Imaginary and Complex Numbers

Notes, Examples, and Practice Quiz (with Solutions)

Topics include i, conjugates, order of operations, quadratic formula, and more.

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Notes on Imaginary and Complex Numbers

Part I: Introduction

Real number--- A value that represents a quantity..

The set of real numbers contains rational and irrational numbers.

A rational number can be written as a fraction.

25 $\quad$ .4056 $\rightarrow \frac{4056}{10000}$ $\quad$ $\frac{1}{3} \rightarrow .3333\overline{3}$ $\quad$ -46

Irrational numbers include π or $\sqrt{2}$

Imaginary number--- The square root of a negative real number

$i = \sqrt{-1}$ $\qquad$ $5i = \sqrt{-25}$

Complex Number---A number that consists of a real part and an imaginary part

"standard form" $\rightarrow$ **a + bi** $\qquad$ **7 + 9i**

Part II: Implications

a) $i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

$i^5 = 1 \cdot i = i$

$\vdots$

$i^{12} = 1$

Then, $i^{48} = 1$

$i^{50} = i^{48} \cdot i^2 = 1 \cdot -1 = -1$

$i^{999} = i^{996} \cdot i^3 = 1 \cdot -i = -i$

Notes on Imaginary and Complex Numbers (continued)

b) $i = \sqrt{-1}$

$\sqrt{-16} = \sqrt{-1 \cdot 16}$
$= 4\sqrt{-1}$
$= 4i$

$\sqrt{-3x} = \sqrt{-1 \cdot 3 \cdot x}$
$= (\sqrt{3x})\, i$

c) "i behaves like most variables"

$3i + 6i = 9i$

$3i^2 \cdot 4i = 12i^3 = -12i$ (reminder: $i^3 = -i$)

$12i - 14i = -2i$

$\frac{8i^3}{6i^2} = \frac{4i}{3}$

d) Multiplying complex numbers (and "using conjugates")

$3i \cdot 7i = 21i^2 = -21$

$(4i + 3) \cdot (5i + 6) = 20i^2 + 15i + 24i + 18 = -20 + 39i + 18$
$= -2 + 39i$

$(3i + 2) \cdot (3i - 2) = 9i^2 + 6i - 6i - 4 = -9 - 4 = -13$
('conjugates')

$(20i + 12)(20i - 12) = -400 - 144 = -544$

notice 'difference of squares' where $(a + b)(a - b) = a^2 - b^2$

$20i^2 - 12^2$

Notes on Imaginary and Complex Numbers (continued)

e) $x^2 + 4 = 0$ To find x: $x^2 = -4$

$x = \sqrt{-4}$

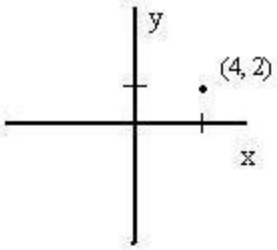
$x = \pm 2i$

$x^2 + x + 6 = 0$ To find x: (quadratic formula) $\dfrac{-1 \pm \sqrt{1 - 24}}{2}$

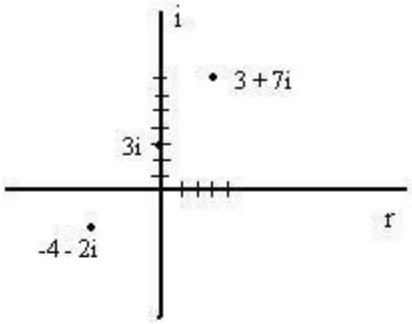
$\dfrac{-1 \pm \sqrt{-23}}{2} = \dfrac{-1 \pm i\sqrt{23}}{2}$

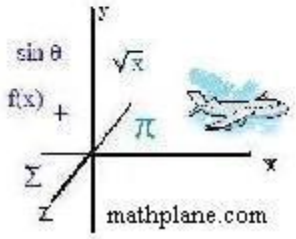
Part III: Graphing

real numbers



complex numbers





A few more examples:

$-(i)^2$ $\quad -1 \cdot (i)(i) = -1 \cdot -1 = 1$

VS.

$(-i)^2$ $\quad (-i)(-i) = (-1 \cdot i)(-1 \cdot i)$

$(-1)(-1)(i)(i) = -1$

Other reminders:

$-5^2 = -25$ $\qquad (-5)^2 = 25$

$-1 \cdot (5)^2$ $\qquad (-5)(-5)$

What is $\sqrt{-2} \cdot \sqrt{-3}$?

$\sqrt{6}$ or $-\sqrt{6}$???

$i\sqrt{2} \cdot i\sqrt{3} \rightarrow -\sqrt{6}$

Imaginary numbers

$2\sqrt{-6} \cdot -1\sqrt{-3}$

Incorrect...

$-2\sqrt{18} = -6\sqrt{2}$

Correct...

$2i\sqrt{6} \cdot -1i\sqrt{3}$

$-2i^2\sqrt{18} = 6\sqrt{2}$

What is i^{-7} ?

$i^{-7} \cdot i^8 = i^1$

1

$\frac{1}{i^7} = \frac{1}{-i}$

$\frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{1}$ i

Simplify:

$$i^{17}(2+4i)$$

(Solution on next page)

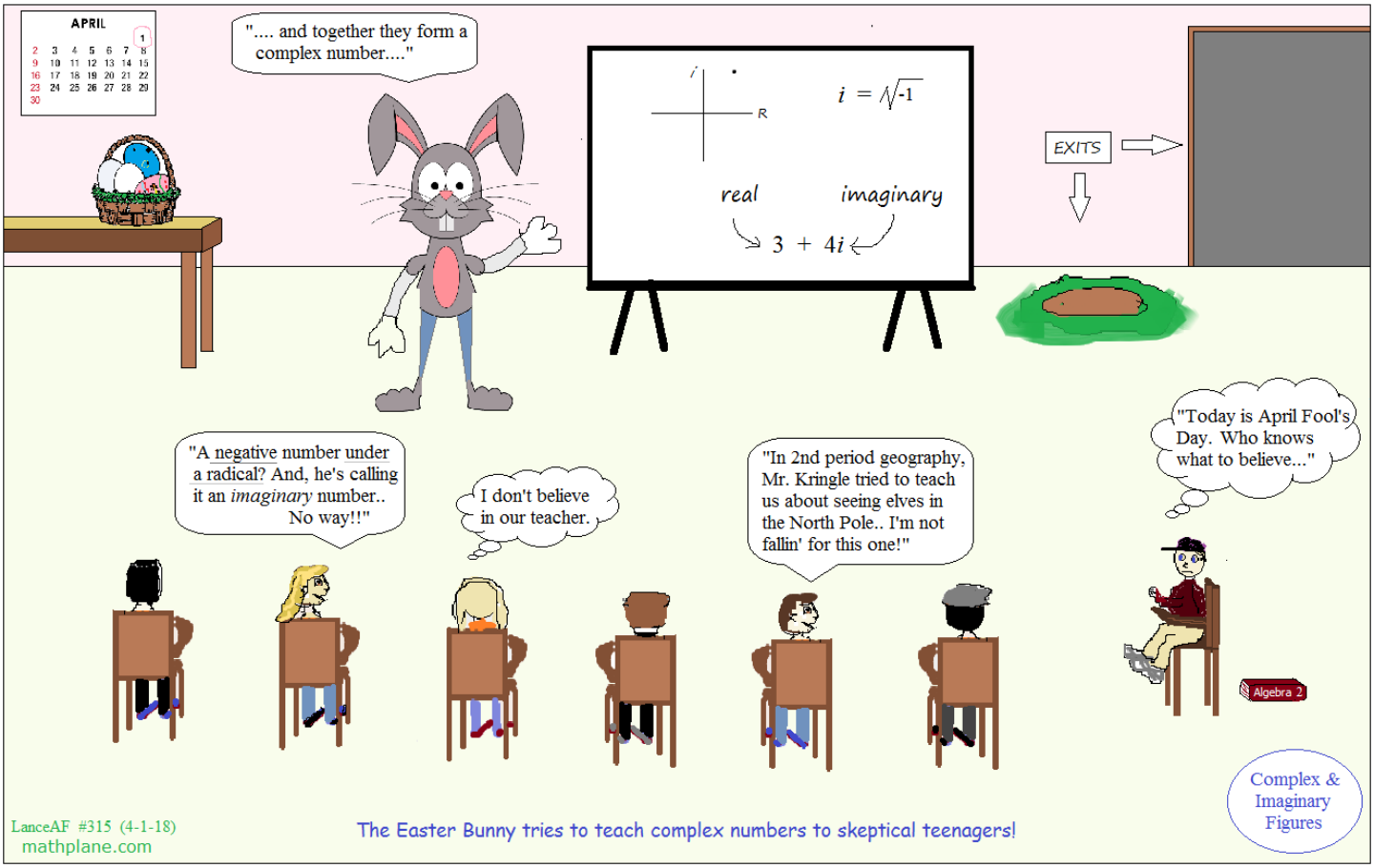
Evaluate: $i^{17}(2+4i)$

$i^{16} \cdot i^{1}\ (2+4i)$

$1 \cdot i \cdot (2+4i)$

$2i + 4i^{2}$

$\boxed{-4+2i}$



Practice Quiz-→

Imaginary & Complex Numbers: Quick Quiz

Part I: Simplify

1) $i^2 =$ 2) $i^{51} =$ 3) $i^8 =$ 4) $i^{-5} =$

Part II: Simplify

1) $\sqrt{-25} =$ 2) $\sqrt{-72} =$ 3) $\sqrt[3]{-8} =$ 4) $\sqrt{-4ab^3} =$

Part III: Complex numbers

Given: $w = 3i + 7$
$v = 2i - 5$

Find: 1) $w + v$ 2) $3w$ 3) vw

Solutions must be in standard form: $a + bi$

4) w^2 5) $\frac{1}{v}$ 6) v^3

Part IV: Solve

1) $x^2 + 3x + 10 = 0$ 2) $3(x + 8)^2 = -15$ 3) $\frac{3i + 4}{4i - 9} =$

4) $(5i - 6)^2 =$ 5) $(7 - 8i)(7 + 8i) =$

Imaginary & Complex numbers: Quick Quiz

SOLUTIONS

Part I: Simplify

1) $i^2 = -1$

2) $i^{51} = i^{48} \cdot i^3$

$= 1 \cdot i^3 = -i$

3) $i^8 = 1$

4) $i^{-5} = i^{-8} \cdot i^3$

$= \frac{1}{i^8} \cdot i^3$

$= \frac{1}{1} \cdot -i = -i$

Part II: Simplify

1) $\sqrt{-25} = 5i$

2) $\sqrt{-72} = \sqrt{(-1)(2)(36)}$

$6i\sqrt{2}$

3) $\sqrt[3]{-8} = -2$

$(-2)(-2)(-2) = -8$

4) $\sqrt{-4ab^3} = 2bi\sqrt{ab}$

Part III: Complex numbers

Given: $w = 3i + 7$
$v = 2i - 5$

Find: 1) $w + v$

$3i + 7$
$2i - 5$
$5i + 2$

Solutions must be in standard form: $a + bi$

2) $3w$ $3(3i + 7)$

$9i + 21$

3) vw

$(2i - 5)(3i + 7)$

$6i^2 - 15i + 14i - 35$

$6(-1) - i - 35 = -41 - i$

4) w^2

$(3i + 7)(3i + 7)$

$9i^2 + 21i + 21i + 49$

$40 + 42i$

5) $\frac{1}{v}$

$\frac{1}{(2i - 5)} \cdot \frac{(2i + 5)}{(2i + 5)} =$

$\frac{2i + 5}{4i^2 - 25} = \frac{5 + 2i}{-29} =$

$\frac{-5}{29} - \frac{2}{29}i$

6) $v^3 = (2i - 5)(2i - 5)(2i - 5)$

$(2i - 5)(2i - 5) = -4 - 20i + 25$

$= 21 - 20i$

then, $(2i - 5)(-20i + 21)$

$-40i^2 + 100i + 42i - 105$

$= 40 + 142i - 105 = -65 + 142i$

Part IV: Solve

1) $x^2 + 3x + 10 = 0$

(use quadratic formula)

$\frac{-3 \pm \sqrt{9 - 4(1)(10)}}{2(1)} =$

$\frac{-3 \pm i\sqrt{31}}{2}$

2) $3(x + 8)^2 = -15$

$(x + 8)^2 = -5$

$(x + 8) = \pm\sqrt{-5}$

$x = -8 \pm i\sqrt{5}$

3) $\frac{3i + 4}{4i - 9} =$

$\frac{3i + 4}{4i - 9} \cdot \frac{4i + 9}{4i + 9} =$

$\frac{12i^2 + 16i + 27i + 36}{16i^2 - 81} =$

$\frac{24 + 43i}{-97} = \frac{-24}{97} - \frac{43i}{97}$

4) $(5i - 6)^2 =$

$(5i - 6)(5i - 6) =$

$25i^2 - 30i - 30i + 36 =$

$-25 - 60i + 36 =$

$11 - 60i$

5) $(7 - 8i)(7 + 8i) =$

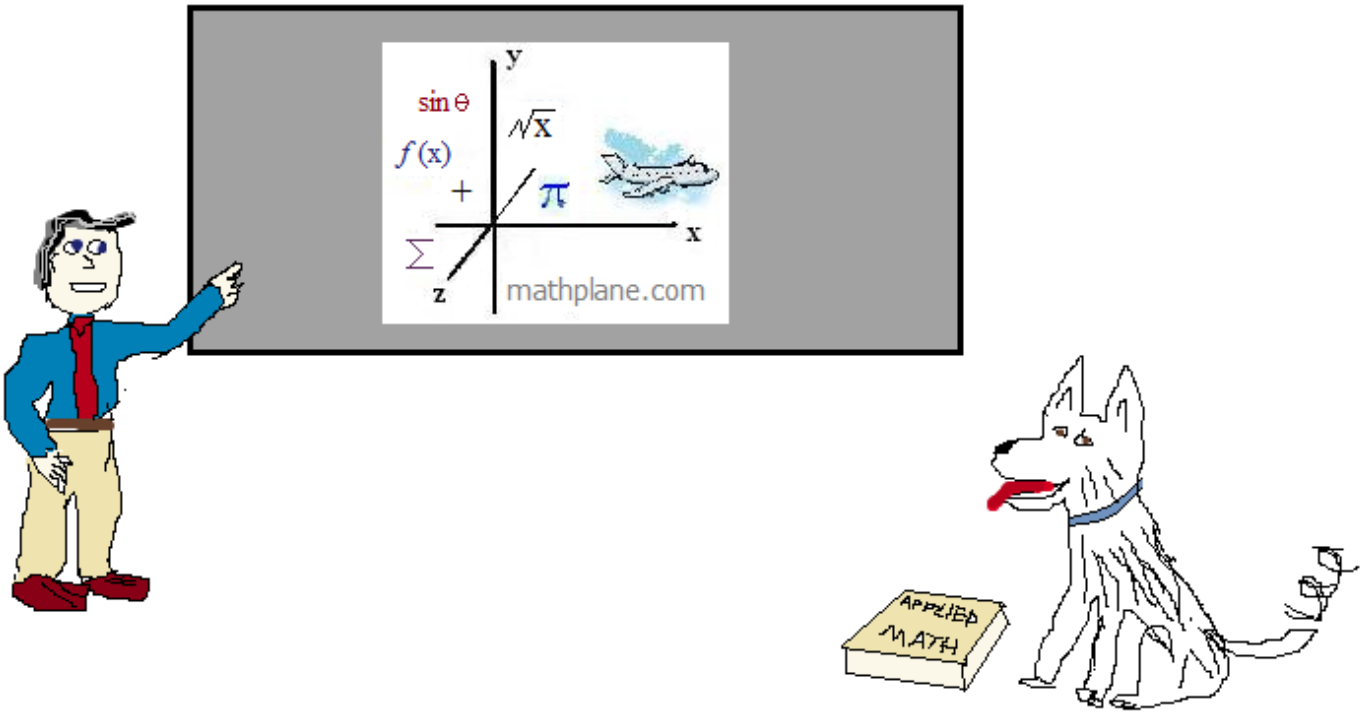
$49 - 56i + 56i - 64i^2 =$

$49 + 64 = 113$

Thanks for downloading the packet. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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