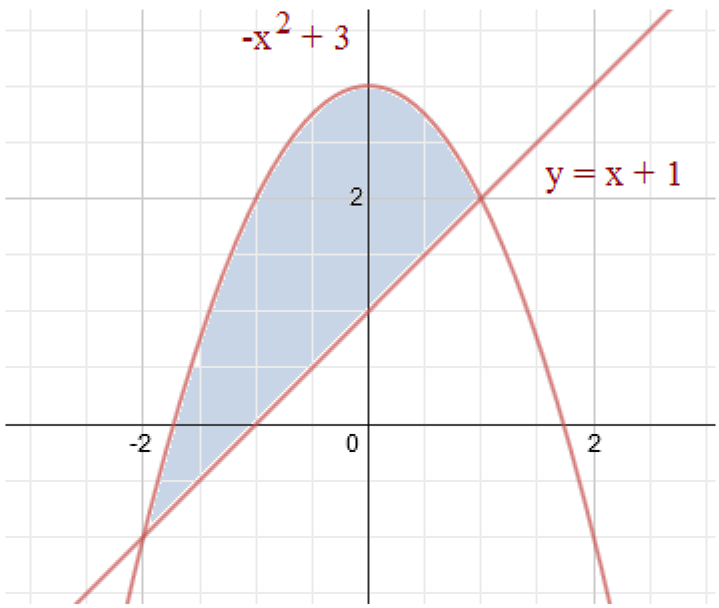
# Calculus: Introduction to Definite Integrals

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*Using Definite Integrals*

A <u>derivative</u> determines the slope at a given point (or instantaneous rate of change). What can a <u>definite integral</u> do?

Answer: It can find area under a function over a specified interval.

> Fundamental Theorem of Calculus
>
> If a function $f(x)$ is continuous on the interval $[a, b]$, then
>
> $$\int_a^b f(x)\,dx = F(b) - F(a)$$
>
> where $F(x)$ is any function such that $F'(x) = f(x)$ for all x in $[a, b]$

*Example:*

$$\int_2^7 4x + 6\,dx$$

Find the integral: $F(x) = \frac{4x^2}{2} + 6x = 2x^2 + 6x$

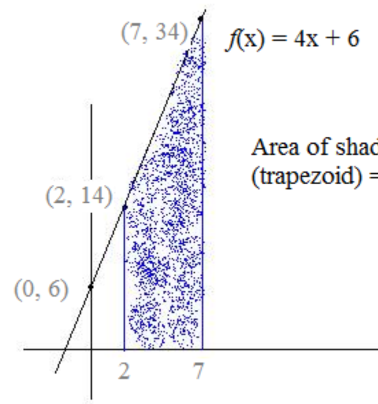
Apply the Fundamental Theorem of Calculus:

$F(7) = 2(7)^2 + 6(7) = 140$

$F(2) = 2(2)^2 + 6(2) = 20$

$F(7) - F(2) = 120$

Conclusion: The area under the function over the interval $[2, 7]$ is 120 (see graph)




Area of shaded region (trapezoid) = 120

*Example:*

$$\int_2^5 x^2 + 7\,dx$$

Find the integral: $F(x) = \frac{x^3}{3} + 7x + C$ (indefinite integral)

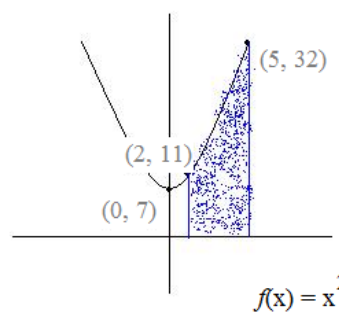
Find the definite integral (apply the Fundamental theorem of Calculus):

$$\left.\frac{x^3}{3} + 7x\right|_2^5 = \frac{5^3}{3} + 7(5) - \left(\frac{2^3}{3} + 7(2)\right)$$

$$= \frac{125}{3} + 35 - \frac{8}{3} - 14$$

$$= 60$$

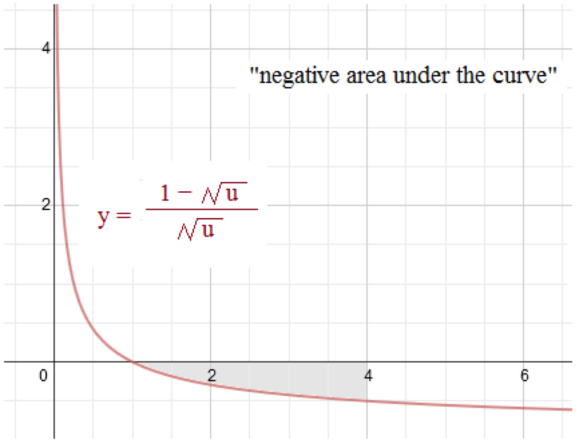
(Notice: we can omit the constant 'C', because F(b) - F(a) would include C - C, which would cancel the constant)




The (blue) area under the function on the interval [2, 5] has an area of 60 square units

*Example:* Evaluate: $\displaystyle\int_1^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du$ "Separate" the function; then, integrate.

$$\int_1^4 \frac{1}{\sqrt{u}} - \frac{\sqrt{u}}{\sqrt{u}}\,du = \int_1^4 \frac{1}{\sqrt{u}}\,du - \int_1^4 1\,du = \left.\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right|_1^4 - \left. u \right|_1^4$$




$$= 4 - 2 - (4 - 1) = \boxed{-1}$$

The definite integral is negative, because the evaluated portion of the function is below the x-axis.

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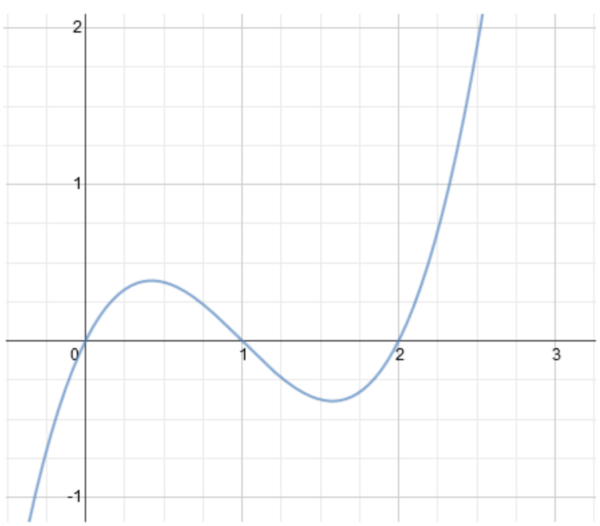
Definite Integral Value vs. Area

A) Evaluate the integral:

$$\int_0^2 x^3 - 3x^2 + 2x \quad dx$$

$$\left.\frac{x^4}{4} - x^3 + x^2\right|_0^2 = 4 - 8 + 4 = \boxed{0}$$




B) Find the area of the region between $x^3 - 3x^2 + 2x$ and the x-axis where $0 \le x \le 2$

First find where the curve is above and below the x-axis!

$$x^3 - 3x^2 + 2x = 0$$

$$x(x^2 - 3x + 2) = 0$$

$$x(x-1)(x-2) = 0$$

$x = 0, 1, 2$ Since there are zeros at 1 and 2, we must split the integral boundaries.

Since there is a zero at 1 (as well as 0 and 2), the curve will go below the x-axis... Since area cannot be negative, we must change the sign in that interval)

$$\int_0^1 x^3 - 3x^2 + 2x \quad dx - \int_1^2 x^3 - 3x^2 + 2x \quad dx$$

$$\left.\frac{x^4}{4} - x^3 + x^2\right|_0^1 - \left.\frac{x^4}{4} - x^3 + x^2\right|_1^2$$

$$\frac{1}{4} - 0 - \left(0 - \frac{1}{4}\right) = \boxed{\frac{1}{2}}$$

NOTE: *Same function, but 2 different integral applications*

"Displacement vs. Distance Travelled"

*Example:* The acceleration of a particle is modeled by the function $a(t) = 2t + 5$ where t is feet/second$^2$

and, the initial velocity of the particle along a line is $v(0) = -6$ feet/second

In the interval $0 \le t \le 3$,

a) Find the velocity at any time t

b) Determine the displacement (i.e. total change in position, or the end point if the particle begins at 0)

c) Find the total distance travelled during the given interval

---

a) What is the velocity function of the particle?
Since we know the accelearation, we can take the antiderivative to find the velocity!

$$a(t) = 2t + 5$$

$$\int 2t + 5 \; dt = t^2 + 5t + C$$

What is C? Since we know $v(0) = -6$

$$v(t) = t^2 + 5t + C$$

$$v(0) = 0^2 + 5(0) + C = -6$$

$$C = -6$$

$$\boxed{v(t) = t^2 + 5t - 6}$$

b) What is the displacement of the particle during the first 3 seconds?
In other words, what is the position of the particle at 3 seconds?

$$v(t) = t^2 + 5t - 6$$

$$\int_0^3 t^2 + 5t - 6 \; dt = \left. \frac{t^3}{3} + \frac{5t^2}{2} - 6t \right|_0^3 = 9 + \frac{45}{2} - 18 - \left( 0 + 0 - 0 \right) = \boxed{13\frac{1}{2} \text{ feet (to the right or in a positive direction)}}$$

c) What is the total distance traveled during the first 3 seconds?
First, we need to determine which direction the particle is going?

$v(t) = t^2 + 5t - 6$ where is the particle at rest?

$$v(t) = t^2 + 5t - 6 = 0$$

$$(t + 6)(t - 1) = 0$$

$$t = -6, 1$$

Since t = -6 is not in the interval [0, 3]
(plus, time isn't negative!),

we'll look at t = 1

The particle is at rest @ t = 1... (also, it changes direction at that time)

In the interval [0, 1), the particle is going in a *negative* direction... (how do we know? try v(1/2).... $v(1/2) < 0$ )

In the interval (1, 3], the particle is going in a *positive* direction... (how do we check? try v(2).... $v(2) > 0$ )

So, to deterine the total distance traveled (backwards and forwards),

$$\int_0^1 t^2 + 5t - 6 \; dt = \left. \frac{t^3}{3} + \frac{5t^2}{2} - 6t \right|_0^1 \quad \frac{1}{3} + \frac{5}{2} - 6 - \left( 0 + 0 - 0 \right) = \frac{-19}{6}$$

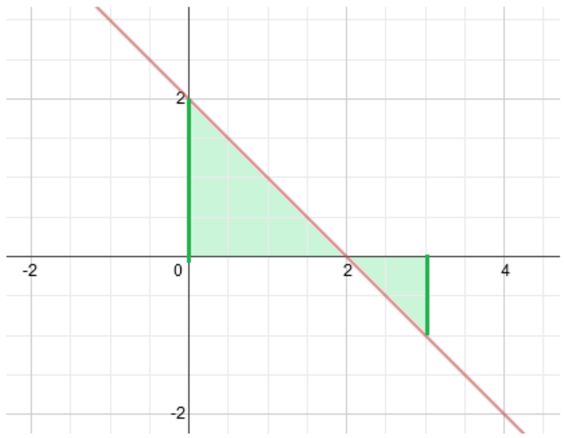
During the first 1 second, the particle moves 19/6 in a negative direction..
However, we're only concerned with the distance traveled (i.e. the absolute value) $\frac{19}{6}$

$$\int_1^3 t^2 + 5t - 6 \; dt = \left. \frac{t^3}{3} + \frac{5t^2}{2} - 6t \right|_1^3 = 9 + \frac{45}{2} - 18 - \left( \frac{1}{3} + \frac{5}{2} - 6 \right) = 13\frac{1}{2} - \frac{-19}{6} = \frac{50}{3}$$

$$\underbrace{\frac{19}{6}}_{\text{distance traveled to the left...}} + \underbrace{\frac{50}{3}}_{\text{distance traveled to the right...}} = \underbrace{\boxed{\frac{119}{6} \text{ feet traveled in the first 3 seconds..}}}_{\text{total movement}}$$

*Example:* Find the area of the region between the 'curve' $y = -x + 2$ where $0 \le x \le 3$

Step 1: Draw a quick sketch




We can see the function is a line.
More importantly, part of the function is negative (below the x-axis). Since area cannot be negative, we must use absolute value in that part of the integral.

Step 2: Set up integral and boundaries

$$\int_0^2 -x + 2\,dx \;-\; \int_2^3 -x + 2\,dx$$

(Since the region between 2 and 3 is negative, we will 'subtract' in order to get a positive value.)

Step 3: Solve

$$\left.\frac{-x^2}{2} + 2x\right|_0^2 \;-\; \left.\frac{-x^2}{2} + 2x\right|_2^3$$

$$(2 + 0) - (3/2 - 2) = \boxed{2\,\frac{1}{2}}$$

Step 4: (if possible) check

Area of triangle: $\frac{1}{2}$(base)(height)

area of left triangle: $\frac{1}{2}(2)(2) = 2$

area of right triangle: $\frac{1}{2}(1)(1) = 1/2$

total shaded area: $2\frac{1}{2}$

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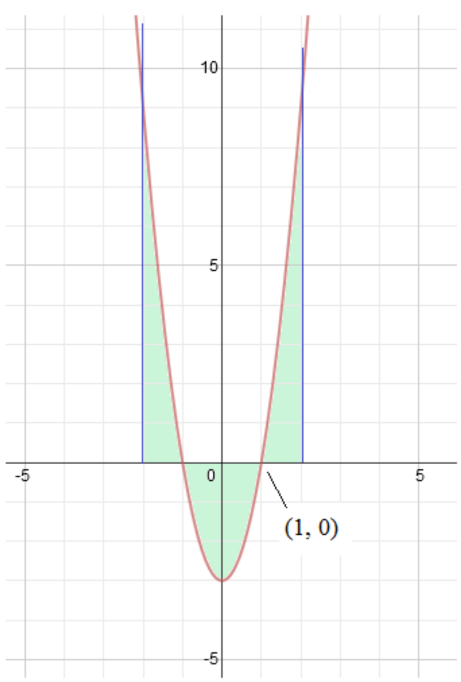
*Example:* Find the area of region between $y = 3x^2 - 3$ and the x-axis on the interval [-2, 2]




***Since this is an even function, we can find the area between 0 and 2, and then double it...

$$-\int_0^1 3x^2 - 3\,dx \;+\; \int_1^2 3x^2 - 3\,dx$$

below the x-axis    above the x-axis

$$-\left. x^3 - 3x\right|_0^1 \;+\; \left. x^3 - 3x\right|_1^2$$

$$-(-2 - 0) + (2 - -2) = 6$$

Since the area between 0 and 2 is 6, the total area between -2 and 2 is $\boxed{12}$

*** Quick check: Count/estimate the number of shaded tiles!

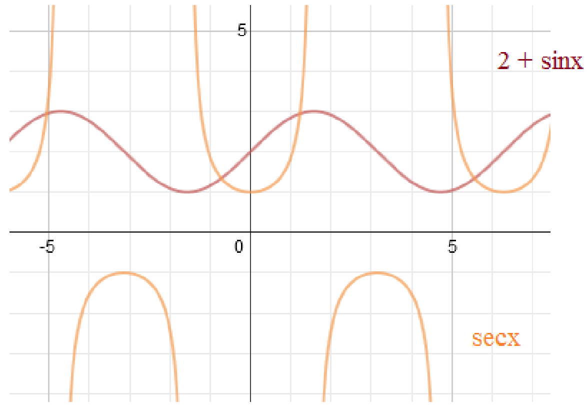
Definite Integrals and Area: Trig & Absolute Value Functions

*Example:* Find the area of the region bounded by

$y = 2 + \sin x$

$y = \sec x$

and, the *positive side* of the y-axis




Step 1: Sketch a graph

Step 2: Determine the boundaries of the integral

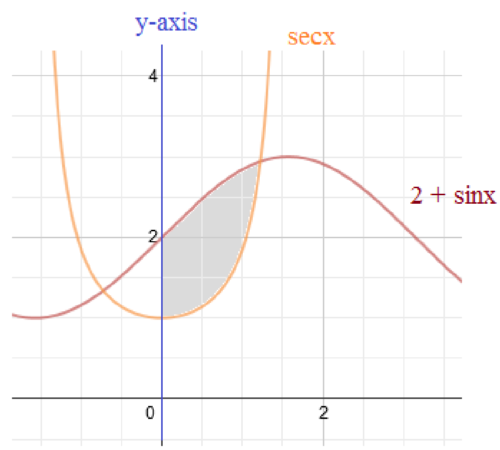
The left boundary will be 0 ('positive side of y-axis')

The right boundary will be the intersection of

$y = \sin x + 2$
and
$y = \sec x$

Using a graphing calculator, we find the intersection occurs at (1.22, 2.94)

Therefore, the right boundary will be 1.22




$$\int_0^{1.22} \underbrace{(2 + \sin x)}_{\text{upper curve}} - \underbrace{(\sec x)}_{\text{lower curve}} \, dx = \boxed{1.23}$$

Using a TI-nspire CX Cas:
Menu
4: Calculus
3: Integral
Enter the boundaries, trig functions, x and dx
The output is approx. 1.23

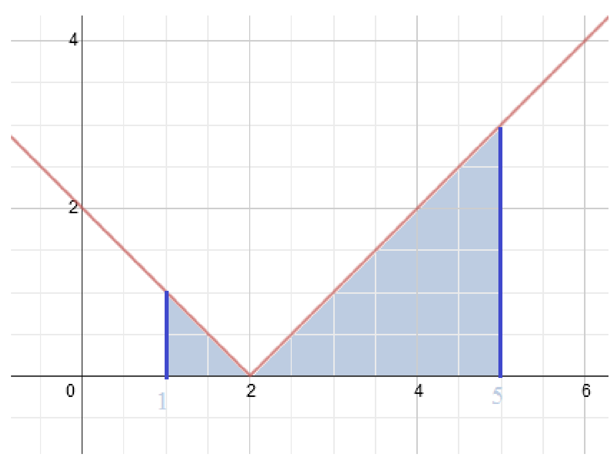
Evaluate $\displaystyle\int_1^5 |x - 2| \, dx$

Rather than trying to integrate an absolute value, it's easier to graph the function.

Then, since it is a definite integral, we're simply looking for the area under the 'curve'!

The area of the little triangle is 1/2, and the area of the big triangle is 9/2

Total area: 5 units

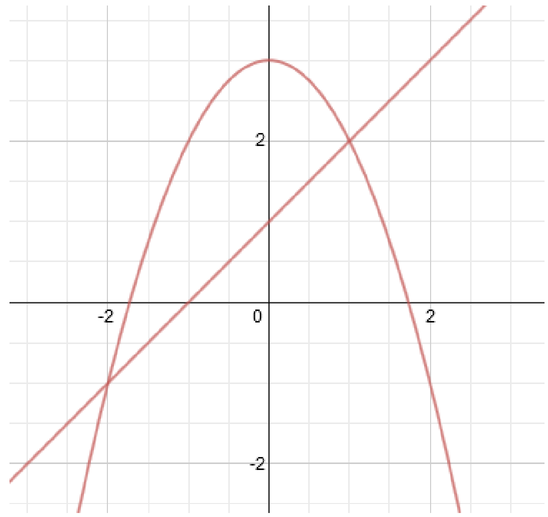


Definite Integrals: Area between functions

*Example:* What is the area of the region bounded by $y = x + 1$ and $y = -x^2 + 3$ ?

Step 1: Sketch the functions

The graph shows a line crossing a downward facing parabola.




Step 2: Determine the boundary of the region

$y = x + 1$

$y = -x^2 + 3$

$$x + 1 = -x^2 + 3$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2 \text{ and } 1$$

intersection at (-2, -1) and (1, 2)

Step 3: Use integration to find the area of the region

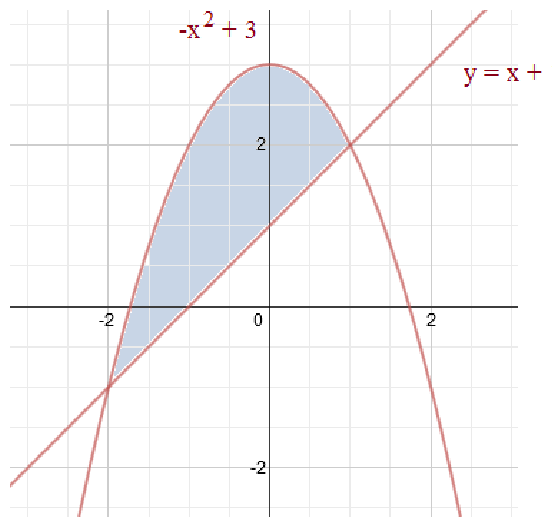
$$\int_{-2}^{1} -x^2 + 3 \; dx \; - \; \int_{-2}^{1} x + 1 \; dx$$

area under the parabola — area under the line

$$\left. \frac{-x^3}{3} + 3x \right|_{-2}^{1} - \left. \frac{x^2}{2} + x \right|_{-2}^{1}$$

$$\frac{-1}{3} + 3 - \frac{8}{3} + 6 - \left( \frac{1}{2} + 1 + 2 - 2 \right)$$

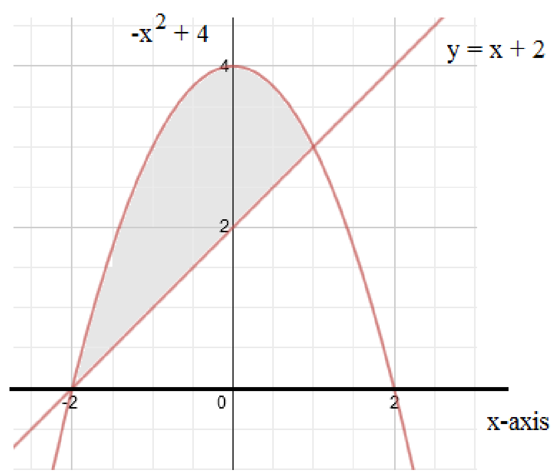
$$6 - 3/2 = \boxed{9/2}$$




Note: Although the shaded region is through the x-axis, integration still determined the area of the region

Why? Because,

If we had shifted the parabola and line up 1 unit (above the x-axis), the definite integral would be the same.



U-Substitution: Definite Integrals

*Example:* $$\int_0^1 r\sqrt{1-r^2}\;dr$$

Let $u = 1 - r^2$

$$\frac{du}{dr} = -2r$$

$$dr = \frac{du}{-2r}$$

Also, we must adjust the boundaries!

If $r = 1$, then $u = 1 - (1)^2 = 0$

If $r = 0$, then $u = 1 - (0)^2 = 1$

$$\int_1^0 r\sqrt{u}\;\frac{du}{-2r}$$

$$\int_1^0 \sqrt{u}\;\frac{du}{-2}$$

$$\frac{1}{-2}\int_1^0 \sqrt{u}\;du$$

$$\frac{1}{\text{-}2}\int_0^1 \sqrt{u}\;du$$

$$\frac{1}{\text{-}2}\cdot\frac{2}{3}u^{3/2}\Big|_0^1 = \boxed{\frac{1}{3}}$$

*Example:* $$\int_{-1}^3 \frac{5x}{(4+x^2)^2}\;dx$$

$$\int_{-1}^3 5x(4+x^2)^{-2}\;dx$$

Let $u = 4 + x^2$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

Then, we adjust the boundaries....

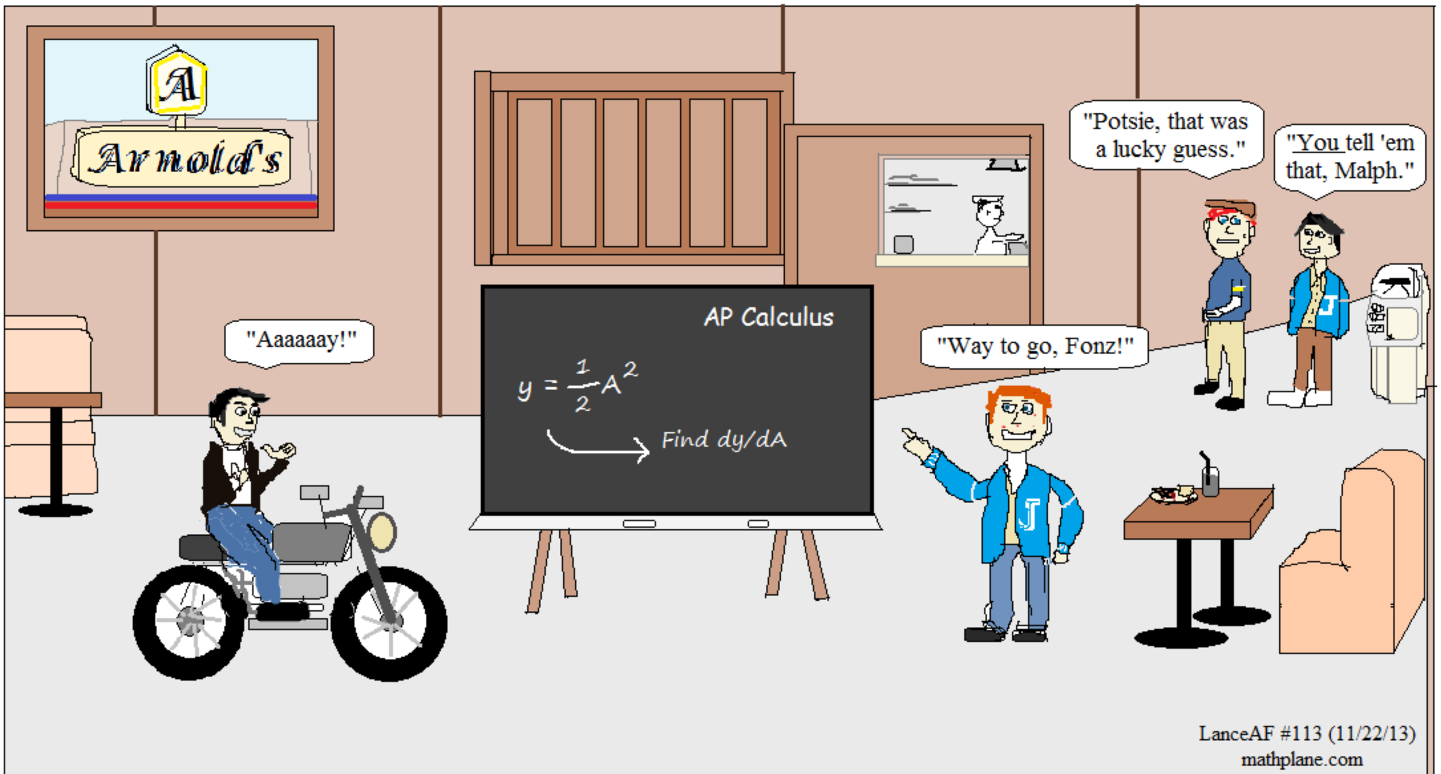
If $x = 3$, then $u = 13$

If $x = -1$, then $u = 5$

$$\int_5^{13} 5x\;(u)^{-2}\;\frac{du}{2x}$$

$$\frac{5}{2}\int_5^{13} (u)^{-2}\;du$$

$$\frac{5}{2}\cdot(-1)(u)^{-1}\Big|_5^{13} = \frac{-5}{26} - \frac{-1}{2} = \boxed{\frac{8}{26}}$$




Happy Days

*Despite Richie's help, Fonzie dropped out of Calculus.*
*(... although he did have some success with velocity and acceleration!)*

# Practice Exercises -→

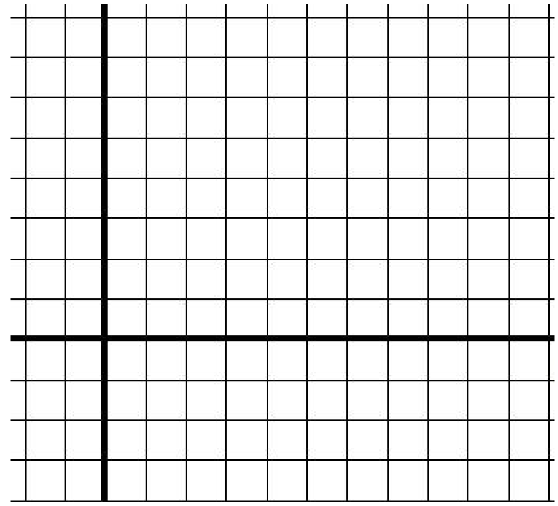
Calculus Area Bound Review

Graph and find the area bounds of the following:

1) $y = |5 - x|$

and the x-axis

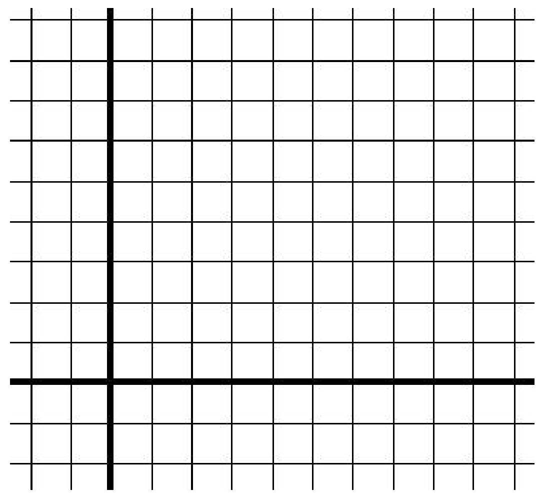
on the interval [0, 8]



2) $y = x + 3$

$y = -x + 6$

x-axis and y-axis

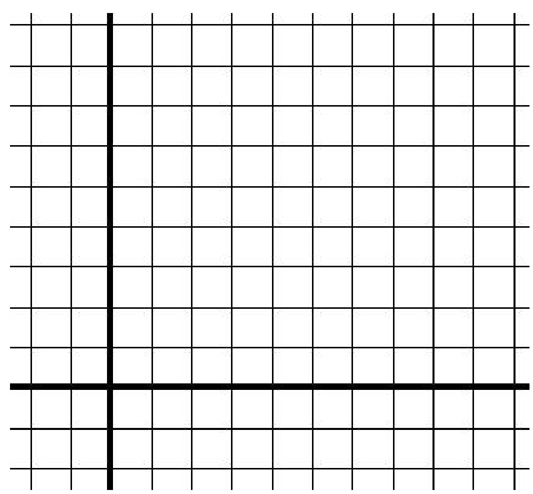


3) $y = e^x$

$y = x$

$x = 2$

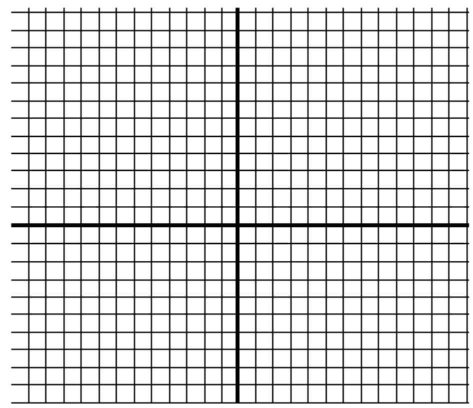
the y-axis



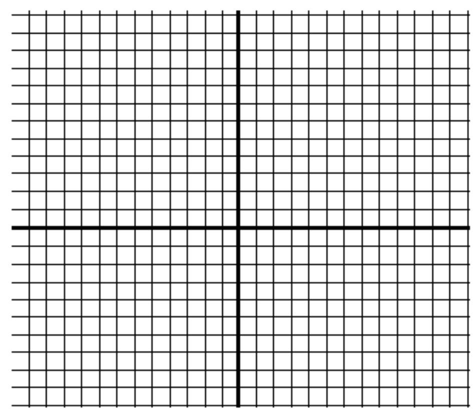
Fundamental Theorem of Calculus/Definite Integrals Exercise

Evaluate the following definite integrals.
Then, sketch a graph, shading the area of the specified range.

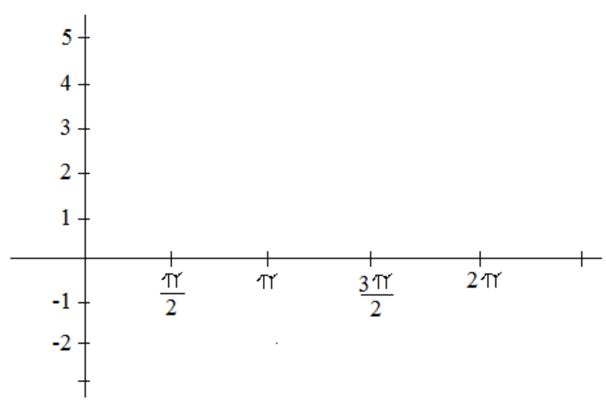
$$\int_{1}^{4} x + 6\,dx$$



$$\int_{3}^{5} -x^2 + 7x - 10\,dx$$



$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x + 3\,dx$$

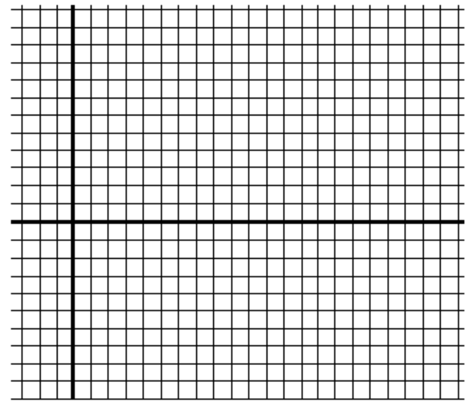


Fundamental Theorem of Calculus/Definite Integrals Exercise

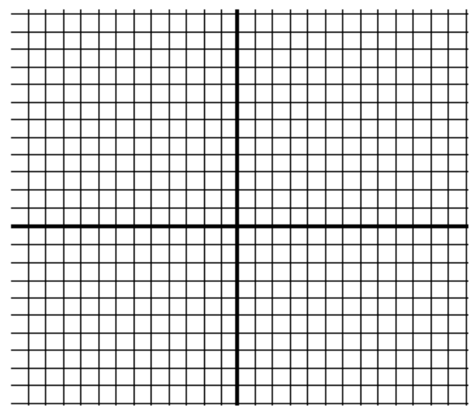


Evaluate the definite integral. Then, sketch the function, shading the area of the specified range.

$$\int_{7}^{14} \sqrt{x+2} - 1 \; dx$$

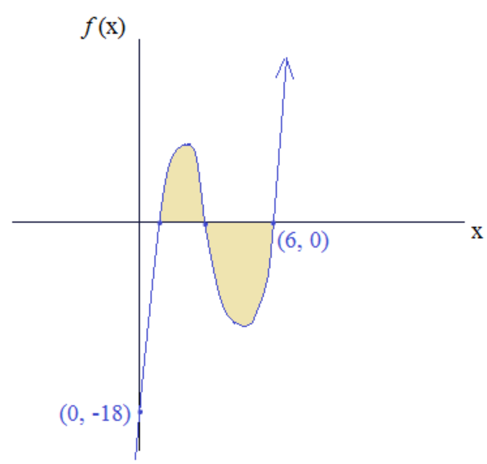


Find the area bounded by $x^2 - 4x - 5$ and the x-axis. Sketch the function and label the area.



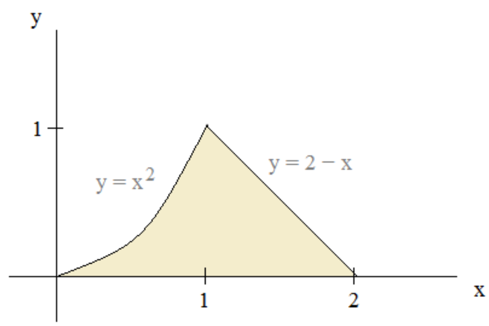
Find the total area enclosed by the x-axis and the cubic function

$$f(x) = (x - 1)(x - 3)(x - 6)$$

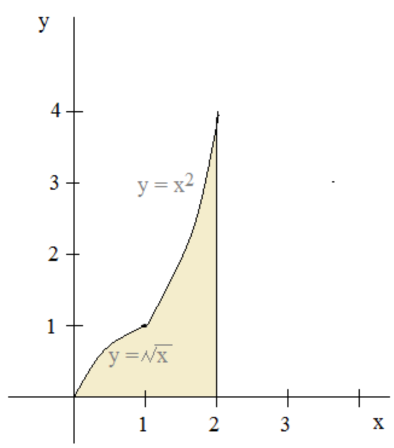


Using Definite Integrals, find the shaded areas:

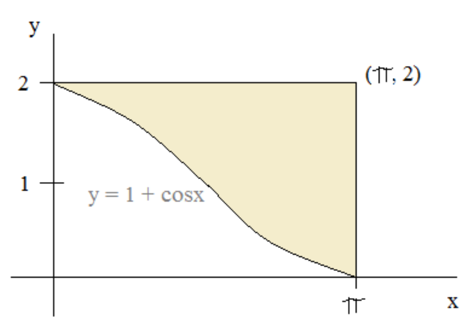
A)

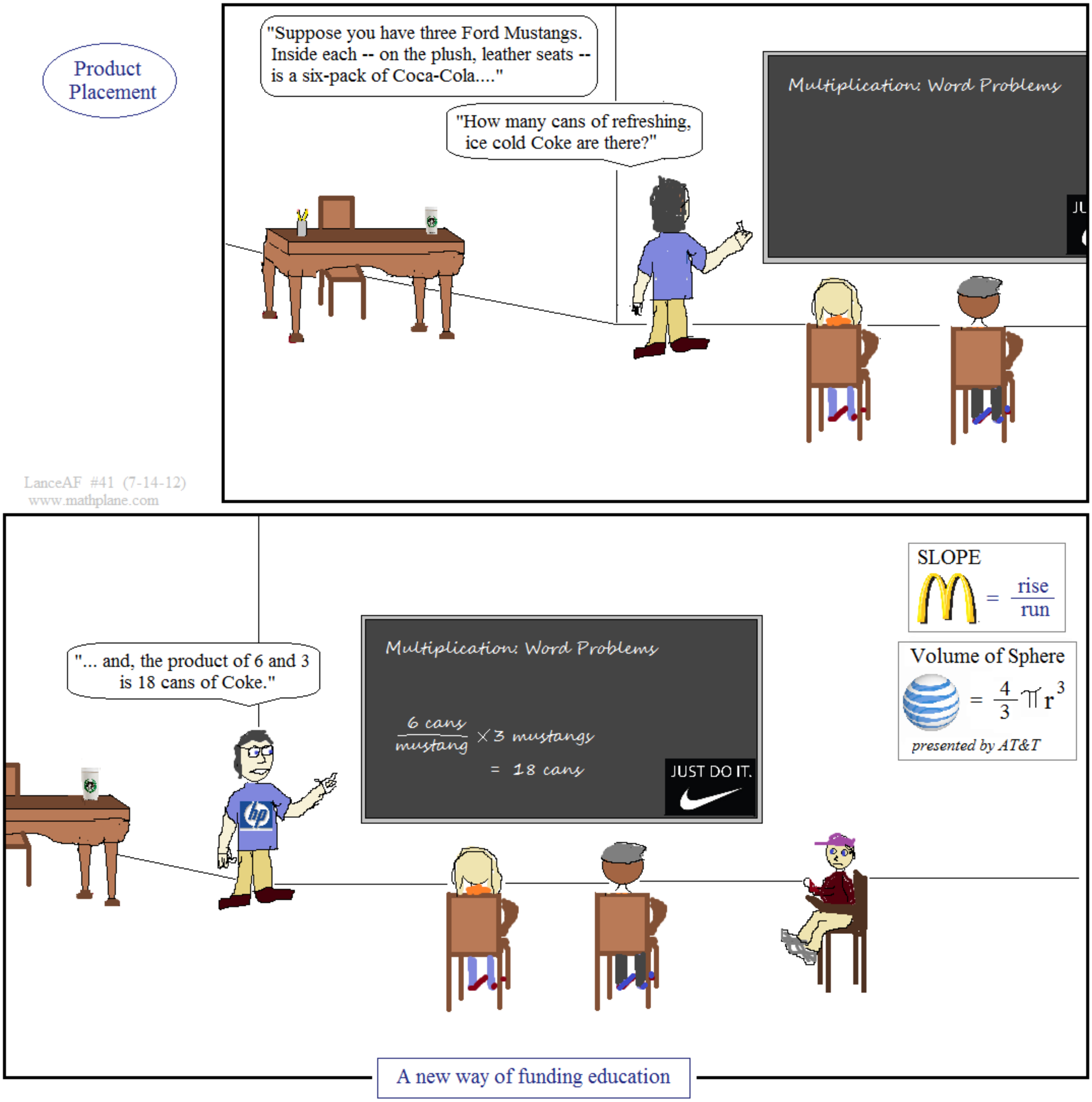



B)




C)




Product Placement
"Suppose you have three Ford Mustangs. Inside each -- on the plush, leather seats -- is a six-pack of Coca-Cola...."
"How many cans of refreshing, ice cold Coke are there?"
Multiplication: Word Problems
LanceAF #41 (7-14-12)
www.mathplane.com
"... and, the product of 6 and 3 is 18 cans of Coke."
Multiplication: Word Problems
$\frac{6 \text{ cans}}{\text{mustang}} \times 3 \text{ mustangs}$
$= 18 \text{ cans}$
JUST DO IT.
SLOPE
$= \frac{\text{rise}}{\text{run}}$
Volume of Sphere
$= \frac{4}{3}\pi r^3$
presented by AT&T
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SOLUTIONS -→

Calculus Area Bound Review

SOLUTIONS

Graph and find the area bounds of the following:

1) $y = |5 - x|$
and the x-axis
on the interval [0, 8]

$y = |5 - x|$

| x | y |
|---|---|
| 3 | 2 |
| 4 | 1 |
| 5 | 0 |
| 6 | 1 |
| 7 | 2 |
| 8 | 3 |

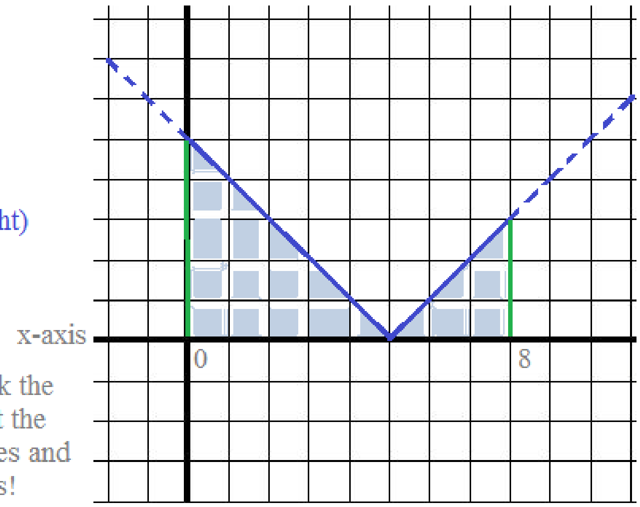
Since the shaded area consists of 2 right triangles, we can use simple area formula: $A = \frac{1}{2}(\text{base})(\text{height})$

$A_1 = \frac{1}{2}(5)(5) = 12.5$

$A_2 = \frac{1}{2}(3)(3) = 4.5$

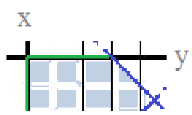
Total = 17 sq. units

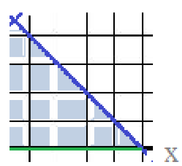
Note: to check the answer, count the shaded squares and partial squares!




2) $y = x + 3$
$y = -x + 6$

x-axis and y-axis




First, find the intersection of the 2 lines:
(combination/elimination method)

$2y = 9$
$y = 9/2$

so, $x = 3/2$

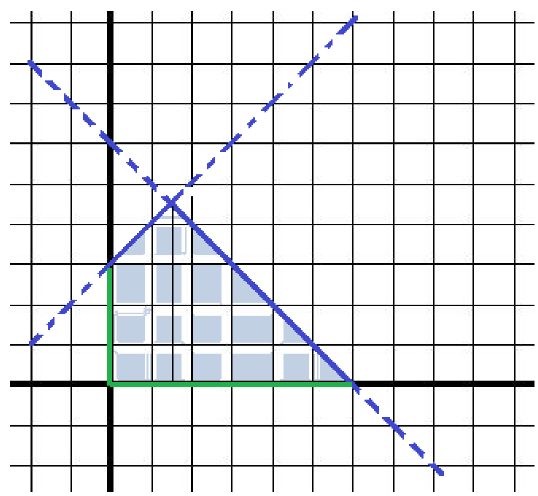
Total: 15.75 sq units

The shaded area consists of a trapezoid and triangle.

$\text{Area}_{tri} = \frac{1}{2}(4.5)(4.5) = 10.125$

$\text{Area}_{trap} = \frac{1}{2}(\text{base1} + \text{base2})(\text{height})$

$= \frac{1}{2}(4.5 + 3)(1.5) = 5.625$



3) $y = e^x$
$y = x$
$x = 2$
the y-axis

The shaded area consists of the area under the log function MINUS the right triangle (i.e. the area under the line $y = x$)

(use definite integral to find area)

$y = e^x$

| x | y |
|---|---|
| -1 | .37 |
| 0 | 1 |
| 1 | 2.72 |
| 2 | 7.39 |

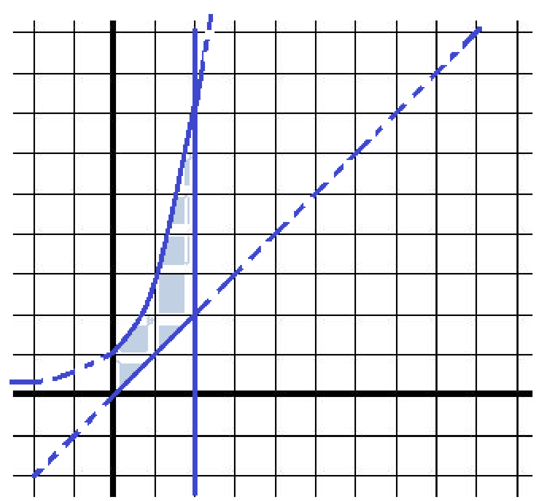
(approx.)

$A_{log} = \int_0^2 e^x\,dx = e^2 - e^0$

$= 7.39 - 1$

$A_{tri} = \frac{1}{2}(2)(2) = 2$

Total Area is approx. 4.39 sq. units



Fundamental Theorem of Calculus/Definite Integrals Exercise

SOLUTIONS

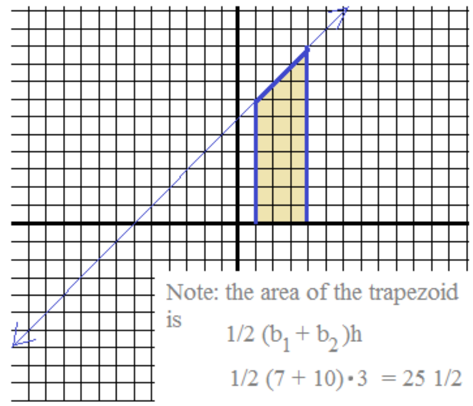
Evaluate the following definite integrals.
Then, sketch a graph, shading the area of the specified range.

$$\int_1^4 x + 6\,dx$$

$$\frac{x^2}{2} + 6x \Big|_1^4 =$$

$$\frac{(4)^2}{2} + 6(4) - \left[\frac{(1)^2}{2} + 6(1)\right]$$

$$8 + 24 - [1/2 + 6] = \boxed{25\ 1/2}$$



Note: the area of the trapezoid is

$1/2\,(b_1 + b_2)h$

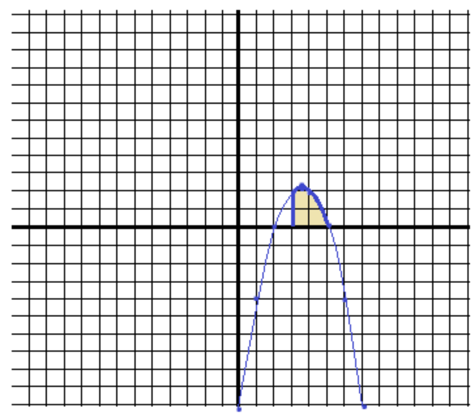
$1/2\,(7 + 10)\cdot 3 = 25\ 1/2$

$$\int_3^5 -x^2 + 7x - 10\,dx \qquad \frac{-x^3}{3} + \frac{7x^2}{2} - 10x \Big|_3^5 =$$

$$\frac{-(5)^3}{3} + \frac{7(5)^2}{2} - 10(5) - \left[\frac{-(3)^3}{3} + \frac{7(3)^2}{2} - 10(3)\right]$$

$$\frac{-125}{3} + \frac{175}{2} - 50 - [-9 + 63/2 - 30] =$$

$$\frac{-125}{3} + \frac{175}{2} - \frac{85}{2} = \boxed{10/3}$$

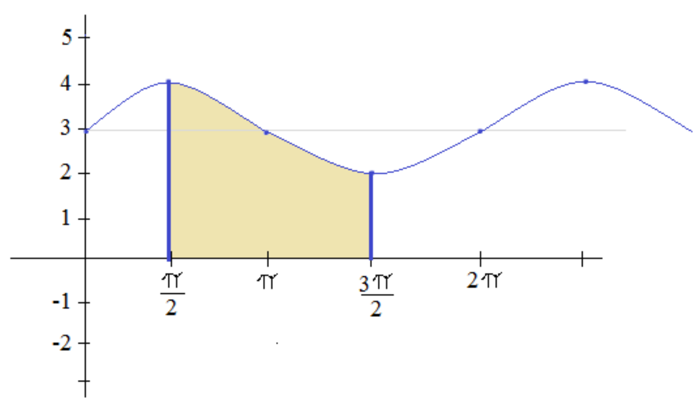


$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x + 3\,dx \qquad -\cos x + 3x \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} =$$

$$-\cos\left(\frac{3\pi}{2}\right) + 3\left(\frac{3\pi}{2}\right) - \left[-\cos\left(\frac{\pi}{2}\right) + 3\left(\frac{\pi}{2}\right)\right]$$

$$0 + \frac{9\pi}{2} - \left[0 + \frac{3\pi}{2}\right] =$$

$$3\pi \boxed{\cong 9.42}$$



Fundamental Theorem of Calculus/Definite Integrals Exercise

SOLUTIONS

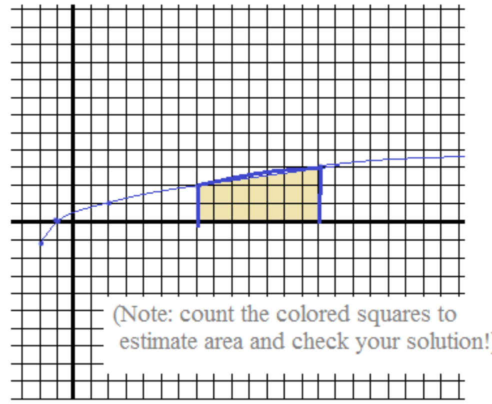


Evaluate the definite integral. Then, sketch the function, shading the area of the specified range.

$$\int_7^{14} \sqrt{x+2} - 1 \; dx \qquad \int_7^{14} (x+2)^{\frac{1}{2}} - 1 \; dx$$

$$\frac{2}{3}(x+2)^{\frac{3}{2}} - x \Big|_7^{14} = \frac{2}{3}(14+2)^{\frac{3}{2}} - 14 - \left[\frac{2}{3}(7+2)^{\frac{3}{2}} - 7\right]$$

$$= \frac{128}{3} - 14 - [11] = \frac{53}{3} = \boxed{17\ 2/3}$$




Find the area bounded by $x^2 - 4x - 5$ and the x-axis. Sketch the function and label the area.

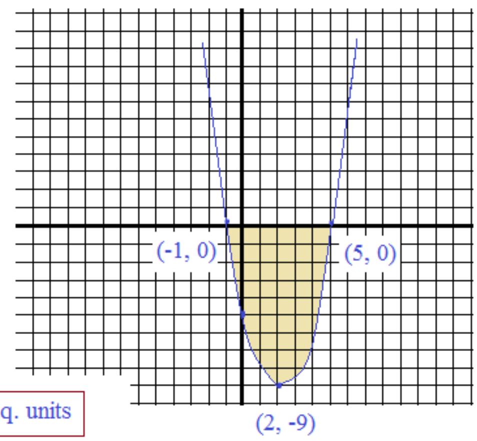
first, find the zeros to determine the bounded range.
Next, construct the definite integral. Then, evaluate.

(factor and set equal to 0): $(x - 5)(x + 1) = 0$ zeros: -1, 5

$$\int_{-1}^{5} x^2 - 4x - 5 \; dx = \frac{x^3}{3} - 2x^2 - 5x \Big|_{-1}^{5}$$

$$= \frac{125}{3} - 50 - 25 - [-1/3 - 2 + 5] = -36 \rightarrow \boxed{36 \text{ sq. units}}$$

(note: the answer is -36 because the function is below the x-axis.. However, the area is *positive* 36)




Find the total area enclosed by the x-axis and the cubic function

$$f(x) = (x - 1)(x - 3)(x - 6)$$

Evaluate each part individually.. then, combine the areas..

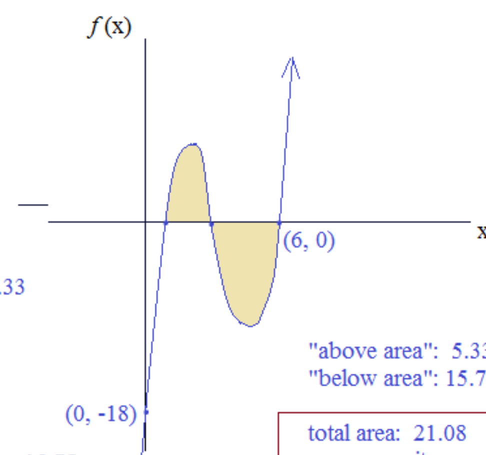
$$(x - 1)(x - 3)(x - 6) = x^3 - 10x^2 + 27x - 18$$

$$\int_1^3 x^3 - 10x^2 + 27x - 18 \; dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{27x^2}{2} - 18x \Big|_1^3$$

$$= \frac{81}{4} - 90 + \frac{243}{2} - 54 - \frac{1}{4} + \frac{10}{3} - \frac{27}{2} + 18 = 5.33$$

$$\int_3^6 x^3 - 10x^2 + 27x - 18 \; dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{27x^2}{2} - 18x \Big|_3^6$$

$$= 324 - 720 + 486 - 108 - \left[\frac{81}{4} - 90 + \frac{243}{2} - 54\right] = -15.75$$




"above area": 5.33
"below area": 15.75

$\boxed{\text{total area: } 21.08 \text{ square units}}$

Using Definite Integrals, find the shaded areas:

SOLUTIONS

A) Identify the boundaries:

-- the lower boundary: x-axis

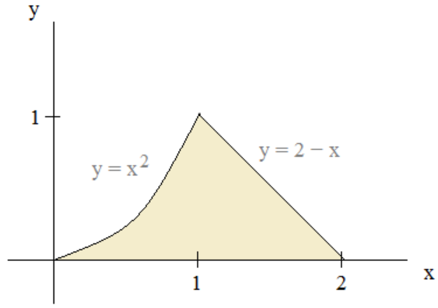
-- the upper boundary:

for $0 \le x \le 1$ $y = x^2$

for $1 \le x \le 2$ $y = 2 - x$

$$\int_0^1 x^2\,dx + \int_1^2 2 - x\,dx$$

Solve to find the area under the functions!




$$\int_0^1 x^2\,dx + \int_1^2 2 - x\,dx$$

$$\left.\frac{x^3}{3}\right|_0^1 + \left.\left(2x - \frac{x^2}{2}\right)\right|_1^2$$

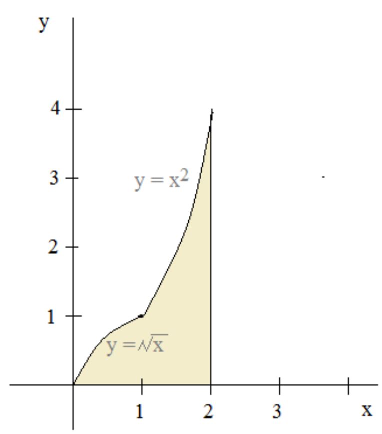
$$\frac{(1)}{3} - \frac{(0)}{3} + \left(2(2) - \frac{(2)^2}{2}\right) - \left(2(1) - \frac{(1)^2}{2}\right)$$

$$\frac{1}{3} + 4 - 2 - (2 - \frac{1}{2}) = \boxed{\frac{5}{6}}$$

B) Again, $y = \sqrt{x}$ and $y = x^2$ meet at (1, 1)

Set up the integrals to find the area under the curves:

$$\int_0^1 \sqrt{x}\,dx + \int_1^2 x^2\,dx$$




$$\int_0^1 \sqrt{x}\,dx + \int_1^2 x^2\,dx$$

$$\left.\frac{x^{3/2}}{3/2}\right|_0^1 + \left.\frac{x^3}{3}\right|_1^2$$

$$\frac{2}{3}(1)^{2/3} - \frac{2}{3}(0)^{2/3} + \frac{(2)^3}{3} - \frac{(1)^3}{3}$$

$$\frac{2}{3} - 0 + \frac{8}{3} - \frac{1}{3} = \boxed{3}$$

C) Strategy: find area of entire rectangle and subtract the area under the curve.

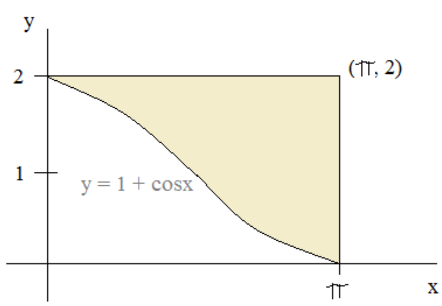
Area of rectangle: lw
length = $\pi$
width = 2 Area$_{\square}$ = $2\pi$

Area under curve:
Find integral for $0 \le x \le \pi$

$$\int_0^{\pi} 1 + \cos x\,dx$$

$$\left. x + \sin x \right|_0^{\pi} = \pi + \sin\pi - (0 + \sin 0)$$

$$= \pi - 0 - 0 - 0 = \pi$$




** Area of entire rectangle = $2\pi$
Area under the curve = $\pi$

Therefore, shaded area is $\pi$

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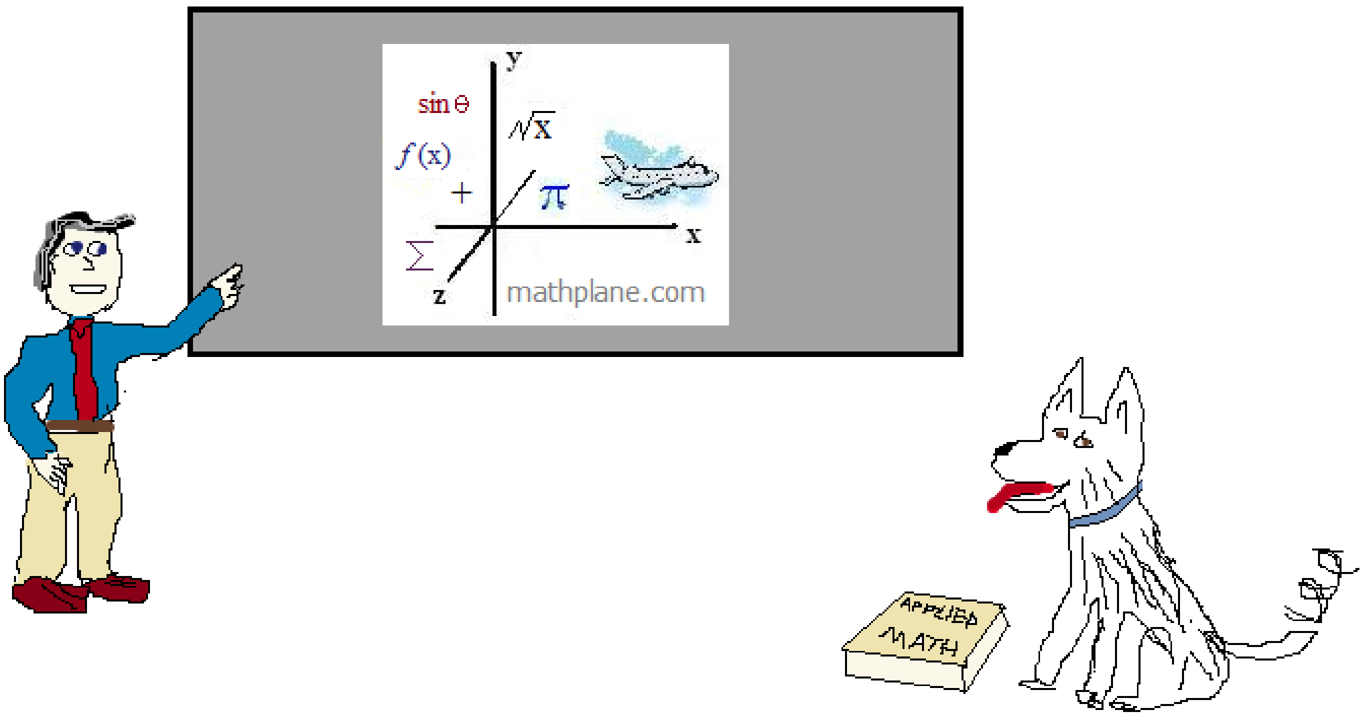
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy

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