x\cos x + \cos^2 x} \qquad \text{factor and rearrange}$$

$$\frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x} \qquad \text{Pythagorean Trig Identity}$$

$$\frac{\sin^2 x - (1 - \sin^2 x)}{1 + 2\sin x\cos x} \qquad \text{Simplify}$$

$$\frac{2\sin^2 x - 1}{1 + 2\sin x\cos x}$$

III. Solve

SOLUTIONS

Trigonometry Identity Review Test

$\csc^2 x - 2\csc(x) = 2 - 4\sin(x)$ where $0^\circ < x < 360^\circ$

$\frac{1}{\sin^2 x} - \frac{2}{\sin(x)} - 2 + 4\sin(x) = 0$

Multiply by $\sin^2 x$

$1 - 2\sin(x) - 2\sin^2(x) + 4\sin^3(x) = 0$

Rearrange

$4\sin^3(x) - 2\sin^2(x) - 2\sin(x) + 1 = 0$

Group/GCF

$2\sin^2 x\,(2\sin x - 1) - 1(2\sin x - 1) = 0$

$\sin x = 1/2$ $\quad$ $\sin^2 x = 1/2$

$x = 30, 150$ $\quad$ $x = \pm \frac{1}{\sqrt{2}}$

$x = 45, 135, 225, 315$

$3\sin^2 x - 1 = \sin^2 x$ for $0 \le x < 2\pi$

Hint: $\sin^2 x = 1\sin^2 x$

$3\sin^2 x - 1\sin^2 x = 1$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$3\cos(x)\cot(x) + 7 = 5\csc(x)$ where $0^\circ < x < 360^\circ$

$3\cos(x)\frac{\cos(x)}{\sin(x)} + 7 = 5\frac{1}{\sin(x)}$

Reciprocal and Quotient Identities

multiply by sin(x)

$3\cos^2(x) + 7\sin(x) = 5$

$3(1 - \sin^2(x)) + 7\sin(x) - 5 = 0$

Pythagorean Identity

$-3\sin^2(x) + 7\sin(x) - 2 = 0$

$3\sin^2(x) - 7\sin(x) + 2 = 0$

Simplify and Factor

$(3\sin(x) - 1)(\sin(x) - 2) = 0$

$3\sin(x) - 1 = 0$ $\quad$ ~~$\sin(x) = 2$~~ Extraneous

$\sin(x) = \frac{1}{3}$

$x = 19.4^\circ$ or 160.6°

Find ALL solutions: $\sin x \tan x + \tan x = 0$

$\tan x(\sin x + 1) = 0$ $\quad$ $\sin x + 1 = 0$

$\tan x = 0$ $\quad$ $\sin x = -1$

$0, \pi, 2\pi$, etc.. $\quad$ $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$ etc

πk $\quad$ $\frac{3\pi}{2} + 2\pi k$

$\cos 4y(\cos y + 1) = 0$ for $0 \le y < 2\pi$

$\cos y + 1 = 0$ $\quad$ $\cos 4y = 0$

$\cos y = -1$ $\quad$ Let $A = 4y$:

$y = \cos^{-1}(-1)$ $\quad$ $\cos(A) = 0$

$A = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ etc...

→ 4y

$y = \pi$

therefore, $y = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

~~$\frac{17\pi}{8}$~~ $> 2\pi$

$2\sin\left(\frac{x}{3}\right) + 1 = 3$ Find ALL solutions

$2\sin\left(\frac{x}{3}\right) = 2$

$\sin\left(\frac{x}{3}\right) = 1$

Let $\left(\frac{x}{3}\right) = U$ $\quad$ $\sin(U) = 1$

$U = \frac{\pi}{2} + 2\pi K$

then, $\left(\frac{x}{3}\right) = \frac{\pi}{2} + 2\pi K$

therefore, $x = \frac{3\pi}{2} + 6\pi K$

IV. Evaluate

SOLUTIONS

Trigonometry Identity Review Test

Compare: $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ and $\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$

$$\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \quad + \quad \frac{1}{2} + \frac{\sqrt{2}}{2}$$

$$\boxed{\frac{\sqrt{3} + 2\sqrt{2} + 1}{2}}$$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{3} + \sqrt{2}}{2}}$$

Find EXACT values (without a calculator)

$\tan(15^\circ)$

$$\tan(45 - 30) = \frac{\tan(45) - \tan(30)}{1 + \tan(45)\tan(30)}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$\boxed{\frac{\sqrt{3} - 1}{\sqrt{3} + 1}}$$

(alternate method: find $\frac{\sin(15)}{\cos(15)}$)

$\cos(75^\circ)$

$$\cos(30 + 45) = \cos(30)\cos(45) - \sin(30)\sin(45)$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$\cos(75^\circ)$ is approximately .2588

$\sin(105^\circ)$ use 1/2 angles

$$\sin\left(\frac{210^\circ}{2}\right) = \pm\sqrt{\frac{1 - \cos(210)}{2}}$$

since the angle will end up in Quadrant II, (and sine is + in Quad II), the 1/2 angle will be positive

$$\sin(105) = +\sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$\boxed{\frac{\sqrt{2 + \sqrt{3}}}{2}}$$

(approx. .966)

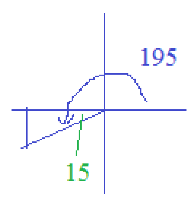
$\sin(195^\circ)$ find $\sin(15)$ for reference angle

$$\sin(15) = \sin(45 - 30)$$

$$\sin(45)\cos(30) - \cos(45)\sin(30)$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$



and, since 195 is in Quad III, sin(195) is negative

$$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

(approx. −.2588)

V. More questions

SOLUTIONS

Trigonometry Identity Review Test

$8 - 2\tan x - 5\sec^2 x = 0$

where $x \in (0^\circ, 360^\circ)$

$5\sec^2 x + 2\tan x - 8 = 0$ — use trig identity / substitution

$5(1 + \tan^2 x) + 2\tan x - 8 = 0$ — distribute and collect 'like' terms

$5\tan^2 x + 2\tan x - 3 = 0$

$5U^2 + 2U - 3 = 0$ — factor

$(5U - 3)(U + 1) = 0$

$(5\tan x - 3)(\tan x + 1) = 0$

$\tan x = 3/5$ or $.6$ $\quad x = \tan^{-1}(.6)$

$\tan x = -1$ $\quad x = \tan^{-1}(-1)$

$x = 30.96^\circ, 210.96^\circ$

$x = 135^\circ, 315^\circ$

$\sin(-x)\tan(-x) + \cos(-x) = ?$

$(-\sin x)(-\tan x) + \cos x$ — odd/even identities

$\sin x \tan x + \cos x$

$\sin x\left(\frac{\sin x}{\cos x}\right) + \cos x$

$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

$\operatorname{Tan} x + \operatorname{Sin}^2 x \operatorname{Sec} x = 0$ where $0 < x < 2\pi$

$\operatorname{Tan} x + \operatorname{Sin} x \cdot \operatorname{Sin} x \cdot \frac{1}{\operatorname{Cos} x} = 0$

$\operatorname{Tan} x + \operatorname{Sin} x \cdot \frac{\operatorname{Sin} x}{\operatorname{Cos} x} = 0$

$\operatorname{Tan} x + \operatorname{Sin} x \cdot \operatorname{Tan} x = 0$

$\operatorname{Tan} x(1 + \operatorname{Sin} x) = 0$

$\operatorname{Tan} x = 0$ $\quad x = 0, \pi$

$\operatorname{Sin} x = -1$ $\quad x = \frac{3\pi}{2}$

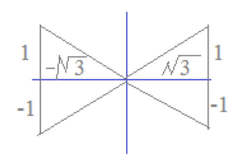
$4\tan^2 x - 1 = \tan^2 x$ (all solutions)

$3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \frac{1}{\sqrt{3}}$ $\qquad \tan x = \frac{-1}{\sqrt{3}}$



30, 210, 390, 570, ... $\qquad$ 150, 330, 510, 690, ...

$30^\circ + 180^\circ n$ $\qquad$ $150^\circ + 180^\circ n$

or, $\frac{\pi}{6} + \pi k$ $\qquad$ $\frac{5\pi}{6} + \pi k$

Find $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ if $\tan\theta = \frac{8}{15}$ in quad I

method 1: find the half angles and compute

$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{2}}$

$= \sqrt{\frac{1 - \frac{15}{17}}{2}} = \sqrt{\frac{1}{17}}$

$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

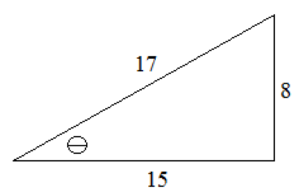
$= \sqrt{\frac{1 + \frac{15}{17}}{2}} = \sqrt{\frac{16}{17}}$

method 2: recognize double angle identity

$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} =$

$\sin 2\left(\frac{\theta}{2}\right) = \sin\theta$

$= \frac{8}{17}$



$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2 \cdot \frac{4}{17}$

$= \frac{8}{17}$

SOLUTIONS

Trigonometry Identity Review Test

Verify: $4\cos^2 x + 3\sin^2 x = \cos^2 x + 3$

$1\cos^2 x + 3\cos^2 x + 3\sin^2 x =$

$\cos^2 x + 3(\cos^2 x + \sin^2 x) =$

$\cos^2 x + 3(1) = \cos^2 x + 3$

Verify: $\dfrac{1 - 2\sin^2 x}{\sin x \cos x} = \cot x - \tan x$

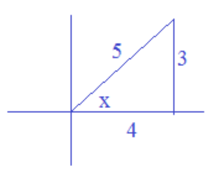
$\dfrac{1 - \sin^2 x - \sin^2 x}{\sin x \cos x}$

$\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}$

$\dfrac{\cos^2 x}{\sin x \cos x} - \dfrac{\sin^2 x}{\sin x \cos x}$

$\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x - \tan x$

Find $\cos\left(2\mathrm{Sin}^{-1}\left(\frac{3}{5}\right)\right)$



$\mathrm{Sin}x = \frac{3}{5}$ $\frac{\text{opposite}}{\text{hypotenuse}}$

find the inverse of 3/5 ... then, remember it's a double angle!

$\cos(2x) = \cos^2 x - \sin^2 x$

$\frac{16}{25} - \frac{9}{25} =$ 7/25

Verify: $\dfrac{1 + \sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1 - \cos\theta}$

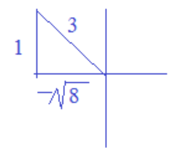
$\dfrac{1}{\sec\theta} + \dfrac{\sec\theta}{\sec\theta}$

$\cos\theta + 1\left(\dfrac{1 - \cos\theta}{1 - \cos\theta}\right)$

$\dfrac{1 - \cos^2\theta}{1 - \cos\theta} = \dfrac{\sin^2\theta}{1 - \cos\theta}$

If $\sin\theta = \frac{1}{3}$

and, the angle is in quadrant II



what is $\cos(\theta + \frac{\pi}{3})$?

Use addition properties

$\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$

$\frac{-\sqrt{8}}{3} \cdot \frac{1}{2} - \frac{1}{3} \cdot \frac{\sqrt{3}}{2}$

$\dfrac{-\sqrt{3} - \sqrt{8}}{6}$

$\sin(\theta - \frac{\pi}{6})$?

Use subtraction properties

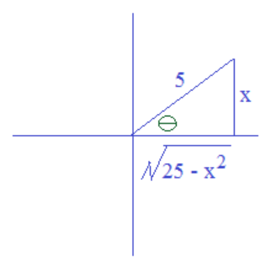
$\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6}$

$\frac{1}{3} \cdot \frac{\sqrt{3}}{2} - \frac{-\sqrt{8}}{3} \cdot \frac{1}{2}$

$\dfrac{\sqrt{3} + \sqrt{8}}{6}$

Find $\tan(\arcsin\frac{x}{5})$

$\sin\theta = \frac{x}{5}$



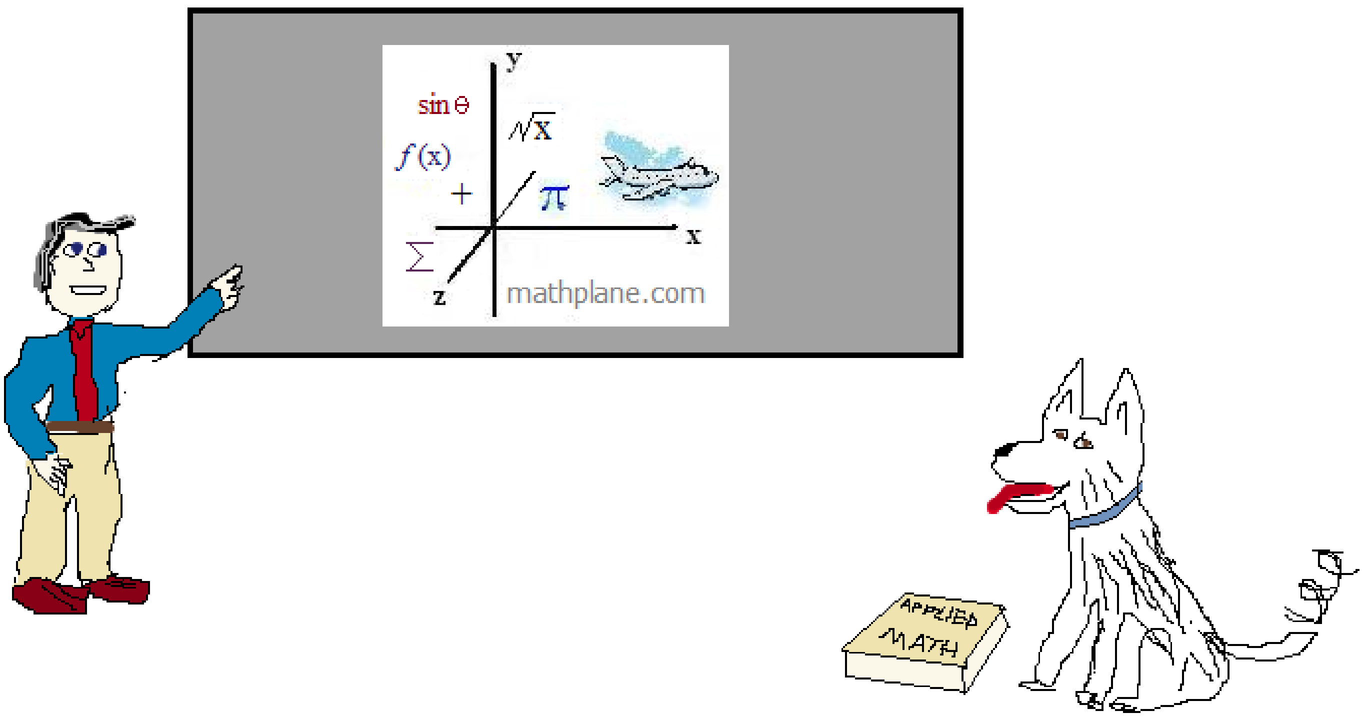
then, find tangent of that angle

$\tan\theta = \dfrac{x}{\sqrt{25 - x^2}}$

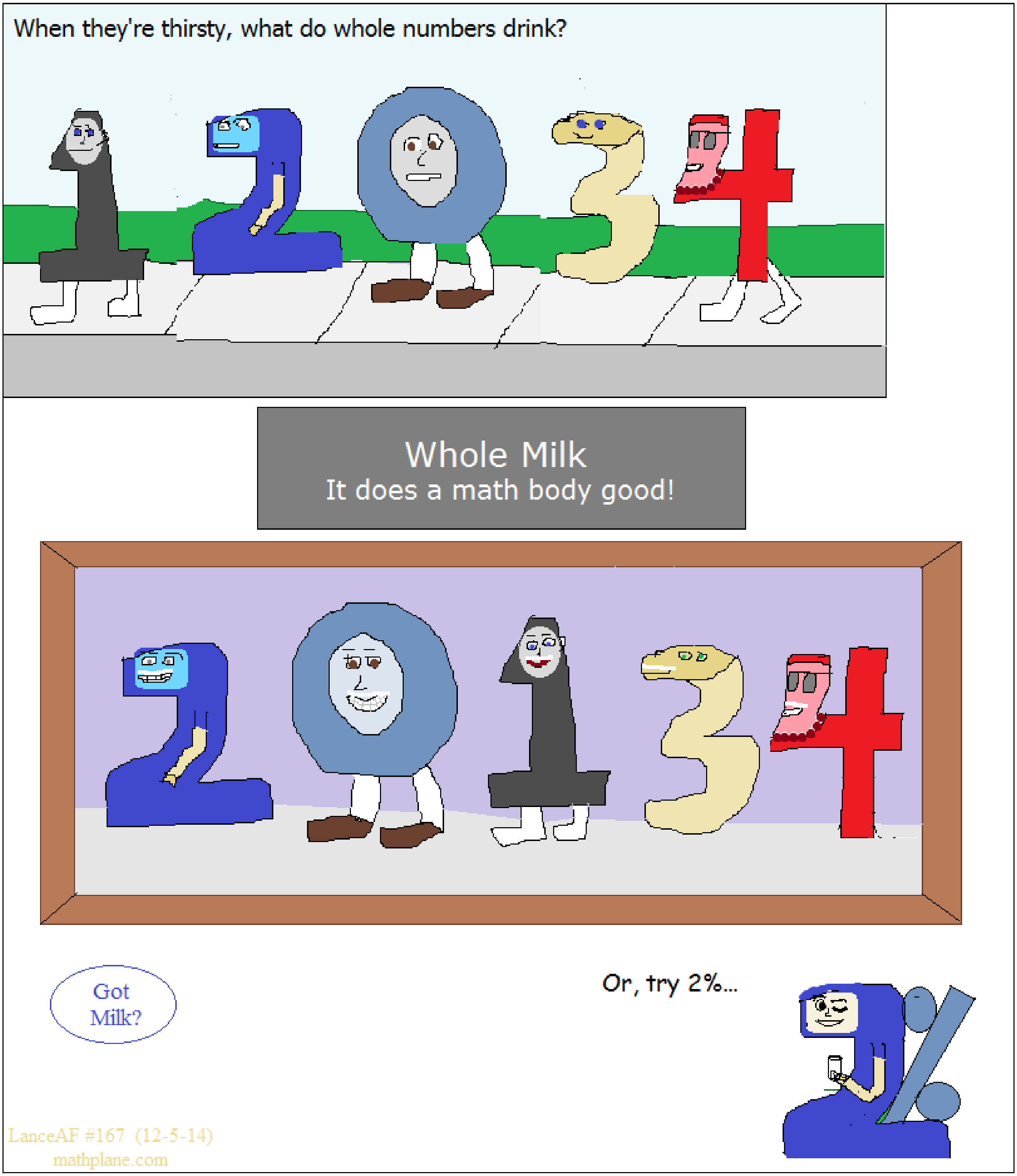
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, Google+, Pinterest, TES, and TeachersPayTeachers


When they're thirsty, what do whole numbers drink?
Whole Milk
It does a math body good!
Got Milk?
Or, try 2%...
LanceAF #167 (12-5-14)
mathplane.com