## Word Problems: Algebra 1 and 2

 Notes, Examples, and Practice Exercises (with detailed solutions)After a long day, a teacher walks into a bar and orders a 10 ounce vodka and cranberry juice. The bartender prepare the drink. After one sip, the teacher says, "It's too strong!"
The bartender explains, "I used 2 parts vodka and 3 parts cranberry." "Can I get one with just $15 \%$ vodka?"
"Of course," the bartender answered. "A new drink? Or, can I pour out some of this one and add cranberry juice?"
The teacher said, "I hate to see a drink go to waste. Just pour out some of this one and add cranberry juice."

Question: How much of the drink should the bartender pour out and replace with pure cranberry juice to produce a $15 \%$ vodka mixture?

|  | total amount | \% | vodka quatity |
| :---: | :---: | :---: | :---: |
| Original drink | 10 | 40\% | 4 |
| pour out | (10-x) | 40\% | . 4 (10-x) |
| add | x | 0\% | 0 x |
| final drink | 10 | 15\% | 1.5 |
| $.4(10-\mathrm{x})+0 \mathrm{x}=1.5$ |  |  |  |
| $4-.4 \mathrm{x}+0=1.5$ |  |  |  |
|  | . $4 \mathrm{x}=2.5$ |  |  |
|  | $\mathrm{x}=6.25$ ounces |  |  |

The bartender should pour out 6.25 ounces of the original drink, and then top it off with cranberry juice....

Topics include translating words to operations, linear systems, mixture, work, and rate problems, area, and more.

1) Write each expression and solve. ("find the number")
a) The sum of 14 and a number is 41 .
b) 8 plus the product of 21 and a number is 71
c) The difference between 104 and a number is 79
d) five minus the product of two and a number is 12
e) Twelve is one-eighth of a number
2) Answer carefully.
a) A number is eight more than another number.

If the sum of the 2 numbers is 70 , what is the smaller number?
b) A number is 10 less than another number.

If their sum is 26 , what is the product of the two numbers?
3) Draw and label each figure. Then, solve.
a) A triangle's height is half the length of its base. If the base is 8 , what is the area?
b) The length of a rectangle is twice the width.

If the width is 6 , what is the perimeter?
c) The length of a rectangle is four times its width. If the area is 100 square feet. What is the width?

## Basic Algebra Word Problems (continued)

4) 50 cars and one locomotive weigh 4825 tons. (Each car is identical.) If the locomotive weighs 225 tons, how much does each car weigh?
5) A car's tank holds 16 gallons of gas. At 1 gallon, you stop at the gas station to refuel. If the car uses 3 gallons per hour, when will you refuel?
6) A farmer has $\$ 755$. One cow costs $\$ 500$ and a flock of chicks costs $\$ 20$. If the farmer buys one cow, how many flocks of chicks can he afford to buy?
7) If 9 friends take a trip, it will cost $\$ 810$. However, during the off-season, there is a discount. If it costs the 9 friends $\$ 729$, how much was the discount per person?
(Bonus: What is the percentage discount?)
8) The sum of 3 consecutive integers is 234 . What is the middle integer?
9) The cable company charges $\$ 59.95$ per month plus $\$ 4$ for each movie rental. If your cable bill is $\$ 111.95$, how many movies did you rent?
10) Write each expression and solve. ("find the number")
a) The sum of 14 and a number is 41 .

$$
\begin{aligned}
& 14+x=41 \\
& -14
\end{aligned}
$$

b) 8 plus the product of 21 and a number is 71

$$
\left.\begin{array}{cc}
8+(21 \cdot n)= & 71 \\
-8
\end{array} \quad \frac{(21}{21} \cdot n\right)=\frac{63}{21} \quad n=3
$$

c) The difference between 104 and a number is 79

$$
\begin{array}{cccc}
104-y=79 & -y=-25 & y=25 \\
-104 & -104 & (-1) \quad(-1) &
\end{array}
$$

(also, if "the number" is larger than 104, then $z-104=79$. and $\mathrm{z}=183$ )
d) five minus the product of two and a number is 12

$$
\begin{array}{llll}
5-(2 \cdot d)= & 12 \\
-5
\end{array} \quad \frac{-2 d}{-2}=\frac{7}{-2} \quad \mathrm{~d}=-7 / 2 \text { or }-3.5
$$

e) Twelve is one-eighth of a number

$$
12=\frac{1}{8} \mathrm{r} \quad \mathrm{r}=96
$$

2) Answer carefully.
a) A number is eight more than another number.

If the sum of the 2 numbers is 70 , what is the smaller number?
let $\mathrm{n}=$ the number
then, $\mathrm{n}-8=$ another number

$$
\begin{aligned}
\begin{array}{r}
\mathrm{n}+(\mathrm{n}-8) \\
+8 \\
+8
\end{array} \quad+8
\end{aligned} \quad \frac{2 \mathrm{n}}{2}=\frac{78}{2} \quad \begin{aligned}
& \mathrm{n}=39 \\
& \text { and } \\
& \mathrm{n}-8=31
\end{aligned}
$$

b) A number is 10 less than another number.

If their sum is 26 , what is the product of the two numbers?

$$
\begin{array}{lrl}
\text { let } x=\text { a number } \\
\text { then, } x+10=\text { another number }
\end{array} \quad \begin{array}{r}
x+(x+10)=26 \\
-10 \quad-10
\end{array} \quad \frac{2 x}{2}=\frac{16}{2} \quad \begin{aligned}
& x=8 \\
& x+10=18
\end{aligned} \quad \begin{aligned}
& \text { the product of } 8 \text { and } 18 \text { is } 144 \\
& \hline
\end{aligned}
$$

3) Draw and label each figure. Then, solve.
a) A triangle's height is half the length of its base. If the base is 8 , what is the area?


$$
\text { area of triangle }=1 / 2(b h)
$$

$$
\begin{aligned}
& b=8 \\
& h=1 / 2 \cdot b=4
\end{aligned}
$$

$$
\text { area }=1 / 2(8)(4)=16
$$

sq. units
b) The length of a rectangle is twice the width.


$$
\begin{aligned}
& \text { perimeter }=2 l+2 w \\
& \quad=2(12)+2(6)=36
\end{aligned}
$$

c) The length of a rectangle is four times its width. If the area is 100 square feet. What is the width?

$$
l=4 w
$$



$$
\begin{aligned}
& \text { area }=l w \\
& 100 \text { sq. feet }=4 w \cdot w \\
& \frac{100 \text { sq. } \mathrm{ft}=\frac{4 w^{2}}{4} \quad w=-5 \text { or } 5 \text { feet }}{} \\
& \sqrt{25 \mathrm{sq} \cdot \mathrm{ft}}=\sqrt{w^{2}} \quad \begin{array}{c}
\text { (length must } \\
\text { be positive) }
\end{array}
\end{aligned}
$$

## Basic Algebra Word Problems (continued)

## SOLUTIONS

4) 50 cars and one locomotive weigh 4825 tons. (Each car is identical.) If the locomotive weighs 225 tons, how much does each car weigh?

| Total weight $=$ loco + cars | 4825 tons $=225$ tons +50 (weight of each car) | $\frac{4600 \text { tons }=\frac{50(\text { weight of car) }}{50}}{\text { let } \mathrm{c}=\text { number of cars }}$Weight $=225$ tons $+\mathrm{c}(\mathrm{wt})$ -225 tons -225 tons |
| :---: | :---: | :---: |

5) A car's tank holds 16 gallons of gas. At 1 gallon, you stop at the gas station to refuel. If the car uses 3 gallons per hour, when will you refuel?

$$
\begin{array}{ccc|c|}
\text { Fill-up }=\text { total gas - gas use } & 1 \mathrm{~g}=16 \mathrm{~g}-\mathrm{t}(3 \mathrm{~g}) & \frac{-15 \mathrm{~g}}{-3 \mathrm{~g}}=-3 \mathrm{tg} \\
\text { let } \mathrm{t}=\text { time driving } & -16 \mathrm{~g}-16 \mathrm{~g} & \mathrm{t}=5 \text { hours } \\
\hline
\end{array}
$$

$$
1 \text { gallon }=16 \text { gallon }-\mathrm{t} \text { hours( } 3 \text { gallons/hour })
$$

6) A farmer has $\$ 755$. One cow costs $\$ 500$ and a flock of chicks costs $\$ 20$.

If the farmer buys one cow, how many flocks of chicks can he afford to buy?
let $\mathrm{c}=\#$ of cows
$\mathrm{f}=\#$ of flocks of chicks
Farmer's money $\geq \$ 500 c+\$ 20 f$
(Farmer's money must match or exceed the cost of the cow and chicks)

$$
\begin{aligned}
& \mathrm{c}=1 \text { cow } \\
& \text { Farmer's money }=\$ 755
\end{aligned}
$$

$$
\begin{aligned}
& \text { Farmer's money }=\$ 755 \quad \frac{\$ 255}{\$ 20} \geq \$ 20 \mathrm{f} \\
& \$ 755 \geq \$ 500(1)+\$ 20 \mathrm{f}
\end{aligned}
$$

$$
-\$ 500 \quad-\$ 500
$$

$12.75 \geq \mathrm{f}$
since flocks can't be divided, the farmer can afford 12 flocks of chicks
7) If 9 friends take a trip, it will cost $\$ 810$. However, during the off-season, there is a discount. If it costs the 9 friends $\$ 729$, how much was the discount per person?
(Bonus: What is the percentage discount?)

$$
\text { discount/person }=\frac{\text { total discount }}{\text { total persons }}=\frac{\$ 810-\$ 729}{9 \text { people }}=\frac{\$ 81}{9 \text { people }} \quad \text { discount: } \$ 9 / \text { person }
$$

$\$ 810$ cost for 9 people... then, cost per person: $\$ 90 \ldots$ If the discount is $\$ 9$, then the percentage discount is $10 \%$
8) The sum of 3 consecutive integers is 234 . What is the middle integer?

$$
\mathrm{n}=77
$$

$$
\begin{array}{cc}
\text { let } \mathrm{n}=\text { first integer } & \mathrm{n}+(\mathrm{n}+1)+(\mathrm{n}+2)=234 \\
\mathrm{n}+1=\text { second (consecutive) integer } & 3 \mathrm{n}+3=234 \\
\mathrm{n}+2=\text { third (consecutive) integer } & -3 \\
& -3 \\
& \frac{3 \mathrm{n}}{3}=\frac{231}{3}
\end{array}
$$

so, the three integers are
77, 78, 79

$$
78 \text { is the middle integer }
$$

9) The cable company charges $\$ 59.95$ per month plus $\$ 4$ for each movie rental. If your cable bill is $\$ 111.95$, how many movies did you rent?

$$
\begin{array}{rl}
\text { cable bill }=\text { monthly fixed cost }+\$ 4 \mathrm{M} & \frac{\$ 52}{\$ 4}=\frac{\$ 4 \mathrm{M}}{\$ 4} \\
\text { where } \mathrm{M}=\text { \# of movies rented } & 13=\mathrm{M} \\
\$ 111.95=\$ 59.95+\$ 4 \mathrm{M} & 13 \text { movies }
\end{array}
$$



More examples and practice- $\rightarrow$

## Solving Word Problems

Basic Strategy:

1) "Let Statements" - Establish Variables
2) Draw a Picture
3) Write Relevant Formulas
4) Solve (and check solutions)
5) Answer the question

Example 1: The product of two consecutive whole numbers is 72.
What are the numbers?

1) Let $X=$ first whole number

Let $\mathrm{X}+1=$ second whole number
3) $X \cdot(X+1)=72$
4) $x^{2}+x=72$
$\mathrm{X}^{2}+\mathrm{x}-72=0$
$(\mathrm{X}+9)(\mathrm{X}-8)=0$
$\mathrm{X}=-9,8 \quad$ Then, $\mathrm{X}+1=-8,9$
5) -9 and -8 are not whole numbers 8 and 9 are consecutive, and $8 \times 9=72$

Example 2: The length of a rectangular backyard is 3 times its width. If the area is 588 square feet, what is the length of the backyard?

1) Let $L=$ length

Let $\mathrm{W}=$ width
2)

3) $\mathrm{LW}=$ Area of rectangle (area formula) $\mathrm{L}=3 \mathrm{~W}$
$\mathrm{L}=3 \mathrm{~W}$
Area $=588$ sq. feet (given)
(given)
4) $\mathrm{LW}=588$

$$
3 \mathrm{~W}(\mathrm{~W})=588
$$

$$
3 w^{2}=588
$$

$$
W^{2}=196
$$

$$
\mathrm{W}=\mathbf{- 1 4}, 14 \quad \text { (since length/width/area cannot be }
$$

negative, we eliminated -14)
5) Width is $\mathbf{1 4}$ feet

Length is $3 \times 14=42$ feet $\quad(14 \times 42=588)$

Solving Algebra Word Problems (continued)

Basic Strategy: 1) "Let Statements" - Establish Variables
2) Draw a Picture
3) Write Relevant Formulas
4) Solve (and check solutions)
5) Answer the question

Example 3: A school play produced $\$ 28,860$ in revenue. Student tickets cost \$30; Adult tickets cost \$45. 736 total tickets were sold. How many adults attended?

1) Let $\mathrm{S}=$ \# of student tickets

Let $A=\#$ of adult tickets
3) Revenue $=$ ticket $x$ price
$28,860=S(30)+A(45)$
Tickets sold $=$ Adult tickets + Student tickets
$736=A+S$
4) (use substitution)
$28,860=30 \mathrm{~S}+45 \mathrm{~A}$
$\mathrm{S}=736-\mathrm{A}$
$28,860=30(736-\mathrm{A})+45 \mathrm{~A} \quad 452 \times \$ 45=\$ 20,340$
$28,860=22,080-30 \mathrm{~A}+45 \mathrm{~A} \quad 284 \times \$ 30=\$ 8,520$
$6,780=15 \mathrm{~A} \quad$ total tickets: 736

Questions for you to try!

1) Five times a number decreased by six is equal to the number squared.

What is the number?
2) As I was flying over the farm, I noticed there were only rabbits and chickens. I could only spot 18 heads and 58 feet. How many rabbits were there? How many chickens? (assumption: rabbits have 4 feet and chickens have 2 feet)
3) Mr. Planter had a square garden in his backyard. He increased each side by 2 feet. After he enlarged the garden, it had an area of 196 square feet. What was the measure of the sides before he enlarged the garden? How much did the change in length increase the area?

## Solving Algebra Word Problems

 (continued)1) Five times a number decreased by six is equal to the number squared.

What is the number?

| Let $\mathrm{X}=$ the number | "5 times a number decreased by six" | $5 \mathrm{X}-6$ |
| :--- | :--- | :--- |
| $5 \mathrm{X}-6=\mathrm{X}^{2}$ | "number squared" |  |
| $\mathrm{X}^{2}$ |  |  | | $\mathrm{X}^{2}-5 \mathrm{X}+6=0$ | $5(2)-6=4=2^{2}$ |
| :--- | :--- |
| $(\mathrm{X}-2)(\mathrm{X}-3)=0$ | $5(3)-6=9=3^{2}$ | | $\mathrm{X}=2$ or 3 |  |
| :--- | :--- |

2) As I was flying over the farm, I noticed there were only rabbits and chickens. I could only spot 18 heads and 58 feet. How many rabbits were there? How many chickens? (assumption: rabbits have 4 feet and chickens have 2 feet)

$$
\begin{aligned}
\text { Let } \mathrm{R} & =\# \text { of rabbits } \\
\mathrm{C} & =\# \text { of chickens }
\end{aligned} \quad \begin{array}{ll}
4 \mathrm{R} & =\# \text { of rabbit feet } \\
\mathrm{R}+\mathrm{C}= & =18 \text { (heads) } \\
4 \mathrm{R}+2 \mathrm{C}=58 \text { (feet) } &
\end{array}
$$

(use substitution) $\mathrm{R}=18-\mathrm{C}$

$$
\begin{gathered}
4(18-C)+2 \mathrm{C}=58 \\
72-4 \mathrm{C}+2 \mathrm{C}=58 \\
14=2 \mathrm{C} \\
\mathrm{C}=7
\end{gathered}
$$

Since $\mathrm{R}+\mathrm{C}=18$, $\mathrm{R}=11$
7 Chickens $=7$ heads, 14 feet
11 Rabbits $=11$ heads, 44 feet
18 heads; 58 feet
3) Mr. Planter had a square garden in his backyard. He increased each side by 2 feet. After he enlarged the garden, it had an area of 196 square feet. What was the measure of the sides before he enlarged the garden? How much did the change in length increase the area?

Let $S=$ side of original garden

$$
S+2=\text { side of enlarged garden }
$$

Area of square $=S \times S=s^{2}$


Area of enlarged square $=196=(\mathrm{S}+2)^{2}$

$$
\begin{aligned}
& \sqrt{(\mathrm{S}+2)^{2}}=\sqrt{196} \\
& \mathrm{~S}+2=14 \quad \begin{array}{l}
\text { (S cannot be }-14, \text { because length of } \\
\mathrm{S}=12
\end{array} \quad \text { sides cannot be negative) }
\end{aligned}
$$



Original side is 12 feet Enlarged side is 14 feet

Original area: $144 \mathrm{sq} . \mathrm{ft}$
Enlarged area: 196 sq. ft

## Word Problems: Practice Quiz

1) A video store charges $\$ 3$ per rental. The store offers the following "frequent movie plan":
$\$ 96$ annual fee and movies are $\$ 1$ per rental.
a) Should a person use the "frequent movie plan" if he rents 2 movies per month?
b) What is the "break even point"? (i.e. the cost of rentals w/o plan $=$ cost of rentals with plan)
c) The video store is offering the movie plan for $\$ 50$. What is the new "break even point"?
2) An investor lost $10 \%$ last year. What percentage increase must he have this year to end up with his original investment?
3) You have 14 feet of wood.
a) How many different types of rectangular enclosures could you make? (Each side must be a whole number)
b) Which will create the largest area?
**Bonus Questions
c) How many different types of isosceles triangles could you make? (each side must be a whole number)
d) Which will create the largest area?
4) Assume you run a 5 K race in 20 minutes.
a) Express your speed in miles per hour.
b) Determine the average rate of each mile.
5) The length of a rectangle is twice the width. If the length is decreased by 1 , and the width is increased by 3 , the area is 72 square meters. Find the original length and width.

## Word Problems: Practice Quiz <br> SOLUTIONS

1) A video store charges $\$ 3$ per rental. The store offers the following "frequent movie plan":
$\$ 96$ annual fee and movies are $\$ 1$ per rental.
a) Should a person use the "frequent movie plan" if he rents 2 movies per month?
b) What is the "break even point"? (i.e. the cost of rentals w/o plan $=$ cost of rentals with plan)
c) The video store is offering the movie plan for $\$ 50$. What is the new "break even point"?

Let $\mathrm{M}=\#$ of movies rented
Cost w/o plan $=3 \mathrm{M}$
Cost with plan $=96+1 \mathrm{M}$
a) $\mathrm{M}=24$ movies per year

Cost w/o plan $=3(24)=\$ 72$
No, he should not use the movie plan.
Cost with plan $=96+1(24)=\$ 120$
b) Break even point: Cost w/o plan $=$ Cost with plan

$$
\begin{aligned}
& 3 \mathrm{M}=96+1 \mathrm{M} \\
& 2 \mathrm{M}=96 \\
& \mathrm{M}=48
\end{aligned}
$$

Cost of 48 movies w/o plan: $3(48)=\$ 144$
Cost of 48 movies with plan: $96+1(48)=\$ 144$
c) Cost w/o plan $=3 \mathrm{M}$

Cost with new plan $=50+1 \mathrm{M}$
Break even point: $\quad 3 \mathrm{M}=50+1 \mathrm{M}$

$$
\begin{aligned}
& 2 \mathrm{M}=50 \\
& \mathrm{M}=25 \text { movies }
\end{aligned}
$$

Cost of 25 movies w/o plan: $\$ 75$ Cost of 25 movies with plan: $\$ 75$
2) An investor lost $10 \%$ last year. What percentage increase must he have this year to end up with his original investment?

Let $\mathrm{X}=$ original investment
Then,
amount after first year is: $\mathrm{X}-(.10 \mathrm{X})=.90 \mathrm{X}$
Now, let $\mathrm{r}=$ rate of return for 2 nd year..
Therefore, we want $\mathrm{X}=.90 \mathrm{X}+\mathrm{r}(.90 \mathrm{X})$

The "new break even point" is 25 movies. If he rents 24 or less, he should go w/o the new plan.. If he rents 26 or more, he would save money using the new plan.

$$
\begin{gathered}
\mathrm{X}=.90 \mathrm{X}(1+\mathrm{r}) \\
\frac{\mathrm{X}}{.90 \mathrm{X}}=1+\mathrm{r} \\
\frac{1}{.90}-1=\mathrm{r} \\
\mathrm{r}=.11 \overline{1}
\end{gathered}
$$

Therefore, the 2nd year must have an $11.11 \%$ return to end up with the original amount...

Word Problems: Practice Quiz

## SOLUTIONS

3) You have 14 feet of wood.
a) How many different types of rectangular enclosures could you make? (Each side must be a whole number)

$2 l+2 w=14$

$$
\begin{aligned}
& \qquad \begin{array}{ll}
1=1 & \mathrm{w}=6 \\
\mathrm{l}=2 & \mathrm{w}=5 \\
\mathrm{l}=3 & \mathrm{w}=4
\end{array} \\
& \text { (three types) }
\end{aligned}
$$

Note: $3 \times 4=4 \times 3$
$2 \times 5=5 \times 2$
$1 \times 6=6 \times 1$
b) Which will create the largest area?

$$
\begin{aligned}
& 1 \times 6=6 \text { square feet } \\
& 2 \times 5=10 \text { square feet } \\
& 3 \times 4=12 \text { square feet }
\end{aligned}
$$

## **Bonus Questions

c) How many different types of isosceles triangles could you make? (each side must be a whole number)

b
$b+2 s=14$
$2 \mathrm{~s}>\mathrm{b}$


1

$$
\begin{array}{lll}
s=1 & b=12 & \text { no } \\
s=2 & b=10 & \text { no } \\
s=3 & b=8 & \text { no } \\
s=4 & b=6 & \text { yes } \\
\hline s=4 & b=4 & \text { yes } \\
s=5 & b=6 & b=2
\end{array} \text { yes }
$$

3 different types (with whole numbers)
d) Which will create the largest area?


$$
\begin{aligned}
\begin{aligned}
\mathrm{b} & =6 \\
\mathrm{~h} & =\sqrt{16-9}=\sqrt{7} \\
\text { Area } & =3 \sqrt{7} \\
& \fallingdotseq 7.93
\end{aligned}
\end{aligned}
$$


$\mathrm{b}=4$
$\mathrm{h}=\sqrt{25-4}=\sqrt{21}$

$$
\begin{gathered}
\text { Area }=2 \sqrt{21} \\
\cong 9.16
\end{gathered}
$$


$\mathrm{b}=2$
$\mathrm{h}=\sqrt{36-1}=\sqrt{35}$
Area $=\sqrt{35}$
$\cong 5.91$

(pythagorean theorem)
4) Assume you run a 5 K race in 20 minutes.
a) Express your speed in miles per hour.
b) Determine the average rate of each mile.

a)

$$
\begin{aligned}
& 5 \mathrm{~K} \text { in } 20 \text { minutes }=15 \mathrm{~K} \text { in } 1 \text { hour } \\
& 1 \text { Kilometer } \cong .62 \text { miles } \\
& 15 \mathrm{~K} \cong 15(.62) \cong 9.3 \text { miles/hour }
\end{aligned}
$$

b)

Assuming 9.3 miles/hour.. then, 9.3 miles/ 60 minutes.
or,
1 mile/ 6.45 minutes $\quad .45$ minutes $\cdot \frac{60 \text { seconds }}{1 \text { minute }}=27$ seconds

$$
\begin{gathered}
\text { average rate: } 6 \text { minute } 27 \text { second } \\
\text { per mile }
\end{gathered}
$$

5) The length of a rectangle is twice the width. If the length is decreased by 1 , and the width is increased by 3 , the area is 72 square meters. Find the original length and width.
rectangle 1

$\mathrm{L}=2 \mathrm{~W}$
rectangle 2

$\mathrm{L}-1$ or $2 \mathrm{~W}-1$

Area $=\mathrm{LW}$
Area of rectangle $2=72$ square meters

$$
(2 \mathrm{~W}-1)(\mathrm{W}+3)=72
$$

$$
2 \mathrm{~W}^{2}-W+6 \mathrm{~W}-3=72
$$

$$
2 W^{2}+5 W-75=0
$$

$$
(2 \mathrm{~W}+15)(\mathrm{W}-5)=0
$$

$$
\mathrm{W}=5 \text { or }-1,5 / 2
$$

Original width: 5 meters Original length: 10 meters

Rectangle 2 width: 8
length: 9

$$
\text { Area }=72
$$

## "Convergence Question"

Example: Joe leaves town A at noon, going 40 miles per hour toward town B .
Bill leaves town $B$ at 2:00 pm, going 50 miles per hour toward town $A$.
If town A and town B are 425 miles apart, when and where will Joe and Bill meet?
At 12:00 Joe leaves... At 2:00, Joe has traveled 80 miles.. ( $\mathrm{d}=\mathrm{rt}$ ) so, when Bill leaves at 2:00, they are 345 miles apart...

Since Joe is going 40 mph and Bill is going 50 mph , they are gaining $90 \mathrm{mph} . .$.

$$
\begin{aligned}
& 345 \text { miles }=90 \frac{\text { miles }}{\text { hour }} \times \text { (time) } \\
& \quad \text { time }=\frac{345}{90}=3 \frac{5}{6} \text { hours } \text { or } 3 \text { hours, } 50 \text { minutes }
\end{aligned}
$$



## "Chase Question"

Example: At 8:00am, Mike starts on a bike ride going 12 mph . Then, at 10:30am, John starts on the same road going 18 mph . What time will John catch up to Mike?

Approach 1: Matching the distances
distance $=$ rate x time
Mike: $\mathrm{d}=12 \mathrm{mph} \mathrm{x}$ (time)
John: $\mathrm{d}=18 \mathrm{mph} \mathrm{x}$ (time -2.5 hours)
(substitution: set $\mathrm{d}=\mathrm{d}$ )

$$
\begin{aligned}
12 t & =18(t-2.5) \\
12 t & =18 t-45 \\
-6 t & =-45 \\
t & =7.5
\end{aligned}
$$

Approach 2: Using related speeds
(Establish Mike's lead)
2.5 hours $\cdot 12 \frac{\text { miles }}{\text { hour }} 30$ miles "head start"

Since John (the 2nd rider) goes 6 mph faster than Mike, the gap will close at $6 \mathrm{mph} .$.

$$
30 \text { miles }=6 \mathrm{mph} \times \text { (time) } \quad \text { time }=5 \text { hours }
$$

John will spend 5 hours catching Mike... And, Mike will ride for 7.5 hours..

## Can you answer this question?

Kelly leaves home at noon going 18 mph . At 4:00, Eric leaves home and rides at 30 mph . What time will Eric catch up to Kelly?


This is a "chase" question...
Kelly leaves 4 hours before Eric...

distance $=$ rate x time

$$
=18 \frac{\text { miles }}{\text { hour }} \times 4 \text { hours }=72 \text { miles } \ldots
$$

| E | K | $6: 00$ |
| :---: | :---: | :---: |
| 60 | 48 miles 108 |  |

When Eric, leaves, he is 72 miles behind Kelly....
Since he rides at 30 mph , Eric gains 12 miles per hour...

$$
\begin{aligned}
\text { distance }=\text { rate } \times \text { time } \quad 72 \text { miles } & =12 \frac{\text { miles }}{\text { hour }} \times \text { time } \\
\text { time } & =6 \text { hours }
\end{aligned}
$$

It takes Eric 6 hours, so he reaches Kelly at 10:00...

36 miles at 7:00
24 miles at 8:00
12 miles at 9:00
0 miles at 10:00

Suppose Charlie and Sam can paint a house together in 6 days.
Separately, it takes Sam 5 days longer than Charlie to paint a house.
How fast can each paint a house when working alone?
Step 1: Establish Variables and Formulas

$$
\begin{array}{cl}
\text { Let } \mathrm{C}=\text { Charlie's Rate } & \text { rate } \cdot \text { time }=\text { house } \\
\mathrm{S}=\text { Sam's Rate } & \text { rate }=\frac{1 \text { house }}{\text { time }}=\frac{1 \text { house }}{\mathrm{x} \text { days }}
\end{array}
$$

Step 2: Construct Equations and Solve

$$
\begin{aligned}
& \underset{(\text { together) }}{\mathrm{C} \text { \& S rate }}=\frac{1 \text { house }}{6 \text { days }} \quad \mathrm{C}=\frac{1 \text { house }}{\mathrm{x} \text { days }} \\
& S=\frac{1 \text { house }}{(x+5) \text { days }} \\
& C(\text { time })+S(\text { time })=(C \& S) \text { time } \\
& \frac{1}{x} t+\frac{1}{(x+5)} t=\frac{1}{6} t \\
& \text { (divide equation by } \mathrm{t} \text { ) } \\
& \frac{1}{x}+\frac{1}{(x+5)}=\frac{1}{6} \\
& \text { (find least common denominator } \\
& \text { on the left side) } \\
& \frac{(x+5)}{x(x+5)}+\frac{x}{x(x+5)}=\frac{1}{6} \\
& \text { (add and consolidate) } \\
& \frac{2 x+5}{x^{2}+5 x}=\frac{1}{6} \\
& \text { (cross multiply and solve for } \mathrm{x} \text { ) } \\
& x^{2}+5 x=6(2 x+5) \\
& \mathrm{x}^{2}-7 \mathrm{x}-30=0 \\
& (x-10)(x+3)=0 \\
& \mathrm{x}=10 \text { or }-3 \longrightarrow \text { Charlie can paint a house in } 10 \text { days.. } \\
& \text { **We eliminate the extraneous solution: -3 } \\
& \text { because Charlie does not paint a house in }-3 \\
& \text { days (unless he is destroying the house, the } \\
& \text { rate can't be negative!) }
\end{aligned}
$$

And, if $x=10$, then $x+5=15 \longrightarrow$ Sam can paint a house in 15 days..
Step 3: Check your answers!
It takes 6 days for them to paint a house together..
Therefore,
6 days $(1$ house $/ 10$ days $)+6$ days $(1$ house $/ 15$ days $)=$

$$
\frac{6 \text { house }}{10}+\frac{6 \text { house }}{15}=1 \text { house! }
$$

## If $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and E are integers, then what are they?

$$
\mathrm{A}^{2}+\mathrm{B}^{2}+\mathrm{C}^{2}=\mathrm{D}^{2}+\mathrm{E}^{2}
$$

## Answer on Next Page- $\rightarrow$



$$
A^{2}+B^{2}+C^{2}=D^{2}+E^{2}
$$

If $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and E are consecutive integers, what are they?

Let $\mathrm{A}=\mathrm{x}$
Then, since they are consecutive integers, $x^{2}+(x+1)^{2}+(x+2)^{2}=(x+3)^{2}+(x+4)^{2}$
A
B
C
D
E

Expand the terms ("FOIL")
$x^{2}+x^{2}+2 x+1+x^{2}+4 x+4=x^{2}+6 x+9+x^{2}+8 x+16$

Collect the 'like' terms
$x^{2}-8 x-20=0$

Factor
$(x-10)(x+2)=0$
$x=-2$ or 10

Therefore, $10,11,12,13,14$
or
$-2,-1,0,1,2$

Example: At 2:00, George leaves Kingstown and drives toward Queensland at 60 miles/hour. At 2:15, Mary leaves Queensland and drives toward Kingstown at 50 miles/hour.

If Kingstown and Queensland are 240 miles apart, what time will they meet?

Step 1: Draw a sketch and describe the sequence


From 2:00 to $2: 15$, George drives 15 miles.

$$
15 \text { minutes } \cdot \frac{1 \text { hour }}{60 \text { minutes }} \cdot \frac{60 \text { miles }}{1 \text { hour }}=15 \text { miles }
$$

Step 2: Solve
At $2: 15$, Mary and George are 225 miles apart.

$$
\begin{array}{cl}
\text { distance }=\text { rate } \cdot \text { time } & 225 \text { miles }=110 \frac{\text { miles }}{\text { hour }} \text { (time) } \quad \begin{array}{l}
\text { since they are going toward each } \\
\text { other, the rates are added. } \\
\text { (i.e. they are approaching each } \\
\text { other at } 110 \text { miles/hour) }
\end{array}
\end{array}
$$

time $=2.045 \overline{45}$ hours
$=2$ hours 2 minutes 44 seconds

Step 3: Answer the question
At 2:00, George leaves and goes 15 miles. Then, at $2: 15$, Mary begins driving... Then, 2 hrs, 2 minutes, 44 seconds later, they meet.

Example: At the Nuthouse, nuts cost $\$ 2.40$ per pound and raisins cost $\$ 3.20$ per pound If you spend $\$ 129.60$ for 50 pounds of a mixture, how many pounds of each did you buy?

Mixture Problem

Step 1: Establish variables and relevant equations

$$
\begin{array}{ll}
\text { Let } \mathrm{R}=\text { \# of raisins (pounds) } & \\
\mathrm{N}=\text { \# of nuts (pounds) } & \mathrm{R}+\mathrm{N}=50 \\
\text { then, } & \begin{array}{l}
3.2 \mathrm{R}=\text { cost per pound of raisins (dollars) } \\
2.4 \mathrm{~N}=\text { cost per pound of nuts (dollars) }
\end{array} \\
3.2 \mathrm{R}+2.4 \mathrm{~N}=129.60
\end{array}
$$

Step 2: Solve

$$
\begin{gathered}
\mathrm{R}+\mathrm{N}=50 \\
3.2 \mathrm{R}+2.4 \mathrm{~N}=129.60 \\
\text { Using combination/elimination method } \\
3.2 \mathrm{R}+2.4 \mathrm{~N}=129.60 \\
2.4 \mathrm{R}+2.4 \mathrm{~N}=120 \\
.8 \mathrm{R}=9.60 \\
\mathrm{R}=12 \\
\text { if } \mathrm{R}=12, \text { then } \mathrm{N}=38
\end{gathered}
$$

Step 3: Answer question and check

The 50 pound mixture consisted of 12 pounds of Raisins and 38 pounds of Nuts

12 pounds of raisins cost $12 \times 3.2=38.40$ 38 pounds of nuts cost $38 \times 2.4=91.20$ total cost: 129.60 V

## Algebra Word Problems

Example: A speeding car goes 80 miles per hour for 1 hour and 12 minutes.
Then, it exits the highway, and drives the last fifteen minutes at 30 miles per hour.
What was the average speed for the entire trip?

```
distance = (rate)(time)
total distance }=\mathrm{ distance at }80\textrm{mph}+\mathrm{ distance at 30 mph
total time = 1.2 hours + . 25 hours
```

$$
\begin{aligned}
& 80 \mathrm{mph} \times 1.2 \text { hours }=96 \text { miles } \\
& 30 \mathrm{mph} \times .25 \text { hours }=7.5 \text { miles } \\
& 103.5 \text { miles }=(\text { total rate })(1.45 \text { hours })
\end{aligned}
$$

approx. 71.38 miles per hour

Example: At the gas station, I paid $\$ 42.51$ to fill my car's gas tank.
The gas station charged $\$ 2.63$ per gallon.
Two days later, the price of gas fell to $\$ 2.52$ per gallon.
If I had waited (to fill up the gas tank), how much could I have saved?
First, let's find out the size of the gas tank.

$$
\begin{aligned}
& \text { total }=(\text { size of tank })(\text { cost per gallon }) \\
& \begin{array}{c}
\$ 42.51=(\text { tank }) \frac{\$ 2.63}{1 \text { gallon }} \\
\text { tank }=16.16 \text { gallons }
\end{array}
\end{aligned}
$$

Second, find the cost of the cheaper fill-up.

$$
\begin{aligned}
& \text { total }=(16.16 \text { gallon }) \frac{\$ 2.52}{1 \text { gallon }} \\
& \text { total }=\$ 40.72
\end{aligned}
$$

Finally, compare the costs...

$$
\$ 42.51-\$ 40.72=\$ 1.79 \text { savings }
$$

Example: Albert invests \$4000@ 3.5\% annual interest.
If he wants to earn $5 \%$ on all his investments,
interest - mixture problem how much more does he need to invest at an $8 \%$ annual interest rate?

There are 2 components: percentage and amount that form the total portions/mixtures.


$$
\begin{aligned}
140+.08 \mathrm{x} & =200+.05 \mathrm{x} \\
.03 \mathrm{x} & =60 \\
\mathrm{x} & =2000
\end{aligned}
$$

Quick check: If he earns $8 \%$ on 2000 dollars, Albert gets 160 dollars in interest. And, he earns 140 dollars from the $3.5 \%$ investment...


Total invested: $\$ 6000$
Total interest: \$300 Investment rate: 5\%

Example: A 50 ounce bottle of detergent contains $5 \%$ chlorine.
If you want to wash your clothes with a $2 \%$ chlorine content, how much non-chlorine solution should you add?

There are 2 components: percentage and amount that form the portions/mixtures

| $95 \%$ |
| :---: |
| 50 oz |$+$| $100 \%$ |
| :---: |
| x | $\mathbf{y}^{98 \%}$| $(50+\mathrm{x}) \mathrm{oz}$ |
| :--- |

bottle and non-chlorine content

| $50(.95)$ | $+$ | $\mathrm{x}(1.00)$ | $=$ | $(50+x)(.98)$ |
| :---: | :---: | :---: | :---: | :---: |
| original bottle |  | dilution amount |  | combined amount at desired ratio |
| 47.5 | + | x | $=$ | $49+.98 \mathrm{x}$ |
|  |  | . 02 x | $=$ | 1.5 |
|  |  |  | $=$ | 75 ounces |

Quick check: "diluted bottle"
amount: $50+75=125$ ounces
chlorine amount: $.02 \times 125=2.5$ ounces...
(and, $5 \%$ of a 50 ounce bottle is $.05 \times 50=2.5$ ounces...)

Example: A homeowner wishes to enclose/fence in 3 plots of land, 80 square feet each. If he has 88 feet of fence, what are the dimesions of each lot?

Area/Perimeter Problem

Step 1: Draw a picture and label parts

Step 2: Develop the equations

$$
\begin{aligned}
& \text { Let } \mathrm{x}=\text { width } \\
& \text { Let } \mathrm{y}=\text { length }
\end{aligned}
$$

There is 88 feet of fence, so
88 feet $=4 x+2 y$
Each plot is 80 sq. feet, so

$$
80 \text { sq. feet }=x \cdot(1 / 3) y
$$

Step 3: Solve
Since we have 2 equations and 2 unknowns, we can solve!

$$
\begin{aligned}
& 88 \text { feet }=4 \mathrm{x}+2 \mathrm{y} \\
& 80 \text { sq. feet }=\mathrm{x} \cdot(1 / 3) \mathrm{y}
\end{aligned} \quad \begin{aligned}
44 \mathrm{ft} & =2 \mathrm{x}+\mathrm{y} \\
\mathrm{y} & =44 \mathrm{ft}-2 \mathrm{x}
\end{aligned}
$$

80 sq. $\mathrm{ft}=\mathrm{x} \cdot(1 / 3)(44 \mathrm{ft}-2 \mathrm{x})$
240 sq. $\mathrm{ft}=\mathrm{x}(44 \mathrm{ft}-2 \mathrm{x})$

$$
\begin{aligned}
& 2 x^{2}-44 x+240=0 \\
& 2(x-10)(x-12)=0 \\
& \quad x=10 \text { or } 12
\end{aligned}
$$



Step 4: Answer the question and check
If $x=10$, then $y=24$ and $(1 / 3) y=8$
$10 \times 8$
area of each plot: 80 total fencing: 88

If $x=12$, then $y=20$
$12 \times 20 / 3 \quad$ and $(1 / 3)=20 / 3$
area of each plot: $12 \times(20 / 3)=80$ total fencing: $4(12)+2(20)=88$

## Algebra Word Problems: "Work Problem Example"

Example: During a one-hour workout at the gym, a woman cycles 4 miles and runs $21 / 2$ miles. If she cycles 8 miles/hour faster than she runs, how fast is her running rate?

SOLUTION:

$$
\begin{aligned}
\text { distance } & =(\text { rate }) \times(\text { time }) \quad \mathrm{r}=\frac{\mathrm{d}}{\mathrm{t}} \quad \mathrm{t}=\frac{\mathrm{d}}{\mathrm{r}} \\
\mathrm{~d} & =\mathrm{rt}
\end{aligned}
$$

Step 1: Establish variables and formulas

$$
\begin{aligned}
& \text { running rate: } r_{\text {running }}=\frac{x \text { miles }}{\text { hour }} \\
& \text { cycling rate: } \\
& r_{\text {cycling }}=\frac{(x+8) \text { miles }}{\text { hour }}
\end{aligned}
$$

$$
\begin{gathered}
\text { time: } t_{\text {running }}+t_{\text {cycling }}=1 \text { hour } \\
\text { distance: } \quad d_{\text {running }}=21 / 2 \text { miles } \\
d_{\text {cycling }}=4 \text { miles }
\end{gathered}
$$

Step 2: Set up equation and solve

|  | $\mathrm{t}_{\text {running }}+\mathrm{t}_{\text {cycling }}=1$ hour | $\frac{2.5}{\mathrm{x}}+\frac{4}{\mathrm{x}+8}=1$ |
| :--- | :--- | :--- |
| substitution | $\frac{\mathrm{d}_{\text {running }}}{\mathrm{r}_{\text {running }}}+\frac{\mathrm{d}_{\text {cycling }}}{\mathrm{r}_{\text {cycling }}}=1$ hour | $2.5(\mathrm{x}+8)+4(\mathrm{x})=1(\mathrm{x})(\mathrm{x}+8)$ |
| simplify units |  |  |
| (the miles | $\frac{2.5 \text { miles }}{\mathrm{x} \text { miles }}$ |  |
| and hours <br> will cancel $)$ | $\frac{4 \text { miles }}{\text { hour }} \frac{(\mathrm{x}+8) \text { miles }}{\text { hour }}$ | $2.5 \mathrm{x}+20+4 \mathrm{x}=\mathrm{x}^{2}+8 \mathrm{x}$ |
|  | $\mathrm{x}^{2}+1.5 \mathrm{x}-20=0$ | use quadratic formula |$\quad \mathrm{x}=-5.28$ or 3.78

Step 3: Answer question and check
Since $x=3.78$,

| the running rate is $\frac{3.78 \text { miles }}{1 \text { hour }}$ |
| :---: |
| and cycling rate is $\frac{11.78 \text { miles }}{1 \text { hour }}$ |

Check: cycling 4 miles @ rate of 11.78 miles/hour

$$
4 \text { miles }=(11.78 \text { miles } / \text { hour })(\text { time })
$$

$$
\text { time }=.34 \text { hours }
$$

running 2.5 miles @ rate of 3.78 miles/hour
2.5 miles $=(3.78 \mathrm{miles} /$ hour $)($ time $)$

$$
\text { time }=.66 \text { hours }
$$

Total time: 1 hour!

A math center charges $\$ 400$ for a course, and they get 750 students. For every $\$ 25$ increase in price, they lose 30 students.

## Quadratic vertex example (finding maximum)

What price would maximize revenue?
What is the domain and range?
$(400+25 \mathrm{x})(750-30 \mathrm{x})=\mathrm{y}$
price quantity revenue
(where x is the number of $\$ 25$ increases)
method 1: use midpoint of zeros

$$
\begin{array}{ll}
400+25 \mathrm{x}=0 & \mathrm{x}=-16 \\
750-30 \mathrm{x}=0 & \mathrm{x}=25
\end{array}
$$

axis of symmetry of 4.5
method 2: $-\mathrm{b} / 2 \mathrm{a}$
change to standard form and find $(-b / 2 a, f(-b / 2 a))$

$$
\begin{gathered}
300000+6750 \mathrm{x}-750 \mathrm{x}^{2} \\
\frac{-6750}{2(-750)}=4.5
\end{gathered}
$$

4.5 increases would lead to a price of

$$
(400+25(4.5))=512.50
$$

4.5 increases would lead to a quantity of

$$
(750+30(4.5)=615
$$

$$
\text { revenue }=\$ 315,187.5
$$


domain: $-16<\mathrm{x}<25$ After 16 price decreases, the price would be $0 \ldots$ (Free items don't have revenue!) After 25 price increases, there will be no sales...

## Word Problems Practice Quiz 2

1) A bag containing only dimes and nickels holds 800 coins. If the bag contains $\$ 49.70$, how many dimes are in the bag?
2) The tortoise leaves home at $1: 00 \mathrm{pm}$, traveling east at 2 miles per hour. The hare leaves 10 hours later and travels east at 9 miles per hour. What time does the hare catch the toroise?
3) Tom can paint a fence in 5 hours. Huck can paint a fence in 8 hours. If they work together, how long would it take for them to paint three fences?
4) Caramel popcorn costs $\$ 1.50$ per pound. Butter popcorn costs $\$ 1.10$ per pound. A customer purchases 20 pounds of a mixture, paying $\$ 27.70$
How much of each flavor of popcorn did he buy?
5) Sal has 20 ounces of a $25 \%$ salt water solution. How many ounces of water must he add to dilute the solution to a $15 \%$ salt water solution?
6) A bag containing only dimes and nickels holds 800 coins. If the bag contains $\$ 49.70$, how many dimes are in the bag?

| (list variables) | (set up equations) | (solve) $\mathrm{d}=800-\mathrm{n}$ | (check answer) |
| :---: | :---: | :---: | :---: |
| Let $\mathrm{d}=$ \# of dimes | $\mathrm{d}+\mathrm{n}=800$ | $.10(800-n)+.05 n=49.7$ |  |
| $\mathrm{n}=$ \# of nickels | $.10 \mathrm{~d}+.05 \mathrm{n}=49.70$ |  | 194 dimes: $\$ 19.40$ |
|  |  | 80-. $10 \mathrm{n}+.05 \mathrm{n}=49.7$ | 606 nickels: \$30.30 |
| so, $.10 \mathrm{~d}=$ value of the dimes <br> $.05 n=$ value of the nickels | (2 equations, 2 unknowns; solve using substitution) | $\begin{aligned} -.05 \mathrm{n} & =-30.3 \\ \mathrm{n} & =606 \end{aligned}$ | 800 coins: \$49.70 |
|  |  | if $\mathrm{n}=606$, then $\mathrm{d}=194$ |  |

2) The tortoise leaves home at $1: 00 \mathrm{pm}$, traveling east at 2 miles per hour. The hare leaves 10 hours later and travels east at 9 miles per hour. What time does the hare catch the toroise?
(list variables)
distance $=$ rate x time
tortoise: $\mathrm{d}=2$ miles/hour x time
hare: $d=9$ miles/hour x time
let $\mathrm{t}=$ time (in hours)
(set up equations)

Since hare leaves 10 hours later, its travel time is 10 hours less than the tortoise.

The distances will be the same when they meet.
$\frac{2 \text { miles }(\mathrm{t})}{\text { hour }}=\frac{9 \text { miles ( } \mathrm{t}-10)}{\text { hour }}$
(solve)
2 t miles $=9 \mathrm{t}$ miles -90 $90=7 \mathrm{t}$ $\mathrm{t}=12.86$ hours
(approx. 12 hours, 51 minutes)
Since tortoise left at 1:00pm, the hare catches him at $1: 51 \mathrm{AM}$
3) Tom can paint a fence in 5 hours. Huck can paint a fence in 8 hours. If they work together, how long would it take for them to paint three fences?
(list variables/formulas)
(set up equations)
work $=$ rate x time
Tom: 1 fence $=$ rate $\times 5$ hours

$$
\text { Tom's rate }=\frac{1 \text { fence }}{5 \text { hours }}
$$

To paint one fence together:
(solve)

| (multiply by 40 hours) |
| ---: |
| 8 fences $(\mathrm{t})+5$ fences $(\mathrm{t})$ |$=120$ fences(hours)

13 fences $(\mathrm{t})=120$ fences(hours)
$\mathrm{t}=9.23$ hours

9 hours 14 minutes

$$
\text { Huck's rate }=\frac{1 \text { fence }}{8 \text { hours }}
$$

(check answer)
12.86 hours x 2 miles $/ \mathrm{hr}$

$$
\xlongequal{\cong} 25.72 \text { miles }
$$

2.86 hours x 9 miles $/ \mathrm{hr}$

$$
\stackrel{\cong}{=} 25.74 \text { miles }
$$

(check)
Tom: 9.23 hours paints 1.846 fences

Huck: 9.23 hours
paints 1.15 fences
together: 3 fences!
4) Caramel popcorn costs $\$ 1.50$ per pound. Butter popcorn costs $\$ 1.10$ per pound.

A customer purchases 20 pounds of a mixture, paying $\$ 27.70$
How much of each flavor of popcorn did he buy?

| rate |  | (pounds) amount | cost | The mixture rate $=\frac{\$ 27.70}{20}=\$ 1.385$ per pound | caramel: 14.25 pounds ${ }^{\text {\$21.375 }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| caramel | \$1.50 | X | \$1.50x | (cost) | butter: 5.75 pounds |
| butter | \$1.10 | (20-x) | \$1.10(20-x) | \$1.50x + \$1.10(20-x) $=\$ 27.70$ | \$6.325 |
| mixture | ? | 20 | \$27.70 | \$0.40x $=\$ 5.70$ | total pounds; \$27.70 |

5) Sal has 20 ounces of a $25 \%$ salt water solution. How many ounces of water must he add to dilute the solution to a $15 \%$ salt water solution?

| (ounces) <br> amount |  |  | rate |
| :--- | :--- | :--- | :--- |
| salt |  |  |  |
| salt $25 \%$ | 20 | $.25 /$ ounce | 5 |
| water | x | $0 /$ ounce | 0 |
| salt $15 \%$ | $20+\mathrm{x}$ | $.15 /$ ounce | $3+.15 \mathrm{x}$ |

salt amount: $5+0=3+.15 \mathrm{x}$
$(25 \%+$ water $=15 \%)$
$2=.15 x$

$$
x=13.333 \text { ounces of water }
$$

33.33 ounces of mixture ---
$15 \%$ of 33.333 is 5 ounces of salt

The volume of a metal box is 30 cubic feet. If the length is 5 feet greater than the height and the width is 2 feet less than the height, what are the dimensions of the box?

Volume $=$ length x width x height

$$
\begin{aligned}
& 30 \mathrm{ft}^{3}=(\mathrm{h}+5) \text { feet } \cdot(\mathrm{h}-2) \text { feet } \cdot(\mathrm{h}) \text { feet } \\
& \mathrm{h}(\mathrm{~h}+5)(\mathrm{h}-2)=30 \quad \text { combine 1st and } 2 \text { nd terms } \\
& \left(h^{2}+5 h\right)(h-2)=30 \quad \text { FOIL } \\
& h^{3}+3 h^{2}-10 h=30 \quad \text { Set equal to zero } \\
& h^{3}+3 h^{2}-10 h-30=0 \quad \text { solve for } h \text { (by grouping) } \\
& h^{2}(h+3)-10(h+3)=0 \quad h+3=0 \quad h=-3 \\
& (h+3)\left(h^{2}-10\right)=0 \\
& \text { combine } 1 \text { st and } 2 \text { nd terms } \\
& \text { FOIL } \\
& \begin{array}{ll}
\mathrm{h}+3=0 & \mathrm{~h}=-3 \\
\mathrm{~h}^{2}-10=0 & \mathrm{~h}=\sqrt{10}
\end{array} \\
& \mathrm{~h}=-\sqrt{10}
\end{aligned}
$$



Since height cannot be negative, our solution is $\mathrm{h}=$
the solution is $\mathrm{h}=\sqrt{10} \quad$ (approximately
so, the dimensions of the box are approximately $8.16 \times 1.16 \times 3.16$


Thanks for visiting. (Hope it helps!)
If you have questions, suggestions, or requests, let us know.
Cheers


Also, Mathplane Express for mobile and tablets at Mathplane.ORG
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Or, visit the mathplane stores at TES and TeachersPayTeachers.

Hidden Message

Answer the ten word problems below. Then, convert numbers to letters to reveal the hidden answer!

Clue: Where you may find x and y in 2 squares?

1) The sum of 2 consecutive even integers is 86 .

What is the difference of the digits of the smaller integer?
2) If the area of a square backyard is 64 feet, how long is each side?
3) A tortoise leaves home at 1:00 and travels east at 4mph... A hare leaves the same home at 4:00 and travels east at 6 mph . How many hours will it take the hare to catch the tortoise?
4) Find the number of a's and o's in the (green) directions above.
5) Liz has an appointment at $3: 15 \mathrm{pm}$.

If she appears at $2: 36 \mathrm{pm}$, how many minutes early is she?
6) Mark drinks a gallon of milk each week.

How many quarts does he consume each year?
7) The lengths of a rectangle are twice the widths.

If the perimeter is 372 feet, what is the area of the rectangle?
8) How many 175 pound people can fit into an elevator with capacity of $3 / 4$ ton?
9) A taxi cost $\$ 1.25$ per person plus $\$ 0.20$ per quarter mile. If it cost you and your friend $\$ 4.90$ for a cab ride downtown, how many miles did you travel?
10) A farmer is raising chickens and cows. If he has 23 total animals which have a total of 74 legs, how many chickens does he have?


Answer the ten word problems below. Then, convert numbers to letters to reveal the hidden answer!

## Clue: Where you may find x and y in 2 squares?

SOLUTIONS

Number/Letter Key
$\begin{array}{llllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0\end{array}$
$\begin{array}{llllllllll}\text { A } & \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{M} & \mathrm{O} & \mathrm{P} & \mathrm{R} & \mathrm{S} & \mathrm{W}\end{array}$

1) The sum of 2 consecutive even integers is 86 . What is the difference of the digits of the smaller integer?

$$
\begin{array}{cll}
\text { let } x=1 \text { st integer } & x+(x+2)=86 & x=42 \\
x+2=2 \text { nd integer } & 2 x+2=86 & x+2=44
\end{array}
$$

the digits of the smaller integer are 4 and 2. Their difference is 2 .

$$
\mathrm{s} \begin{aligned}
& 64 \\
& \mathrm{~S}
\end{aligned} \quad \sqrt{64}=8
$$

3) A tortoise leaves home at 1:00 and travels east at 4mph... A hare leaves the same home at 4:00 and travels east at 6 mph . How many hours will it take the hare to catch the tortoise?

When the hare leaves home, the tortoise will have traveled 12 miles ( $4 \mathrm{mph} \times 3$ hours)..
At 6 mph vs. 4 mph , the hare will gain 2 miles/hour on the tortoise.. so, it will take 6 hours to catch

8) How many 175 pound people can fit into an elevator with capacity of $3 / 4$ ton?

$$
3 / 4 \text { ton }=1500 \text { pounds }
$$

$$
1500 \div 175=8.57
$$

8 people can fit... (the 9th would be too much)
$175 \mathrm{p}<1500$

39 total minutes early
6) Mark drinks a gallon of milk each week.

How many quarts does he consume each year?
Mark drinks 1 gallon/week... 1 gallon $=4$ quarts, so he drinks 4 quarts/week.. Since there are 52 weeks/year, he drinks 208
7) The lengths of a rectangle are twice the widths.

If the perimeter is 372 feet, what is the area of the rectangle?


$$
\begin{array}{r}
2 \mathrm{w}+2 \mathrm{w}+\mathrm{w}+\mathrm{w}=372 \mathrm{ft} \\
\mathrm{w}=62 \mathrm{ft} \\
2 \mathrm{w}=1=124 \mathrm{ft}
\end{array}
$$

$$
\begin{aligned}
\text { Area } & =l \mathrm{w} \\
& =124 \times 62 \\
& =7688
\end{aligned}
$$

$$
4 \mathrm{mph}(\mathrm{t}+3)=6 \mathrm{mph}(\mathrm{t})
$$

5) Liz has an appointment at $3: 15 \mathrm{pm}$.

If she appears at $2: 36 \mathrm{pm}$, how many minutes early is she?
2:36 is 24 minutes before $3: 00 \ldots$ And, $3: 15$ is 15 minutes after 3:00..
quarts per year
9) A taxi cost $\$ 1.25$ per person plus $\$ 0.20$ per quarter mile.

If it cost you and your friend $\$ 4.90$ for a cab ride downtown, how many miles did you travel?
If you and your friend take a cab, the fixed cost is $\$ 2.50 \ldots$ therefore, you spent $\$ 2.40$ for x quarter miles..

At $\$ 0.20$ per quarter mile, you traveled 12 quarter miles...
12 quarter miles $=$ 3 miles...

10) A farmer is raising chickens and cows. If he has 23 total animals which have a total of 74 legs, how many chickens does he have?

```
let k=# of cows
    c=# of chickens }\quad\textrm{k}+\textrm{c}=2
    4k+2c=74
4k = # of cow legs
2c = # of chicken legs
(using substitution)
    k}=23-\textrm{c
```

$$
\begin{aligned}
4(23-c)+2 c & =74 \\
92-4 c+2 c & =74 \\
-2 c & =-18 \\
c & =9
\end{aligned}
$$

9 chickens and 14 cows


You may find x and y in the square boxes of a crossword puzzle!


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