Triangle Characteristics

180 degrees

Notes, Illustrations, and practice quiz (& Solutions)
Triangle Introduction

Definition: A 2-dimensional, enclosed figure containing 3 line segments linked end to end (at the vertices)

Classification: Sides

- Scalene
  - (no equal sides)
- Isosceles
  - (2 equal sides)
- Equilateral
  - (3 equal sides)

Classification: Angles

- Right
  - (1 angle - 90 degrees)
- Acute
  - (all angles < 90 degrees)
- Obtuse
  - (1 angle > 90 degrees)
All Triangles are $180^\circ$

The sum of the interior angles of a triangle is $180$ degrees

$$a^\circ + b^\circ + c^\circ = 180^\circ$$

Observation: for any polygon, the sum of the interior angles is $(n - 2) \times 180^\circ$
where $n$ is the number of sides.

Why? Because, polygons can be cut into triangles.

**Examples:**

![Diagram of a quadrilateral](image)

$n = 4$ (sides)
$(n - 2) \rightarrow 2$ triangles
$(n - 2) \times 180^\circ = 360^\circ$

Interior angles of a quadrilateral add up to $360^\circ$ degrees...

![Diagram of a hexagon](image)

$n = 6$ (sides)
$(n - 2) \rightarrow 4$ triangles
$(n - 2) \times 180^\circ = 720^\circ$

The sum of the interior angles of a hexagon is $720^\circ$ degrees.

Observation: for any regular polygon, each interior angle is

$$\frac{(n - 2) \times 180^\circ}{n}$$

**Example:**

$n = 5$

$(n - 2) \times 180^\circ = 540^\circ$

then, $\frac{180(n - 2)}{n} = 108^\circ$

Each interior angle of a regular pentagon is $108^\circ$.
Triangle Inequality Theorem

Definition: The sum of the lengths of any 2 sides of a triangle is always greater than the length of the 3rd side.

Why? Because if a 3rd side is too long, then the others can't reach!

Example:
If A = 6 and B = 13, find C.
To get the 3rd side, "find the sum & find the difference"

\[ 13 + 6 = 19 \]
\[ 13 - 6 = 7 \]

\[ C \text{ is the largest side ("the sum")} \]
\[ C \text{ is the largest side ("the difference")} \]

Case 1: C is largest side
\[ A = 6 \quad B = 13 \]
\[ 13 < C < 19 \]

Case 2: isosceles
\[ A = 6 \quad B = C = 13 \]

Case 3: B is largest side
\[ A = 6 \quad B = 13 \]
\[ 7 < C < 13 \]

Note: If \( C = 7 \) or \( C = 19 \), then it's a line!

Note: If \( C < 7 \), then A and C won't touch!

If \( C > 19 \), then A and B won't touch!
Classifying and Identifying Triangles

I. Identify the following:

\[ \triangle ABC \text{ where } m \angle A = 25^\circ \]
\[ m \angle B = 35^\circ \]

sides: 
angles: 

II. Classify the following triangles:

"30-60-90 triangle":

sides: 
angles: 

"45-45-90 triangle":

sides: 
angles: 

"60-60-60 equiangular triangle":

sides: 
angles: 

III. Always, Sometimes, or Never?

1) An equilateral triangle is obtuse.

2) A right triangle is isosceles.

3) The sum of the interior angles of an obtuse triangle is 180°.
1) If the perimeter of an equilateral triangle is 18 feet, then what are the lengths of each side?

2) What is the measure of exterior angle $X$?

3) If $\triangle ABC$ is isosceles and right, what are the measures of each angle?

4) In diagram A, what are the measures of angles D, E, and F?

*5) Trick question: If 2 sides of an isosceles triangle are 6 and 10 inches, what is the length of the 3rd side?

**6) Challenge question: If 2 sides of a triangle are 7 and 12 inches, what is the length of the 3rd side?
1) Given $\triangle ABC \cong \triangle DEF$
In triangle ABC, which side is the smallest?

- $\overline{AC} = x^2$
- $\overline{DE} = 8x - 11$
- $\overline{FD} = 2x + 5$
- $\overline{FE} = 4x + 6$

2) Given $\overline{KM}$ is a perpendicular bisector of $\overline{JL}$; $\overline{JL} = 5x - 5$
What is the length of $\overline{JM}$?

3) Always, Sometimes, or Never?
Two triangles are congruent if
2 sides and 1 angle are congruent to corresponding parts of another.

4) If the perimeter of an equilateral triangle is $6y + 18$ and one side is $4y - 14$, what is the perimeter?
5) In \( \triangle ABC \),
if \( AC > BC > AB \), list the 3 angles in order of size (from largest to smallest).

6) In Circle C, \( PS \perp SR \)
\( \angle P = 38^\circ \)
Find
a) \( \angle PSC \)
b) \( \angle R \)

7) Given: \( AC > AB \)
What are the restrictions of \( x \)?
8) Find $x$

9) Find $y$

10) Find $z$

11) The vertices of a triangle are $(2, -6)$, $(5, -2)$, $(7, -6)$
Is this triangle scalene, isosceles, or equilateral?
SOLUTIONS
I. Identify the following:

\[ \triangle ABC \]
- Sides: scalene
- Angles: acute

\[ \triangle ABC \text{ where } m \angle A = 25^\circ, m \angle B = 35^\circ \]
- Sides: scalene
- Angles: obtuse

II. Classify the following triangles:

- "30-60-90 triangle":
  - Sides: scalene
  - Angles: right

- "45-45-90 triangle":
  - Sides: isosceles
  - Angles: right

- "60-60-60 equiangular triangle":
  - Sides: equilateral
  - Angles: acute

III. Always, Sometimes, or Never?

1) An equilateral triangle is obtuse. **NEVER**

2) A right triangle is isosceles. **SOMETIMES** (if it is a 45-45-90, then it is right)

3) The sum of the interior angles of an obtuse triangle is 180\(^\circ\). **ALWAYS** Sum of interior angles of ALL triangles is 180\(^\circ\)
1) If the perimeter of an equilateral triangle is 18 feet, then what are the lengths of each side?
   Since it is an equilateral triangle, all sides are the same.
   \[ A = B = C \]
   \[ A + B + C = 18' \]
   \[ 6 \text{ feet} \]

2) What is the measure of exterior angle \( X \)?
   Since the triangle has 3 equal sides, it must be equilateral.
   Therefore, all angles are 60°
   \[ \angle X + 60 = 180 \]
   \[ \angle X = 120° \]

3) If \( \triangle ABC \) is isosceles and right, what are the measures of each angle?
   (Isosceles) \( AB = BC \)
   (Right) \( \angle B = 90° \)
   \[ \angle A + \angle B + \angle C = 180 \]
   \[ \angle A = \angle C = 45° \]

4) In diagram A, what are the measures of angles D, E, and F?
   Sum of angles in a \( \triangle \) is 180°.
   And, vertical angles are congruent.
   Therefore, \( F = 80° \)
   \( \triangle DEF \) is isosceles because \( DF = EF \)
   so, \( \angle D = \angle E = 50° \)

**5) Trick question**: If 2 sides of an isosceles triangle are 6 and 10 inches, what is the length of the 3rd side?

**6) Challenge question**: If 2 sides of a triangle are 7 and 12 inches, what is the length of the 3rd side?
   If largest side is 12 inches, then \( X \) must be larger than 5
   If \( X \) is the largest side, then \( X \) cannot be larger than 19
   \[ 5 < X < 19 \text{ inches} \]
1) Given \( \triangle ABC \cong \triangle DEF \)
In triangle ABC, which side is the smallest?

\[
\begin{align*}
\overline{AC} &= x^2 \\
\overline{DE} &= 8x - 11 \\
\overline{Fe} &= 4x + 6 \\
\overline{FD} &= 2x + 3
\end{align*}
\]

Corresponding Parts Congruent Triangles Congruent
\(AC = DF\) \(x^2 = 2x + 3\) \(x^2 - 2x - 3 = 0\) \((x - 3)(x + 1) = 0\)
\(x = 3, -1\) If \(x = -1\), then \(DE = 8(-1) - 11 = -19\)
Side cannot be negative, so \(x \neq -1\)

Since \(x = 3\), \(DF = AC = 9\) \(EF = BC = 18\) \(DE = AB = 13\)

2) Given \(\overline{KM}\) is a perpendicular bisector of \(\overline{JL}\); \(\overline{JL} = 5x - 5\)
What is the length of \(\overline{JM}\)?

Quick proof:
- \(\overline{KM} \cong \overline{KM}\) (reflexive property)
- \(\angle JMK \cong \angle LMK\) (def. of perpendicular)

Therefore, \(\overline{JK} \cong \overline{KL}\) CPCTC

\[
\begin{align*}
10x + 3 &= 13x - 12 \\
15 &= 3x \\
x &= 5
\end{align*}
\]

If \(x = 5\), then \(\overline{JL} = 20\) and \(\overline{JM} = 10\)

3) Always, Sometimes, or Never?
Two triangles are congruent if
2 sides and 1 angle are congruent to corresponding parts of another.

SOMETIMES...
If the included angles are congruent, then the triangles must be congruent.

4) If the perimeter of an equilateral triangle is \(6y + 18\) and one side is \(4y - 14\), what is the perimeter?

If one side is \(4y - 14\), then all 3 sides are \(4y - 14\)

Therefore, the perimeter is \(3(4y - 14)\)

\[
\begin{align*}
6y + 18 &= 3(4y - 14) \\
6y + 18 &= 12y - 42 \\
60 &= 6y \\
y &= 10
\end{align*}
\]

Since \(y = 10\), each side is 26 and the perimeter is 78
5) In $\triangle ABC$,
if $\overline{AC} > \overline{BC} > \overline{AB}$, list the 3 angles in order of size (from largest to smallest).

Draw a diagram and assign values:

Since $B$ is opposite the largest side, it is the largest angle...
And, since $C$ is opposite the smallest side, it is the smallest angle...

B (largest), A (middle), C (smallest)

6) In Circle C, $PS \perp SR$

If $\angle P = 38^\circ$ then, $\angle PSC = 38^\circ$

Find a) $\angle PSC$

b) $\angle R$

If $\angle PSC$ is 38 degrees, then $\angle CSR = 52^\circ$

Therefore, $\angle R = 52^\circ$

**All radii are congruent**
(angles-sides theorem if sides are congruent, then opposite angles are congruent)

7) Given: $\overline{AC} > \overline{AB}$

What are the restrictions of $x$?

If $AC > AB$, then $\angle B > \angle C$

$5x - 42 > 18 + x$

$4x > 60$

$x > 15$

Since the sum of interior angles of triangle is 180, $B + C < 180$

$5x - 42 + 18 + x < 180$

$6x - 24 < 180$

$6x < 204$

$x < 34$

$15 < x < 34$
8) Find x

Sum of angles must be 180 degrees

a + a + 40 = 180
2a = 140
a = 70

Since a = 70, b = 110 (supplementary)
If b = 110, then x + x = 70... therefore x = 35

9) Find y

"sides-angles theorem", so other angle is 68...

therefore, 3rd angle is 44 degrees
(2y) = 136
y = 68

10) Find z

b + b + 42 = 180
2b = 138
b = 69

a = b (vertical angles)
a = 69
then, z = 42

11) The vertices of a triangle are (2, -6) (5, -2) (7, -6)

Is this triangle scalene, isosceles, or equilateral?

To determine sides, use the distance formula:

(2, -6) to (5, -2)
d = \sqrt{(5 - 2)^2 + (-2 - (-6))^2}
= \sqrt{9 + 16} = 5

(5, -2) to (7, -6)
d = \sqrt{(7 - 5)^2 + (-6 - (-2))^2}
= \sqrt{4 + 16} = 2\sqrt{5}

(7, -6) to (2, -6)
d = 5
(horizonal line segment)

Since 2 sides are the same length, the triangle is isosceles
6 more triangle questions:

1) What is the name of a triangle where all sides have different lengths?

2) A triangle has sides of length 8 and 13. What are the possible lengths of the 3rd side?

3) Two sides of an isosceles triangle are 5 and 7 feet. What is the perimeter of the triangle?

4) DEF is a right triangle. If angle E is 37 degrees, what are the measures of D and F?

5) Where do the altitudes of a right triangle intersect?

6) If the 3 altitudes of triangle ABC intersect outside the triangle, what type of triangle is ABC?
Answers to *6 more triangle questions*:

1) *Scalene* Triangle

2) Length of 3rd side (S):

\[ 5 < S < 21 \] (i.e. any length between 5 and 21)

3) a) \[ 5 + 5 + 7 = 17 \] OR,
   b) \[ 7 + 7 + 5 = 19 \]

4) \[ D = 53 \quad F = 90 \quad OR, \]
   \[ F = 53 \quad D = 90 \]

5) At the right angle vertex

![Diagram of a right triangle with altitudes](image1)

(3 altitudes of right triangle)

6) ABC is *Obtuse*

![Diagram of an obtuse triangle with altitudes](image2)

(3 altitudes of obtuse triangle)
Triangle Characteristics

1) What are the restrictions of x?

\[ m \angle A > m \angle B \]

Since \( \angle A > \angle B \),

\[ \overline{BC} > \overline{AC} \]

\[ (18 - x) > (3x + 30) \]

\[ -12 > 4x \]

\[ x < -3 \]

Also, since a side cannot be less than or equal to zero,

\[ \overline{BC} \quad 18 - x > 0 \quad x < 18 \]

\[ \overline{AC} \quad 3x + 30 > 0 \quad x > -10 \]

Therefore, the restrictions for x are \(-10 < x < -3\)

2) If the perimeter is less than 45, which side is the base?

If 10 is the base:

\[ x + 7 = 2x - 8 \]

\[ x = 15 \]

Therefore, the legs are 22

(If the legs are 22, then the perimeter exceeds 45)

If 2x - 8 is the base:

\[ x + 7 = 10 \]

\[ x = 3 \]

Therefore, the legs are 10 and the base is -6

(a segment cannot be negative!)

If x + 7 is the base:

\[ 2x - 8 = 10 \]

\[ x = 9 \]

Therefore, the legs are 10 and the base is 16

The base is \( \overline{TV} = 16 \)