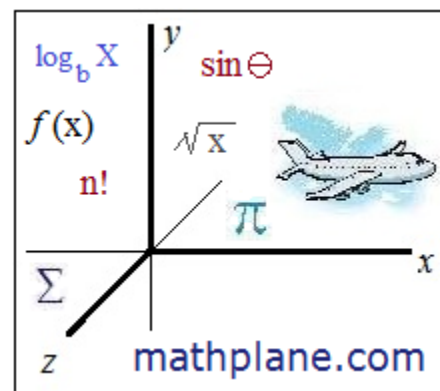


Algebra: Introduction to Polynomials

Definition, Notes, Examples, and Quizzes (w/Solutions)



Defining Polynomial Expressions

What is a 'polynomial'?

A sum/difference of terms that have variables raised to positive integers and coefficients that are real or complex.

Examples:	$x^3 + 3x^2 - 2x + 7$	yes	
	$3xy^2 + 6$	yes	
	$.445z^3 - 3i$	yes	
	$\frac{3}{x^2}$	no	$\frac{3}{x^2} = 3x^{-2}$ (Variables must have exponents with positive integers.)
	5^x	no	

Classifying Polynomial Expressions

There are 2 ways to describe polynomials:

- 1) The number of monomials ('number of non-zero terms')

# of terms	Classification	Examples
1	Monomial	xy^2 7 $-2z^3$
2	Binomial	$3x + 4$ $-9y^7 - y^2$ $xyz + x^2z^3$
3	Trinomial	$7 + xyz - 3i$ $x^2 - xy + 3y^4$
Any	Polynomial	$4x$ $x + 3y + 7z - 23xy^2$

Comment: When counting terms, consider "like terms"

$3x + 2y^2 + 4y^2$ is a binomial (because there is $3x$ and $6y^2$)

- 2) The largest degree of any one term

common polynomials

Degree	Classification	Examples
0	Constant	3 -5 $4/5$
1	Linear	$y + 6$ $3x$ $.23z - 12$
2	Quadratic	$x^2 + 2x - 7$ $6 + y^2$ $x^2 + y^2 - 6y$
3	Cubic	$x^3 + 1$ $2 + 3y - 4y^2 + y^3$ $xy^2 + x + 3$
4	Quartic	$x^4 + x^2y + 4y + 1$ $yz - 6x^4$
5	Quintic	$x^2y^3 + 7x - 3$

Comments: — xy^2 has degree 3 and x^2y^3 has degree 5

— The order of the terms does not matter.

— There are polynomials with larger degrees that are not listed above.
EX: 'octic' 'nonic' 'decic'

Defining/Classifying Polynomials

Examples:

	Number	Degree	Expression
$x^2 + 6x + 5$	3	2	Quadratic Trinomial
$x^3y^5 - 3y + 7x$	3	8	Octic Trinomial
$y - 2$	2	1	Linear Binomial
$z + 2z^2 - 7z^3$	3	3	Cubic Trinomial
$x^2 + 3x + 8 - x$ (note: you must combine "like terms")	3	2	Quadratic Trinomial

Quick Quiz:

Define any polynomials in the following expressions:

- 1) $x + 3y - 2$
- 2) $2 + 3x - 4x^2$
- 3) $\frac{1}{y^3}$
- 4) $12y + 3/4$
- 5) $z^3 + 1$
- 6) $3x^2y^5 + 2xy + 8$
- 7) $x^2y + \sqrt{7}$
- 8) $4^2r^3s^2$
- 9) $x^2 + xy - 10x + 3$
- 10) $x^2\sqrt{x+2}$

****Challenge:** (Using Examples)

Show that multiplying 2 linear binomials can produce a quadratic trinomial.
Then, show that multiplying 2 linear binomials can produce a quadratic binomial.

Quick Quiz:

Define the following Expressions:

	Number of monomials	Highest Degree	Definition
1) $x + 3y - 2$	3	1	Linear Trinomial
2) $2 + 3x - 4x^2$	3	2	Quadratic Trinomial
3) $\frac{1}{y^3}$	0	0	-----
4) $12y + 3/4$	2	1	Linear Binomial
5) $z^3 + 1$	2	3	Cubic Binomial
6) $3x^2y^5 + 2xy + 8$	3	7	Trinomial of degree 7
7) $x^2y + \sqrt{7}$	2	3	cubic binomial
8) $4^2r^3s^2$	1	5	5th degree monomial
9) $x^2 + xy - 10x + 3$	4	2	quadratic polynomial
10) $x^2\sqrt{x+2}$	0	0	-----

****Challenge:** (Using Examples)

Show that multiplying 2 linear binomials can produce a quadratic trinomial.

$$(X + 5)(X + 3) = X^2 + 8X + 15 \quad \text{Quadratic Trinomial}$$

Then, show that multiplying 2 linear binomials can produce a quadratic binomial.

$$(X + 7)(X - 7) = X^2 - 49 \quad \text{Quadratic Binomial}$$

also,

$$(X + 5)(Y + 3) = XY + 3X + 5Y + 15 \quad \text{Quadratic with 4 terms}$$

"Distribute" -- Multiply *each term* in the polynomial

Working with Polynomials

Example: $5x(3x + 2x^2 - 1)$

$$5x \quad (3x + 2x^2 - 1)$$

$$5x \cdot 3x = 15x^2$$

$$5x \cdot 2x^2 = 10x^3$$

$$5x \cdot (-1) = -5x$$

(then, write in descending order)

$$10x^3 + 15x^2 - 5x$$

Example: $-2xy(x^3 + 2y^2 + 4y - 1)$

$$-2xy \quad (x^3 + 2y^2 + 4y - 1)$$

$$-2xy \cdot x^3 = -2x^4y$$

$$-2xy \cdot 2y^2 = -4xy^3$$

$$-2xy \cdot 4y = -8xy^2$$

$$-2xy \cdot (-1) = 2xy$$

(multiply carefully, then combine all the terms....)

$$-2x^4y - 4xy^3 - 8xy^2 + 2xy$$

Taking out the Greatest Common Factor (GCF)

A useful way to simplify a polynomial is to take out the GCF:

Example: $9x^5 + 12x^3 + 6x$

The greatest common factor of 9, 12, and 6 is 3
and
the greatest common factor of x^5 , x^3 , and x is x

The GCF of the polynomial is $3x$

**So divide *each term* by $3x$

$$9x^5 + 12x^3 + 6x$$

/3x

$$3x(3x^4 + 4x^2 + 2)$$

Example: $4a^3bc - 10ab^4 + 20ac^2$

The greatest common factor of 4, 10, and 20 is 2
and
the greatest common factor of a^3 , a , and a is a
then..

the GCF of b , b^4 and no b is 1

the GCF of c , no c , and c^2 is 1

The GCF of the polynomial is $2a$

(divide each term by $2a$)

$$4a^3bc - 10ab^4 + 20ac^2$$

/2a

$$2a(2a^2bc - 5b^4 + 10c^2)$$

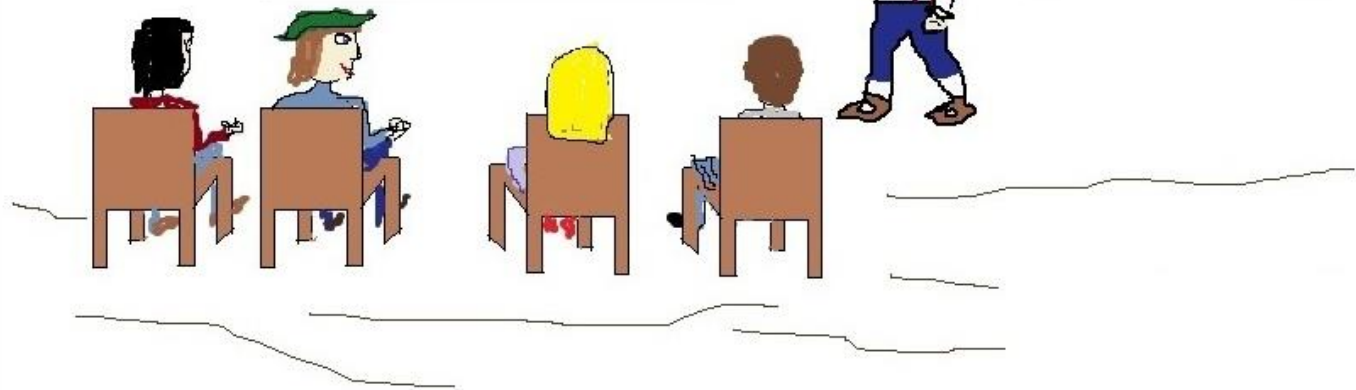
Math
Poet

19 January
MDLXXXIV

$$\sqrt{4b^2}$$

"2b or not 2b?
That is the question."

"Romeo, pay attention!
stop staring at Juliet."



LAF #15 (1-22-12)
mathplane.com

To earn a little extra coin, Bill Shakespeare
works as a substitute math teacher.

Practice Quiz (and Solutions)-→

Introduction to Polynomials Quiz

I. Addition/Subtraction: Simplify the following

a) $(2x^2 - 4x + 6) + (x^2 + 13x - 5)$ b) $(2x + 5 - 7x^2) + (x^2 - 11x + 3)$ c) $(x^2 - 3x + 6) - (2x^2 + 6x - 5)$

d) $(3ab^2 + 2a^2b - b^3) + (5a^2 + 6ab^2 + 2b^3)$

e) $3(x + y) + 2c(x + y) + d(x + y)$

II. Multiplication: Expand the following

a) $(x + 3)(x + 7)$

b) $(x + 4)^2$

c) $(x - 11)(x + 4)$

d) $(x + 2y)(3x - y)$

e) $-(x - 3)(x + 8)$

f) $(x + 5)(x^2 + 7x - 1)$

g) $(x + 9)(x - 9)$

h) $(x + 1)(x + 2)(x + 3)$

i) $(x^2 - 2)(x^3 + 6x + 14)$

III. Classifying Polynomials: Determine which are polynomials; then, identify the type, degree, and lead coefficients

a) $3x + 3$

b) $5 + 4x - 5x^3$

c) $4^Y + 1$

d) $z^3 + z^2 + z + \frac{1}{z}$

e) $4c^3d + 3cd^2 + d^3$

f) $9x^3y^3$

IV. Simplify and Classify: Solve and identify each polynomial (type and degree)

a) $(2x + 3)^2 + (3x^2 + 6)$

b) $(2a^2 + b)(2b + a)$

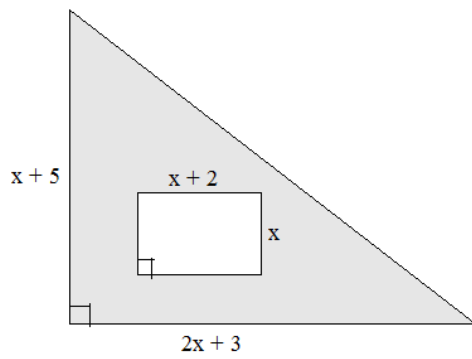
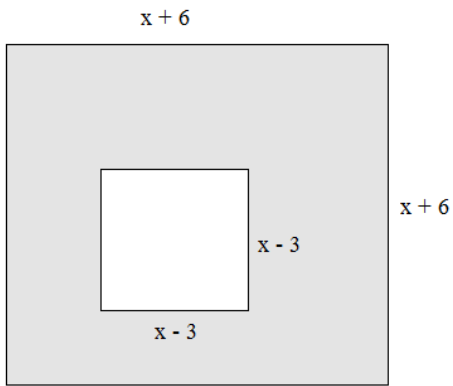
c) $-3(6 - 2s^2 + 5s^3)$

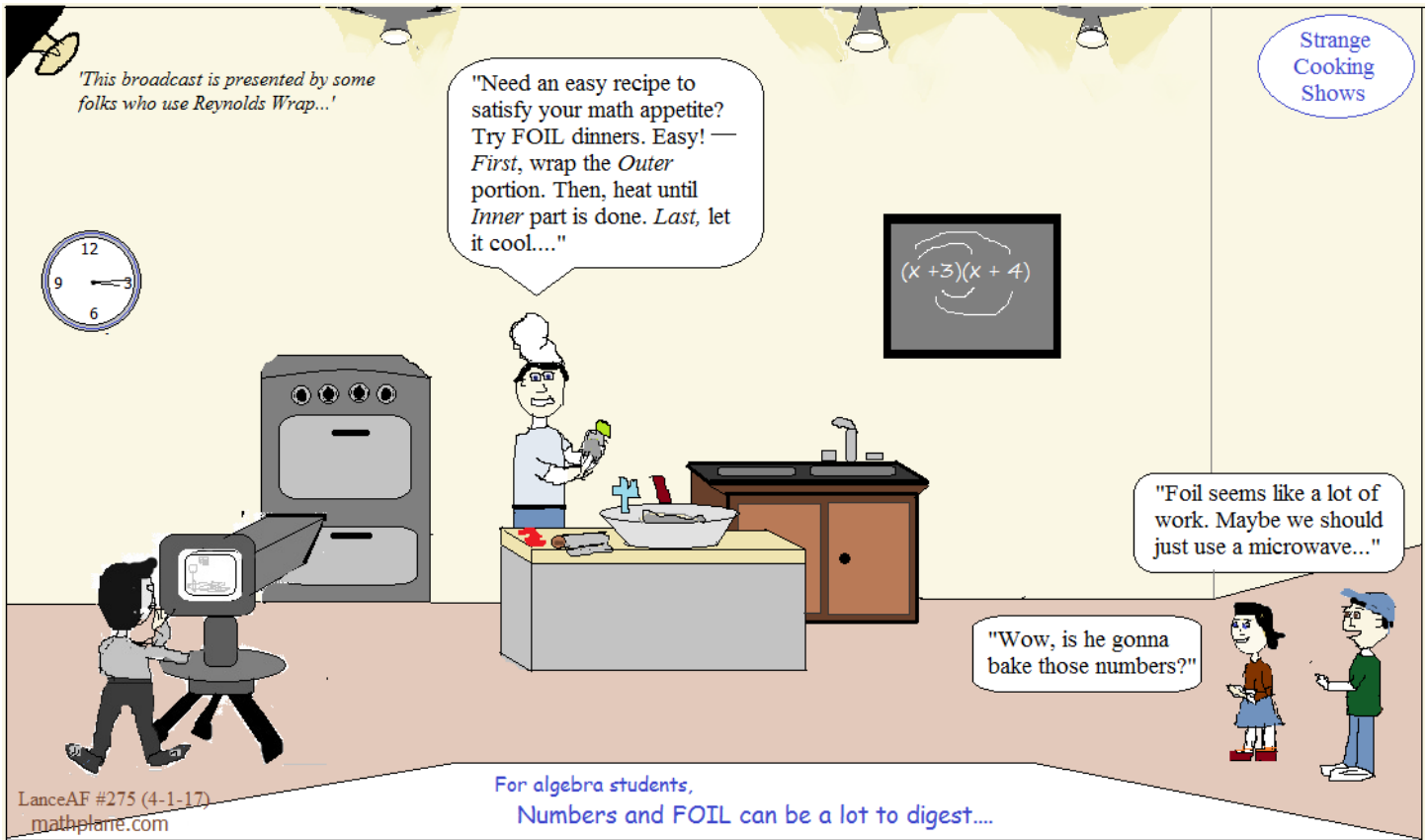
V. More Questions

a) Classify the following $3x^2y^A + 42x$

b) If $f(x)$ is a polynomial, describe the domains of A and B $f(x) = 5x^{A-7} - 2x^{(B/2)}$

c) What is the area of each shaded region? (Write an expression in simplified form)





ANSWERS-→

I. Addition/Subtraction: Simplify the following

a) $(2x^2 - 4x + 6) + (x^2 + 13x - 5)$

Collect "like terms"

$$3x^2 + 9x + 1$$

b) $(2x + 5 - 7x^2) + (x^2 - 11x + 3)$

$$-6x^2 - 9x + 8$$

c) $(x^2 - 3x + 6) - (2x^2 + 6x - 5)$

distribute -1

$$x^2 - 3x + 6 + (-2x^2 - 6x + 5)$$

$$-x^2 - 9x + 11$$

d) $(3ab^2 + 2a^2b - b^3) + (5a^2 + 6ab^2 + 2b^3)$

$$9ab^2 + 2a^2b + 5a^2 + b^3$$

e) $3(x + y) + 2c(x + y) + d(x + y)$

same term

$$(3 + 2c + d)(x + y)$$

II. Multiplication: Expand the following

a) $(x + 3)(x + 7)$

FOIL (First, Outer, Inner, Last)

$$x^2 + 7x + 3x + 21$$

$$x^2 + 10x + 21$$

b) $(x + 4)^2$

$$(x + 4)(x + 4)$$

$$x^2 + 8x + 16$$

c) $(x - 11)(x + 4)$

$$x^2 + 4x - 11x - 44$$

$$x^2 - 7x - 44$$

d) $(x + 2y)(3x - y)$

$$3x^2 - xy + 6xy - 2y^2$$

$$3x^2 + 5xy - 2y^2$$

e) $-(x - 3)(x + 8)$

$$-1 \cdot (x^2 + 5x - 24)$$

$$-x^2 - 5x + 24$$

f) $(x + 5)(x^2 + 7x - 1)$

$$x^3 + 7x^2 - x$$

$$+ \frac{5x^2 + 35x - 5}{x^3 + 12x^2 + 34x - 5}$$

$$x^3 + 12x^2 + 34x - 5$$

g) $(x + 9)(x - 9)$

$$x^2 - 9x + 9x - 81$$

$$x^2 - 81$$

('difference of squares')

h) $(x + 1)(x + 2)(x + 3)$

$$(x^2 + 3x + 2) \cdot (x + 3)$$

$$x^3 + 3x^2 + 2x$$

$$+ \frac{3x^2 + 9x + 6}{x^3 + 6x^2 + 11x + 6}$$

$$x^3 + 6x^2 + 11x + 6$$

i) $(x^2 - 2)(x^3 + 6x + 14)$

$$x^5 + 6x^3 + 14x^2$$

$$+ \frac{-2x^3 - 12x - 28}{x^5 + 4x^3 + 14x^2 - 12x - 28}$$

$$x^5 + 4x^3 + 14x^2 - 12x - 28$$

III. Classifying Polynomials: Determine which are polynomials; then, identify the type, degree, and lead coefficients

Solutions

a) $3x + 3$ binomial (2 terms); degree 1 (linear); lead coefficient: 3

b) $5 + 4x - 5x^3$ trinomial (3 terms); degree 3 (cubic); lead coefficient: -5
 $\rightarrow -5x^3 + 4x + 5$

c) $4^Y + 1$ NOT a polynomial -- Exponent cannot be a variable

d) $z^3 + z^2 + z + \frac{1}{z}$ NOT a polynomial -- All exponents must be whole numbers
 $\rightarrow z^{-1}$

e) $4c^3d + 3cd^2 + d^3$ trinomial (3 terms); degree 4; lead coefficient: 4

f) $9x^3y^3$ monomial (1 term); degree 6 ; lead coefficient: 9

IV. Simplify and Classify: Solve and identify each polynomial (type and degree)

a) $(2x + 3)^2 + (3x^2 + 6)$ $4x^2 + 6x + 6x + 9 + 3x^2 + 6$
 $7x^2 + 12x + 15$

Quadratic Trinomial

b) $(2a^2 + b)(2b + a)$ $4a^2b + 2a^3 + 2b^2 + ba$

Four-term polynomial of degree 3

c) $-3(6 - 2s^2 + 5s^3)$ $-18 + 6s^2 - 15s^3$

$-15s^3 + 6s^2 - 18$

Cubic Trinomial

Solutions

a) Classify the following $3x^2y^A + 42x$

If A is a whole number, then this is a binomial with degree $A + 2$

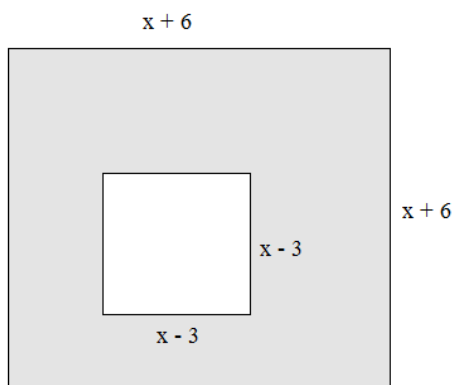
If A is not a whole number, then this is not a polynomial!

b) If $f(x)$ is a polynomial, describe the domains of A and B $f(x) = 5x^{A-7} - 2x^{(B/2)}$

$A \geq 7$ where A is a whole number

$B = 0$ or any positive even number

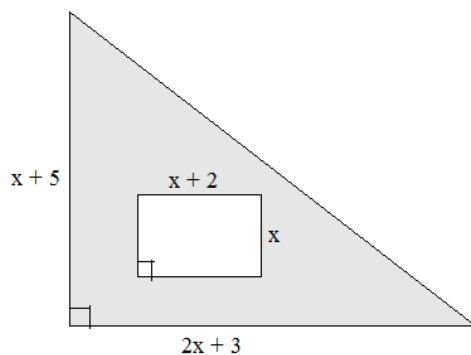
c) What is the area of each shaded region? (Write an expression in simplified form)



$$(x + 6)(x + 6) - (x - 3)(x - 3)$$

$$x^2 + 12x + 36 - (x^2 - 6x + 9)$$

$$18x + 27$$



$$\frac{1}{2}(x + 5)(2x + 3) - (x + 2)(x)$$

$$\frac{1}{2}(2x^2 + 3x + 10x + 15) - (x^2 + 2x)$$

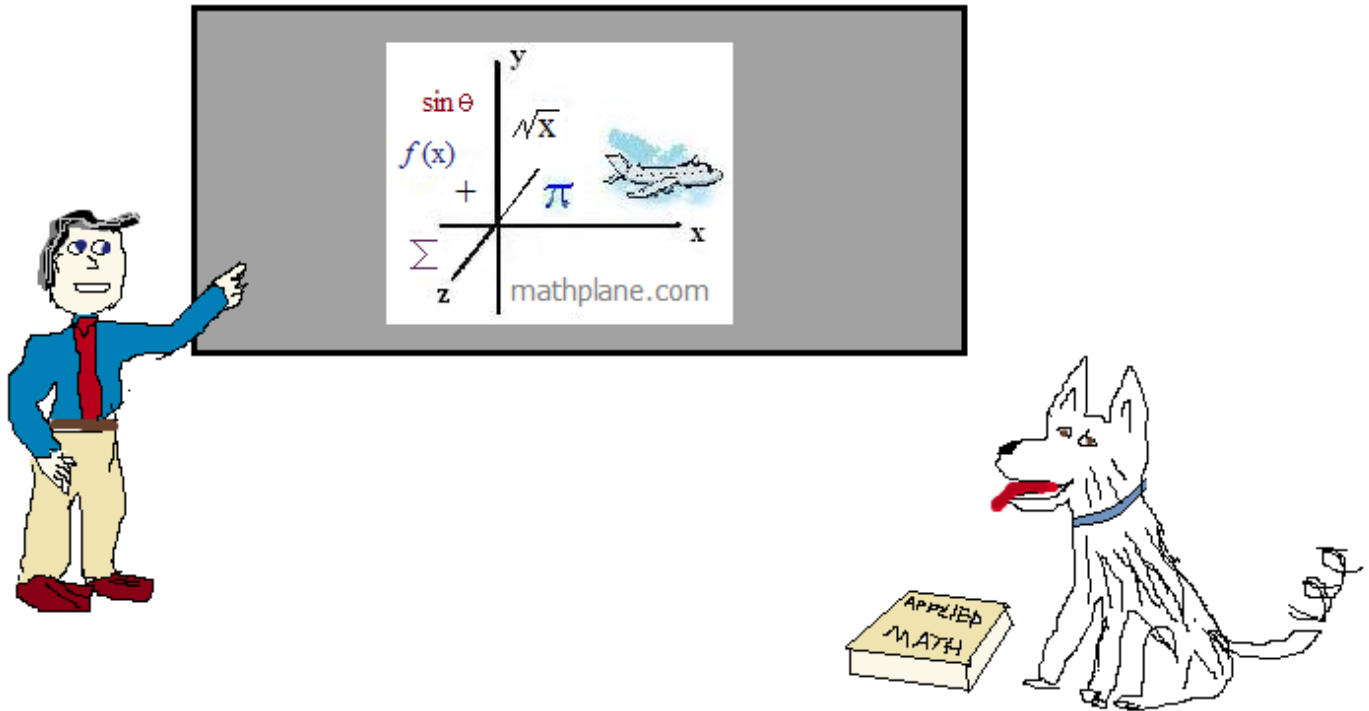
$$x^2 + \frac{13}{2}x + \frac{15}{2} - x^2 - 2x$$

$$\frac{9}{2}x + \frac{15}{2}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Good luck!



Also, at Facebook, Google+, TeachersPayTeachers, TES, and Pinterest

And, Mathplane *Express* for mobile at Mathplane.ORG

Challenge Question:

$$\text{If } (x + y)^2 = 80 \text{ and } -xy = 2,$$

what is $x^2 + y^2$?

The answer is.....

$$\text{If } (x + y)^2 = 80 \text{ and } -xy = 2$$

then what is $x^2 + y^2$?

SOLUTION:

$$x^2 + 2xy + y^2 = 80$$

$$xy = -2$$

$$x^2 + 2(-2) + y^2 = 80$$

$$x^2 + y^2 = 84$$