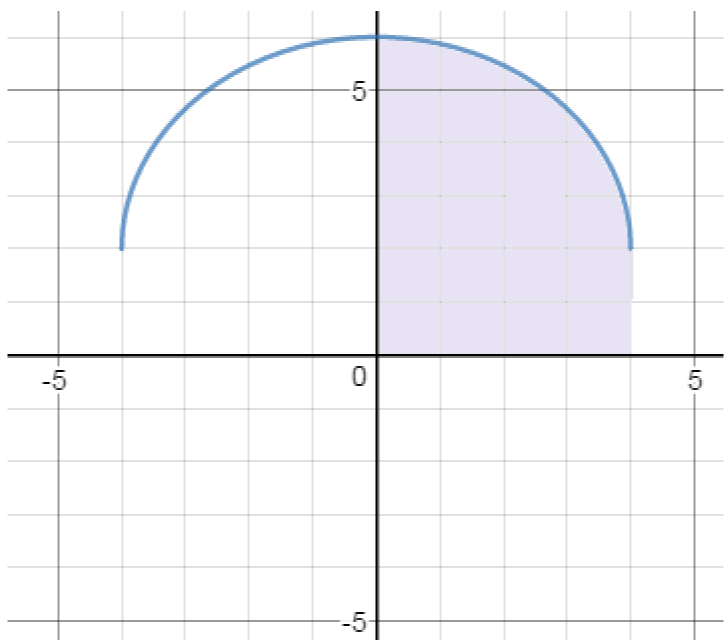
# Definite Integrals:

## Antiderivatives, concepts, and applications

*Notes, Examples, and Practice Exercises*




Topics include velocity, distance traveled, “finding C”, average value, area, geometric shapes, and more.

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Finding antiderivatives and integrals

Integral or Antiderivative?

Antiderivatives and Indefinite Integrals are similar....
If you find an antiderivative, then you find one function.
(there are others)
If you find the indefinite integral, then you find all the functions at once!

A definite integral is different, because it produces an actual value...

Antiderivative: A function that has a given function as its derivative.

$F(x) = x^3 + 7$ is an antiderivative of $f(x) = 3x^2$

Indefinite Integral: A family of functions that have a given function as a common derivative.

$$\int 3x^2 = x^3 + C \quad \text{(where C is any constant)}$$

(Definite) Integral: An integral evaluated over an interval
(which determines area under a curve)

"limit of a Riemann Sum where the partitions approach 0"

$$\int_1^4 3x^2 = x^3 \Big|_1^4 = (4)^3 - (1)^3 = 63$$

Some techniques:

*Example:* $\displaystyle\int_1^4 (x-3)(2x+4)\, dx$

Approach: Multiply the binomials (FOIL)

$$\int_1^4 2x^2 - 2x - 12 \; dx = \frac{2x^3}{3} - \frac{2x^2}{2} - 12x \Big|_1^4$$

$$= \frac{128}{3} - 16 - 48 - \left( \frac{2}{3} - 1 - 12 \right) = -9$$

(Definite) Integral

*Example:* $\displaystyle\int_1^e \frac{x^2 + x + 1}{x}\, dx$

Approach: Divide the rational equation (i.e. separate the terms)

$$\int_1^e \frac{x^2}{x} + \frac{x}{x} + \frac{1}{x}\; dx = \int_1^e x + 1 + \frac{1}{x}\; dx$$

(Definite) Integral

$$= \frac{x^2}{2} + x + \ln x \Big|_1^e = \frac{e^2}{2} + e + \ln e - \left( \frac{1}{2} + 1 + \ln 1 \right)$$

$$\frac{e^2}{2} + e + 1 - (3/2) = \frac{e^2}{2} + e - 1/2$$

*Example:* $\displaystyle\int x\,(\sqrt[3]{x} - \sqrt[4]{x})\; dx =$

Approach: Change radicals to exponent form; then, combine

Indefinite Integral

$$\int x^{\frac{4}{3}} - x^{\frac{5}{4}}\; dx = \frac{x^{\frac{7}{3}}}{\frac{7}{3}} - \frac{x^{\frac{9}{4}}}{\frac{9}{4}} = \frac{3}{7}x^{\frac{7}{3}} - \frac{4}{9}x^{\frac{9}{4}} + C$$

Finding antiderivatives and integrals

*Example:* $\displaystyle\int \frac{\sin(2x)}{\cos^2(2x)}\,dx$

Approach: Change form with trig identity

$$\int \frac{\sin(2x)\cdot 1}{\cos(2x)\cos(2x)}\,dx$$

$$\int \tan(2x)\sec(2x)\,dx =$$

$$\frac{1}{2}\int 2\tan(2x)\sec(2x)\,dx = \frac{1}{2}\sec(2x) + C$$

Approach: U-substitution

$$\int \sin(2x)\cos^{-2}(2x)\,dx \qquad u=\cos(2x)$$

$$du = -2\sin(2x)\,dx$$

$$\int \sin(2x)\,u^{-2}\,dx \qquad \frac{-1}{2}du = \sin(2x)dx$$

$$\int u^{-2}\sin(2x)\,dx$$

$$\int u^{-2}\frac{-1}{2}du = \frac{u^{-1}}{-1}\cdot\frac{-1}{2} = \frac{1}{2u}$$

$$= \frac{1}{2\cos(2x)} = \frac{\sec(2x)}{2} + C$$

*Example:* $\displaystyle\int \frac{1}{x}\,dx = \ln(|x|) + C$ x must be positive! Restrict the Domain

*Example:* $\displaystyle\int \frac{x}{9+x^4}\,dx$ rewrite: $\displaystyle\int \frac{x}{9\left(1+\frac{x^4}{9}\right)}\,dx$

inverse tangent $\displaystyle\frac{1}{9}\int \frac{x}{\left(1+\frac{x^4}{9}\right)}\,dx$ "u" $= \dfrac{x^2}{3}$ $\dfrac{du}{dx} = \dfrac{2}{3}x$

$$\frac{3}{2}\cdot\frac{1}{9}\int \frac{\frac{2}{3}x}{\left(1+\frac{x^4}{9}\right)}\,dx$$

$$\frac{1}{6}\arctan\left(\frac{x^2}{3}\right) + C$$

$$\boxed{\int \frac{dx}{1+x^2} = \arctan(x) + C \qquad \int \frac{1}{1+u^2}\frac{du}{dx} = \arctan(u) + C}$$

*Example:* $\displaystyle\int \frac{1}{1+16x^2}\,dx$ rewrite: $\displaystyle\int \frac{1}{1+(4x)^2}\,dx$ "u" $= 4x$ $\dfrac{du}{dx} = 4$

so, we need a 4 in the equation...

$$\frac{1}{4}\int 4\cdot\frac{1}{1+(4x)^2}\,dx$$

$$\frac{1}{4}\tan^{-1}(4x) + C$$

Antiderivatives and Integrals Quiz



I. First and Second Antiderivatives

1) $f'(x) = 3x^2 + 6x + 5$

If $f(3) = 10$, what is $f(x)$ ?

2) $f''(x) = 2 - 12x + 10x^2$

$f(0) = 4$

$f'(2) = 12$

Find $f(x)$

3) $f'(x) = 20x^3 + 12x^2 - 6x$

Find $f(x)$ and $f''(x)$

4) $f''(x) = x^5 + 4x^3 - x + 1$

Find $f'(x)$ and $f(x)$

5) $\int x^8 + 4\sec x\tan x - \frac{4}{x} + \frac{1}{\sqrt{1-x^2}} + e^{-3x} \; dx$

6) $\int_{-1}^{1} \frac{dx}{1+x^2}$

II. Position, Velocity, Acceleration



1) The particle's movement with respect to time has the following velocity.

$v(t) = \sin t + \cos t$

Find the position function of the particle if $s(0) = 0$

2) $a(t) = 3\cos t - 2\sin t$

$s(0) = 0$

$v(0) = 4$

What is the function $s(t)$ that describes the position of a particle?

What is the function $v(t)$ that describes the velocity of a particle?

3) $a(t) = 2t + 1$

$v(1) = 5$

$s(0) = 7$

What is the position at 2 seconds?

III. Graphing

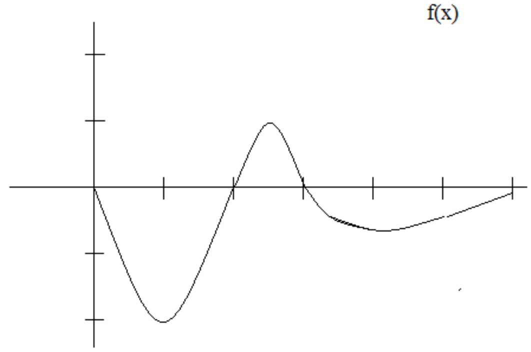
Antiderivatives and Integrals Quiz

1) F is the anti-derivative of f

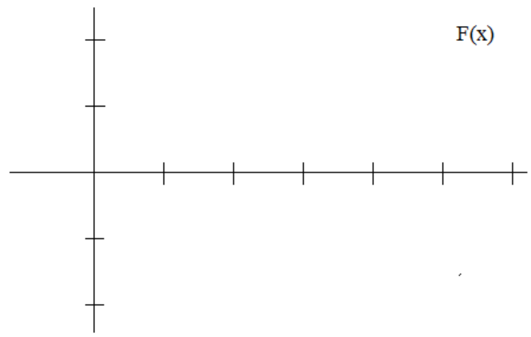
F(0) = 1

Using the graph of f, sketch F

f(x)



F(x)



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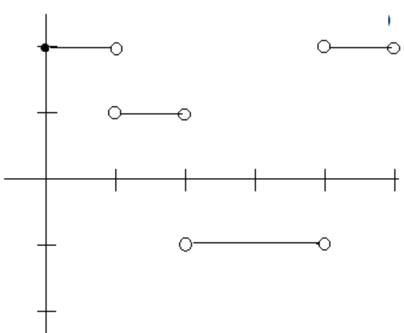
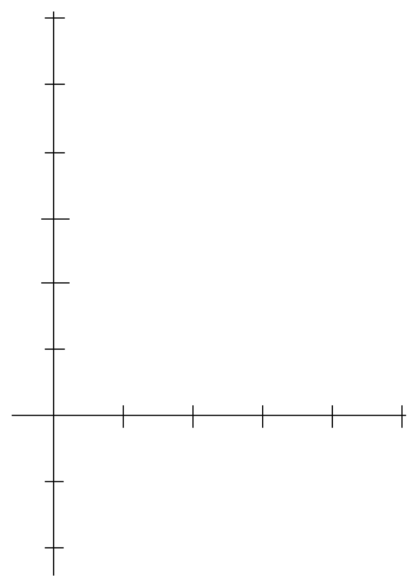


f '(x)

f(x)



if f(0) = 3, using the graph of f '(x), sketch the graph f(x)

## IV. Applications



1) A rock was dropped off of a cliff.
If it hit the ground at a speed of 120 feet per second,
what is the height of the cliff?

2) A math company estimates its marginal cost is $1.92 - .002x$ where x is the number of units.
If the cost of producing 1 unit is 562 dollars, what is the cost of producing 100 units?

3) Find $f(x)$ such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to $f(x)$

Antiderivatives and Integrals Quiz

SOLUTIONS

I. First and Second Antiderivatives

1) $f'(x) = 3x^2 + 6x + 5$

If $f(3) = 10$, what is $f(x)$ ?

The derivative of $f(x)$ is $f'(x)$... So, the antiderivative of $f'(x)$ is $f(x)$...

The antiderivative of $3x^2 + 6x + 5$ is $x^3 + 3x^2 + 5x$

Since $f(3) = 10$, $x^3 + 3x^2 + 5x + C = 10$ when $x = 3$

$(3)^3 + 3(3)^2 + 5(3) + C = 10$

$27 + 27 + 15 + C = 10$

$C = -59$

$\boxed{f(x) = x^3 + 3x^2 + 5x - 59}$

2) $f''(x) = 2 - 12x + 10x^2$

$f(0) = 4$

$f'(2) = 12$

Find $f(x)$

The antiderivative of $f''(x)$ is $f'(x)$... (the derivative of $f'$ is $f''$ )

$f'(x) = 2x - 6x^2 + \frac{10x^3}{3} + C$

since $f'(2) = 12$, $2(2) - 6(2)^2 + \frac{10(2)^3}{3} = 12$

$4 - 24 + 80/3 + C = 12$

$C = 5\ 1/3$

$f'(x) = 2x - 6x^2 + \frac{10x^3}{3} + \frac{16}{3}$

Then, the antiderivative of $f'(x)$ is $f(x)$...

$f'(x) = \frac{10x^3}{3} - 6x^2 + 2x + \frac{16}{3}$

$f(x) = \frac{10x^4}{12} - 2x^3 + x^2 + \frac{16}{3}x + C$

when is $f(0) = 4$ ?

When $C = 4$

$\boxed{\frac{5}{6}x^4 - 2x^3 + x^2 + \frac{16}{3}x + 4}$

3) $f'(x) = 20x^3 + 12x^2 - 6x$

Find $f(x)$ and $f''(x)$

$f''(x)$ is the derivative of $f'(x)$...

$\boxed{f''(x) = 60x^2 + 24x - 6}$

$f(x)$ is the antiderivative of $f'(x)$...

$\boxed{f(x) \text{ is } 5x^4 + 4x^3 - 3x^2 + C}$ (where C is any constant)

4) $f''(x) = x^5 + 4x^3 - x + 1$

Find $f'(x)$ and $f(x)$

$\boxed{f'(x) = \frac{x^6}{6} + x^4 - \frac{x^2}{2} + x + C}$

$\boxed{f(x) = \frac{x^7}{42} + \frac{x^5}{5} - \frac{x^3}{6} + \frac{x^2}{2} + Cx + D}$

where C and D are constants

5) $\int x^8 + 4\sec x\tan x - \frac{4}{x} + \frac{1}{\sqrt{1-x^2}} + e^{-3x}\,dx$

$\boxed{\frac{x^9}{9} + 4\sec x - 4\ln x + \sin^{-1}x - \frac{1}{3}e^{-3x} + C}$

to check: simply take the derivative.
(does it match the indefinite integral? yes!)

6) $\int_{-1}^{1} \frac{dx}{1+x^2}$

answer: use trig substitution....

let $x = \tan(u)$

$\frac{dx}{du} = \sec^2(u)$

$dx = \sec^2(u)\,du$

so $u = \tan^{-1}(x)$

$\int_{-1}^{1} \frac{\sec^2(u)\,du}{1+\tan^2(u)}$

$\int_{-1}^{1} \frac{\sec^2(u)\,du}{\sec^2(u)} = u\Big|_{-1}^{1}$ -----> $\tan^{-1}(x)\Big|_{-1}^{1} = \frac{\pi}{4} - \frac{-\pi}{4} = \boxed{\frac{\pi}{2}}$

II. Position, Velocity, Acceleration

SOLUTIONS

Antiderivatives and Integrals Quiz

1) The particle's movement with respect to time has the following velocity.

$v(t) = \sin t + \cos t$

Find the position function of the particle if $s(0) = 0$

$s'(t) = v(t)$

anti-derivative of $v(t)$ equals $s(t)$

$\int v = s$

$s(t) = -\cos t + \sin t + C$

since $s(0) = 0$

$0 = -\cos(0) + \sin(0) + C$

$0 = -1 + 0 + C$

$1 = C$

$\boxed{s(t) = -\cos t + \sin t + 1}$

2) $a(t) = 3\cos t - 2\sin t$

$s(0) = 0$

$v(0) = 4$

What is the function $s(t)$ that describes the position of a particle?

What is the function $v(t)$ that describes the velocity of a particle?

$a(t)$ is acceleration (second derivative)

$v(t)$ is velocity (first derivative)

$s(t)$ is position (function)

find anti-derivative of a(t)...

$v(t) = 3\sin t + 2\cos t + C$

since $v(0) = 4$,

$4 = 3\sin(0) + 2\cos(0) + C$

$4 = 0 + 2 + C$

$2 = C$

$\boxed{v(t) = 3\sin t + 2\cos t + 2}$

find anti-derivative of v(t)...

$s(t) = -3\cos t + 2\sin t + 2t + C$

since $s(0) = 0$

$0 = -3\cos(0) + 2\sin(0) + 2(0) + C$

$0 = -3 + 0 + 0 + C$

$3 = C$

$\boxed{s(t) = -3\cos t + 2\sin t + 2t + 3}$

3) $a(t) = 2t + 1$

$v(1) = 5$

$s(0) = 7$

What is the position at 2 seconds?

$a(t) = v'(t)$

so, $v(t)$ is anti-derivative of $a(t)$

$v(t) = t^2 + t + C$

$v(1) = (1)^2 + (1) + C$

$5 = 2 + C$

$3 = C$

$v(t) = t^2 + t + 3$

$v(t) = s'(t)$

so, $s(t)$ is anti-deriative of $v(t)$

$s(t) = \frac{t^3}{3} + \frac{t^2}{2} + 3t + C$

$s(0) = 0 + 0 + 0 + C$

$7 = C$

$s(t) = \frac{t^3}{3} + \frac{t^2}{2} + 3t + 7$

at $s(2) = 8/3 + 2 + 6 + 7 = \boxed{17\frac{2}{3}}$

III. Graphing

1) F is the anti-derivative of f

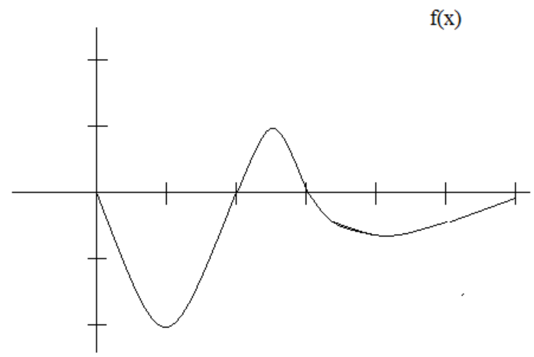
F(0) = 1

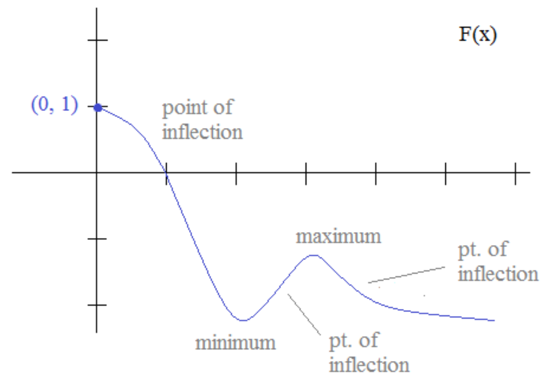
Using the graph of f, sketch F

The max and min points in the f(x) function are points of inflection in the F(x) function

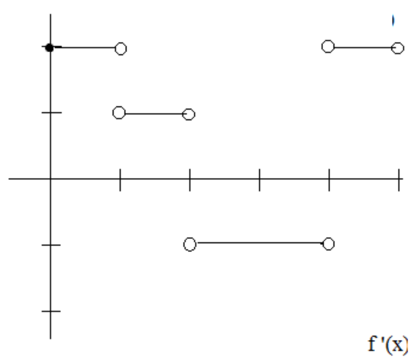
The zeros in the f(x) function are max and min points in the F(x) function

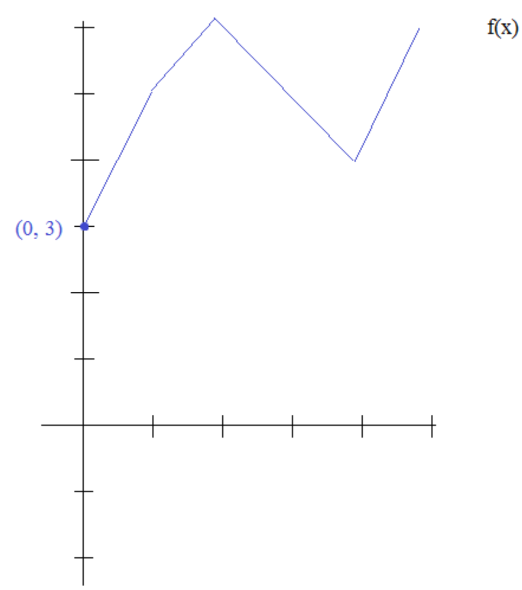
Since the area below the x-axis is larger than the area above the axis, then the graph will end lower than it began... (In fact, (0, 1) is the highest point in the interval..)




2)




if f(0) = 3, using the graph of f '(x), sketch the graph f(x)

IV. Applications

SOLUTIONS

1) A rock was dropped off of a cliff.
If it hit the ground at a speed of 120 feet per second,
what is the height of the cliff?

$h(t) = -16t^2 + v_0 t + s_0$ where $h(t)$ is height in feet
$t$ is time in seconds
$v_0$ is the initial velocity
$s_0$ is the initial height

Since the rock was dropped, the initial velocity is 0

speed of rock: h'(t)... $h'(t) = -32t + v_0 + 0$

$120 = -32t + 0$

$t = -15/4$ The rock hits the ground (at 120 ft/sec) when the time is 15/4

$h(t) = -16(15/4)^2 + 0(15/4) + s_0$

$= -16(225/16) + 0 + s_0$

height must be 225 feet..

2) A math company estimates its marginal cost is $1.92 - .002x$ where x is the number of units.
If the cost of producing 1 unit is 562 dollars, what is the cost of producing 100 units?

Marginal cost is the rate of change of cost, $c'(x) = 1.92 - .002x$

To find the cost function, we need the anti-derivative.

$$c(x) = 1.92x - \frac{.002x^2}{2} + C$$

to find C, we need a value to set...

(1, 562) $562 = 1.92(1) - .001(1)^2 + C$

$562 - 1.919 = C$

$C = 560.081$

$c(100) = 1.92(100) - .001(100)^2 + 560.081$

$= 192 - 10 + 560.081 = \boxed{742.081}$

3) Find $f(x)$ such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to $f(x)$

First, we want to find $f(x)$...

find the anti-derivative of $f'(x)$: $\frac{x^4}{4} + C$

then, figure out C (i.e. what value would make the curve have the tangent $x + y = 0$)

At the tangent point, the slope of the line and the slope of $f(x)$ will be the same...

$y = -x$... slope is -1... therefore, $f'(x) = -1$...

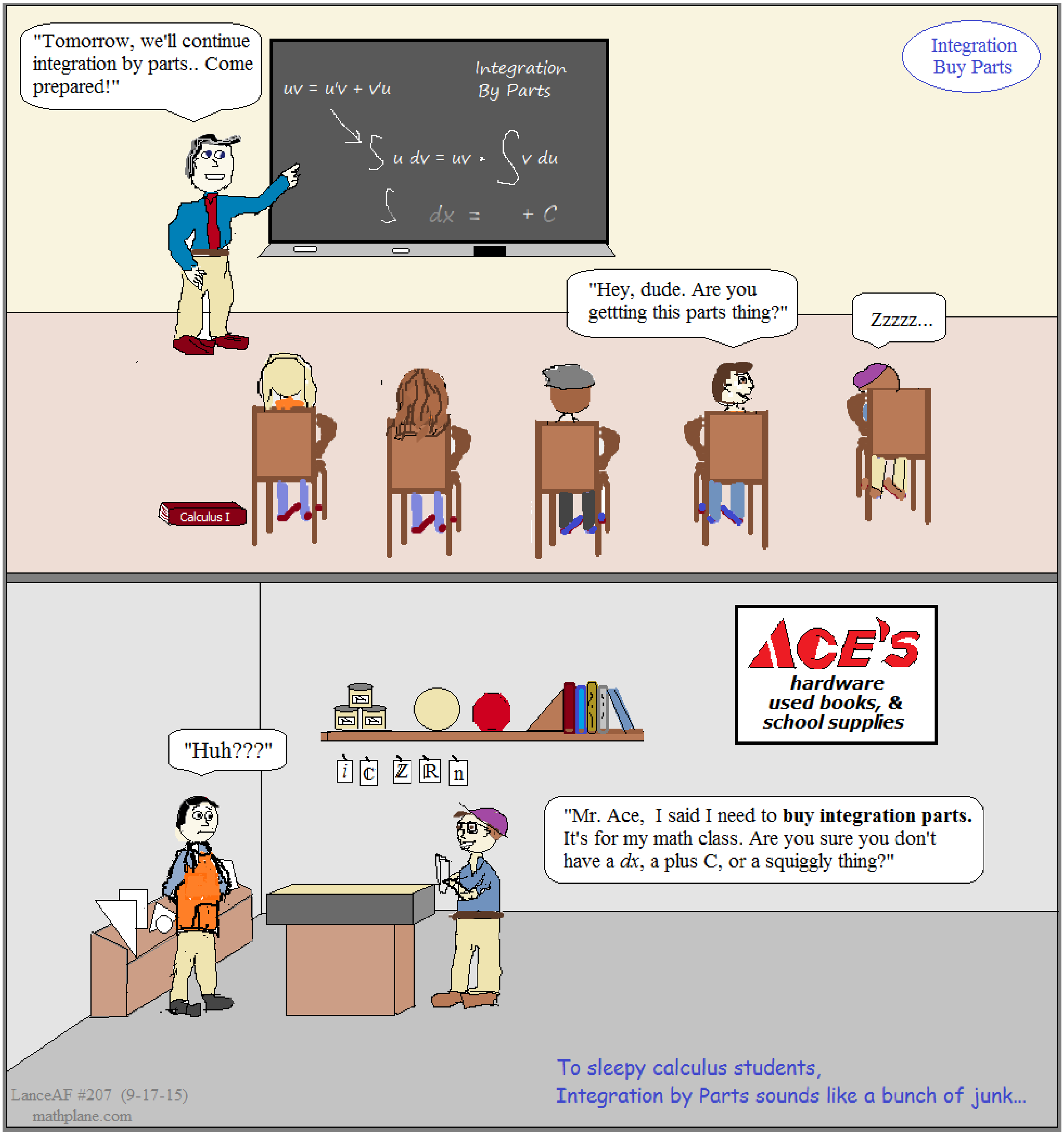
this occurs at $x = -1$.... If $x = -1$, then $y = 1$

$$\frac{x^4}{4} + C = y$$

$$\frac{(-1)^4}{4} + C = y$$

$$\frac{1}{4} + C = 1 \qquad C = 3/4$$

$$\boxed{f(x) = \frac{x^4}{4} + \frac{3}{4}}$$


"Tomorrow, we'll continue integration by parts.. Come prepared!"
$uv = u'v + v'u$
Integration By Parts
$\int u\, dv = uv - \int v\, du$
$\int \quad dx = \quad + C$
Integration Buy Parts
"Hey, dude. Are you gettting this parts thing?"
Zzzzz...
Calculus I
ACE'S hardware used books, & school supplies
"Huh???"
$i$ C Z R n
"Mr. Ace, I said I need to **buy integration parts.** It's for my math class. Are you sure you don't have a *dx*, a plus C, or a squiggly thing?"
To sleepy calculus students, Integration by Parts sounds like a bunch of junk...
LanceAF #207 (9-17-15)
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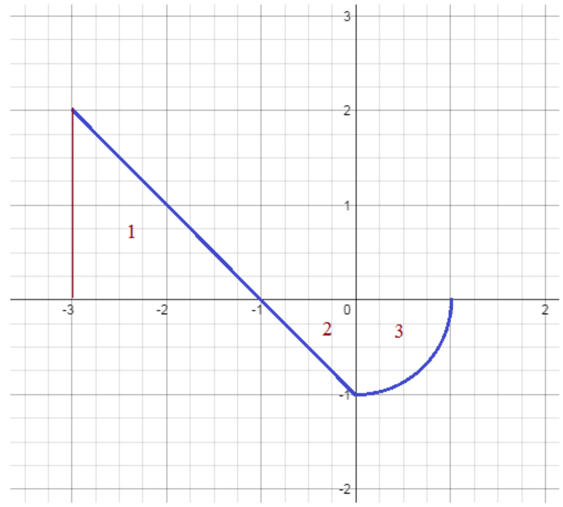
*Example:*

$$f(x) = \begin{cases} -x-1 & \text{if } -3 \le x < 0 \\ -\sqrt{1-x^2} & \text{if } 0 \le x \le 1 \end{cases}$$




a) Evaluate the integral $\displaystyle\int_{-3}^{1} f(x)\,dx$ $\qquad 2 - \frac{1}{2} - \frac{\pi}{4}$

(Area of triangle 1) − (Area of triangle 2) − (Area of quarter circle 3)

above x-axis (+) below x-axis (-) below x-axis (-)

b) On the interval [-3, 1], what is the area of the region bordered by $f(x)$ and the x-axis? $\qquad 2 + \frac{1}{2} + \frac{\pi}{4}$

(Area 1) + (Area 2) + (Area 3)

All the area values are positive!

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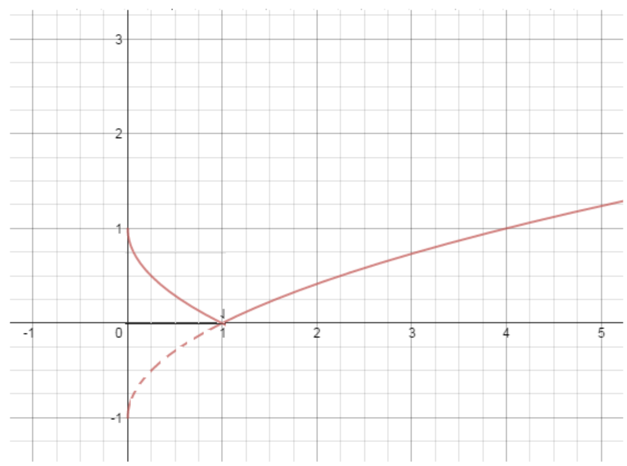
*Example:*

Evaluate the integral $\displaystyle\int_0^4 |\sqrt{x} - 1|\,dx$



$$\int_0^1 \sqrt{x} - 1 \; dx \; + \; \int_1^4 \sqrt{x} - 1 \; dx$$

$-$ $\qquad$ $+$

$$\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - x \,\right|_0^1 = -1/3 \qquad \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - x \,\right|_1^4 = 5/3$$

1/3 and 5/3 = 2

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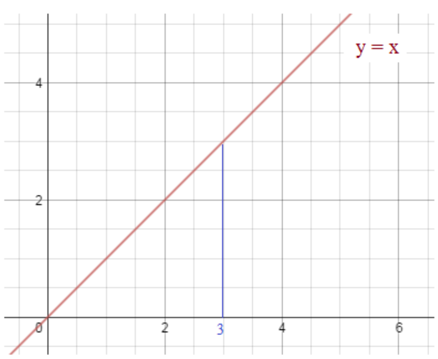


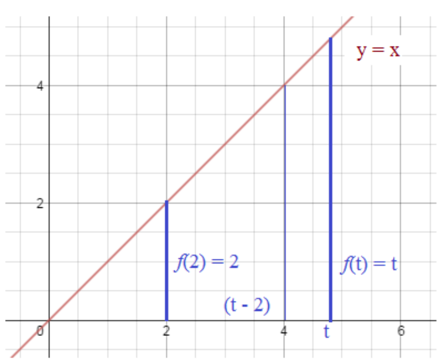


*Example:* Find the area between the line and the x-axis:

a) between the y-axis and x = 3

(Triangle) Area = $\frac{1}{2}$ (base)(height) $\qquad \displaystyle\int_0^3 x\,dx = \frac{9}{2}$

b) between the y-axis and any x = t

$$\int_0^t x\,dx = \left.\frac{x^2}{2}\right|_0^t = \frac{1}{2}t^2$$

c) between x = 2 and x = 4

(Trapezoid) Area = $\frac{1}{2}(b_1 + b_2)$(height) $\qquad \displaystyle\int_2^4 x\,dx = 6$

d) between x = 2 and any t (that is greater than 2)

$$\int_2^t x\,dx = \left.\frac{x^2}{2}\right|_2^t = \frac{1}{2}t^2 - 2 \qquad \frac{1}{2}(f(2) + f(t))(t-2)$$

$$= \frac{1}{2}(t^2 - 4) \text{ or } \frac{1}{2}(t+2)(t-2)$$

Average Value and Integrals

Average area under a curve

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

(Integral) Mean Value Theorem

If function $f(x)$ is continuous on the interval [a, b], then there exists a number "c" in [a, b] where

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

*Example:* $f(x) = x^2 + 1$ on the interval [0, 2]

a) Find the average value of the function

b) Find the value "c" guaranteed by the "mean value theorem"

a) to find average value.... $\displaystyle\int_0^2 x^2 + 1\,dx = \left.\frac{x^3}{3} + x\right|_0^2 = \frac{8}{3} + 2 - (0/3 + 0) = \frac{14}{3}$ area under the curve (i.e. total value on interval [0, 2])

"divide by the length of the interval" $\dfrac{\frac{14}{3}}{(2-0)} = \dfrac{7}{3}$ average value

b) since the function is continuous and closed on the interval, there must be a value "c" such that $f(c)$ = average value

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

so, where does the function equal $\frac{7}{3}$ ?

$$\frac{7}{3} = x^2 + 1$$

$x = \dfrac{2\sqrt{3}}{3}$ approx. 1.15

We don't include -1.15 (because it is not in the interval )

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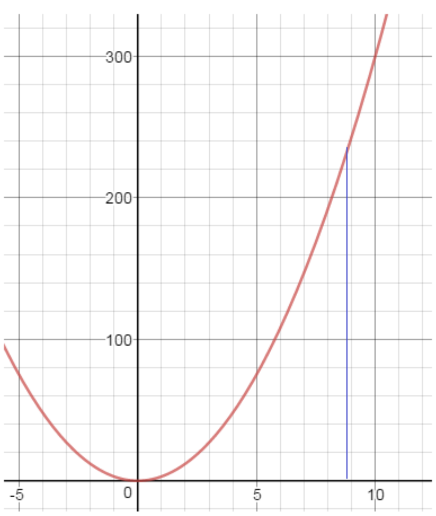
*Example:* $f(x) = 3x^2$ has an average value of 100 on the closed interval [2, k]. What is k?




$$\int_2^k 3x^2\,dx = \left. x^3 \right|_2^k = k^3 - (2)^3 = k^3 - 8$$ (area under the curve)

*average value* $= \dfrac{k^3 - 8}{(k-2)} = 100$ $\qquad \dfrac{(k-2)(k^2+2k+4)}{(k-2)} = 100$

$(k^2 + 2k + 4) = 100$

$k^2 + 2k - 96 = 0$

(quadratic formula) $k = \cancel{-10.85}$ or $\boxed{8.85}$

On the closed interval [2, 8.85], the average value of $3x^2$ is 100

Notice, the average value 100 occurs when c = 5.77 which is in the interval [0, 8.85] (Integral Mean Value Theorem)

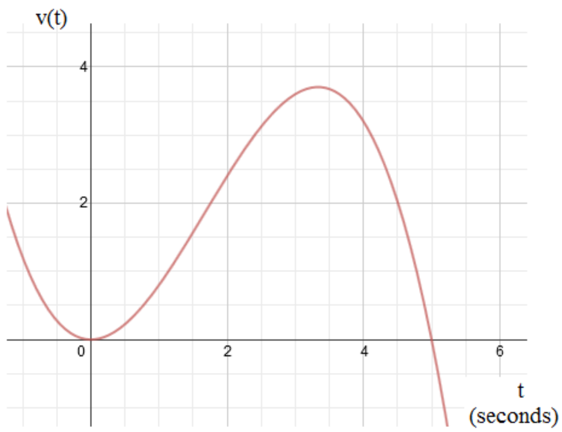
## Distance Traveled

*Example:* A Pinewood Derby car slides down the ramp and stops after 5 seconds. The *velocity* of the car can be modeled by the equation

$$v(t) = t^2 - 0.2t^3$$

t = time in seconds
v(t) = feet/second

How far did the car travel?




The graph shows the (cubic) velocity equation.

At every point on the curve, the velocity of the car is shown at that instant. Therefore, the area under the curve, will represent the total distance traveled. (each instant added up)

$$\int_0^5 t^2 - 0.2t^3 \, dt$$

$$\left. \frac{t^3}{3} - \frac{0.2t^4}{4} \right|_0^5 = \frac{125}{3} - 31.25 = \boxed{10.417}$$

---

## Acceleration, Velocity, Position

*Example:* A bug travels along the x-axis.
The model of its velocity is $v(t) = t^2 - 8t + 15$

where $0 \le t \le 18$ minutes, and the initial position $s(0) = 43$ feet

What is the position of the bug when its acceleration is 6 inches per minute?

To find the acceleration of the bug, we must find v'(t).

$v'(t) = 2t - 8$ $\quad 6 = 2t - 8$, $t = 14$ $\quad$ The accleration occurs at 14 minutes

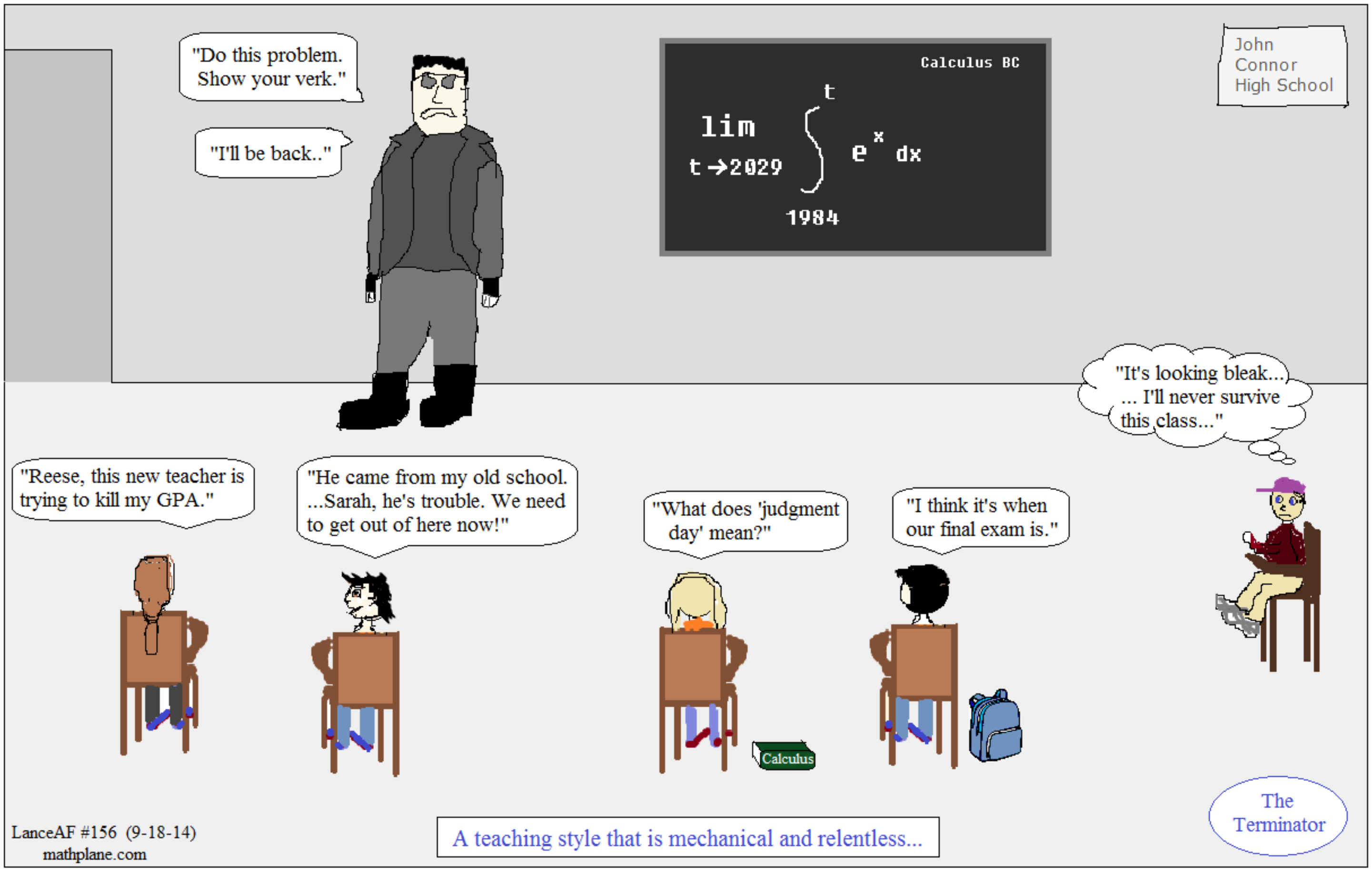
Then, to find the position function, we must find the antiderivative of $v(t)$

$$s(t) = \int t^2 - 8t + 15 = \frac{t^3}{3} - 4t^2 + 15t + C$$

since $s(0) = 43$, $C = 43$

$$s(t) = \frac{t^3}{3} - 4t^2 + 15t + 43$$

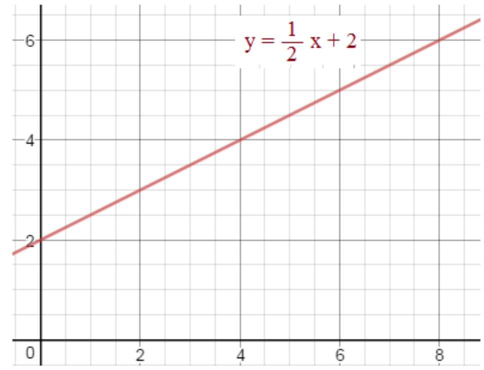
then, $s(14) = 914.67 - 784 + 210 + 43 = \boxed{383.67}$




Practice Quiz-→

Integral and Area Concepts

1) Find the area of the region bordered by the line and the x-axis:

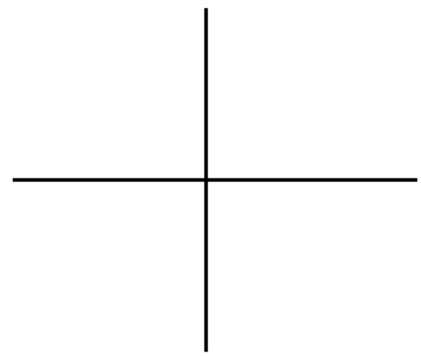



a) between the y-axis and x = 6

b) between x = 2 and x = 6

c) between x = 2 and any t (that is greater than 2)

2) $f(x)$ is an ODD function $\quad \int_0^7 f(x)\,dx = 20 \quad\quad \int_0^{10} f(x)\,dx = 14$



a) Sketch a (possible) graph of $f(x)$ which includes the interval [-10, 10]

b) Evaluate the following:

$\int_7^{10} f(x)\,dx = \quad\quad \int_{-7}^{0} f(x)\,dx = \quad\quad \int_0^{-7} f(x)\,dx = \quad\quad \int_{-10}^{10} f(x)\,dx =$

c) What is the average value between x = -7 and x = 7?

Between x = 0 and x = 10?

Between x = 7 and x = 10?

d) Evaluate the following:

$\int_0^7 f(x) + 3\,dx = \quad\quad \int_0^7 -f(x) - 4\,dx = \quad\quad \int_0^{10} 4f(x)\,dx =$

3) A particle moves along the x-axis.
At any time (t), its *acceleration* is $a(t) = 6t - 18$.

At $t = 0$, the velocity is 24. $v(0) = 24$
At $t = 1$, the position of the particle is 20 $x(1) = 20$

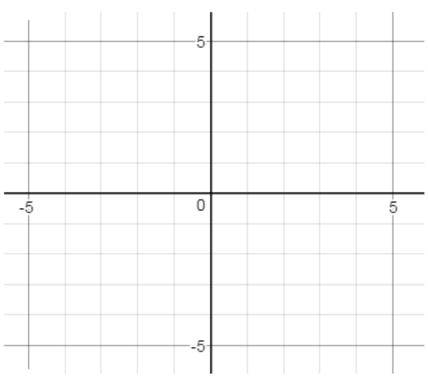
a) What is the function describing the velocity of the particle?

b) Write an expression for the position of the particle in terms of time (t).

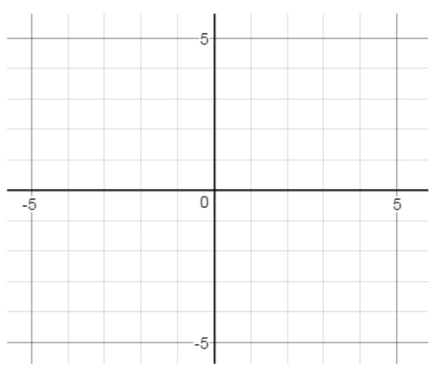
c) When is the particle at rest?

4) Graph and Evaluate:

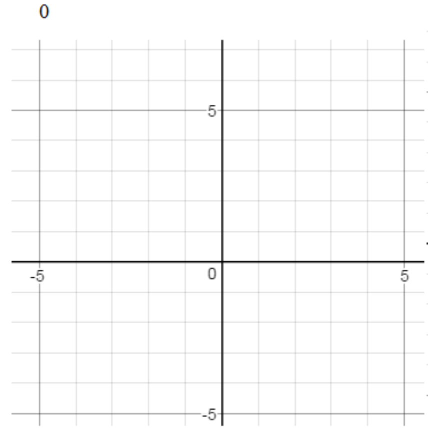
a) $\int_{-2}^{1} |x| \, dx$



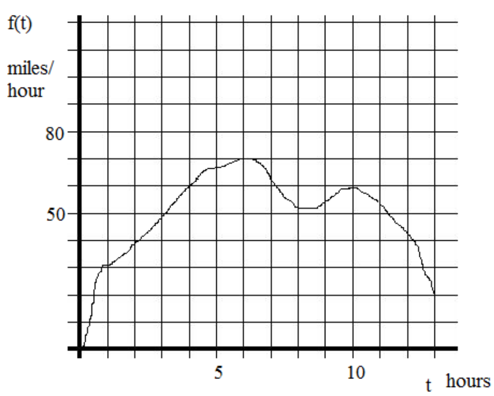

b) $\int_{-4}^{0} \sqrt{16 - x^2} \, dx$




c) $\int_{0}^{4} 2 + \sqrt{16 - x^2} \, dx$



5) The following graph shows the speed of a car traveling cross country...
Estimate how far the car traveled between hour 2 and hour 8.
(***Use trapezoid rule with 6 intervals to approximate)




6) The acceleration of a particle can be modeled by the function $a(t) = 2t - 8$ where t is time in seconds
If the velocity of the particle at 1 second is 5 ft/sec,

a) find the velocity function $v(t)$.

b) Find the *displacement* of the particle over the first 6 seconds.

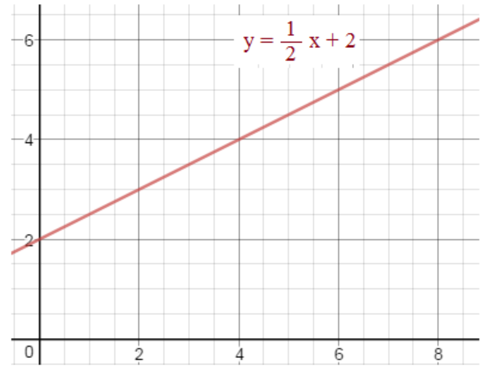
c) Find the *distance traveled* by the particle over the first 6 seconds.

1) Find the area of the region bordered by the line and the x-axis: SOLUTIONS



a) between the y-axis and x = 6

area of the trapezoid = $\frac{1}{2}(2+5)(6) = 21$ $\quad \int_0^6 \frac{1}{2}x + 2\, dx = \left. \frac{x^2}{4} + 2x \right|_0^6 = \boxed{21}$




b) between x = 2 and x = 6

area of the trapezoid = $\frac{1}{2}(3+5)(4) = 16$ $\quad \int_2^6 \frac{1}{2}x + 2\, dx = \left. \frac{x^2}{4} + 2x \right|_2^6 = \boxed{16}$

c) between x = 2 and any t (that is greater than 2)

base1 base2 height

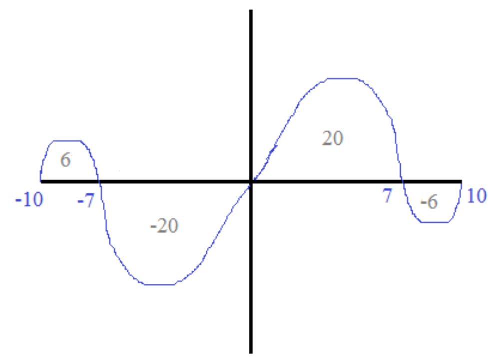
$\frac{1}{2}(f(2) + f(t))(t-2)$

$\boxed{\frac{1}{2}(3 + f(t))(t-2)}$

$\int_2^t \frac{1}{2}x + 2\, dx = \left. \frac{x^2}{4} + 2x \right|_2^t = \frac{1}{4}t^2 + 2t - 5$

$\boxed{\frac{1}{4}(t+10)(t-2)}$

2) $f(x)$ is an ODD function $\quad \int_0^7 f(x)\, dx = 20 \quad \int_0^{10} f(x)\, dx = 14$




a) Sketch a (possible) graph of $f(x)$ which includes the interval [-10, 10]

Odd function reflects over the origin...

Value between 0 and 7 is 20... between 0 and 10 is 14... Therefore, between 7 and 10 is -6

b) Evaluate the following:

$\int_7^{10} f(x)\, dx = \boxed{-6} \quad \int_{-7}^{0} f(x)\, dx = \boxed{-20} \quad \int_0^{-7} f(x)\, dx = -\int_{-7}^{0} f(x)\, dx = \boxed{20} \quad \int_{-10}^{10} f(x)\, dx = \boxed{0}$

c) What is the average value between x = -7 and x = 7? $\frac{0}{14} = \boxed{0}$

Between x = 0 and x = 10? $\frac{14}{10} = \boxed{7/5}$

Between x = 7 and x = 10? $\frac{-6}{3} = \boxed{-2}$

d) Evaluate the following:

$\int_0^7 f(x) + 3\, dx = \int_0^7 f(x) + \int_0^7 3\, dx$

$20 + 21 = \boxed{41}$

$\int_0^7 -f(x) - 4\, dx =$

$-20 - 28 = \boxed{-48}$

$\int_0^{10} 4f(x)\, dx =$

$4 \cdot 14 = \boxed{56}$

3) A particle moves along the x-axis. SOLUTIONS

Integral and Area Concepts

At any time (t), its *acceleration* is $a(t) = 6t - 18$.

At $t = 0$, the velocity is 24. $v(0) = 24$
At $t = 1$, the position of the particle is 20 $x(1) = 20$

a) What is the function describing the velocity of the particle?

Since the derivative of velocity leads to acceleration,
the anti-derivative of acceleration will lead to velocity!

$\int 6t - 18\, dt = 3t^2 - 18t + C$ To determine C, use the given point (0, 24)

then, $24 = 3(0)^2 - 18(0) + C$

$24 = C$

$\boxed{v(t) = 3t^2 - 18t + 24}$

b) Write an expression for the position of the particle in terms of time (t).

Since the derivative of position is velocity, the anti-derivative of velocity is position...

$\int 3t^2 - 18t + 24\, dt = t^3 - 9t^2 + 24t + C$ To determine C, use the position point (1, 20)

then, $20 = (1)^3 - 9(1)^2 + 24(1) + C$

$20 = 16 + C$

$4 = C$

$\boxed{x(t) = t^3 - 9t^2 + 24t + 4}$

c) When is the particle at rest?

The particle is at rest when its velocity is 0. (rate of change = 0)

velocity function: $v(t) = 3t^2 - 18t + 24$

$0 = 3t^2 - 18t + 24$

$0 = 3(t^2 - 6t + 8)$

$0 = 3(t - 2)(t - 4)$

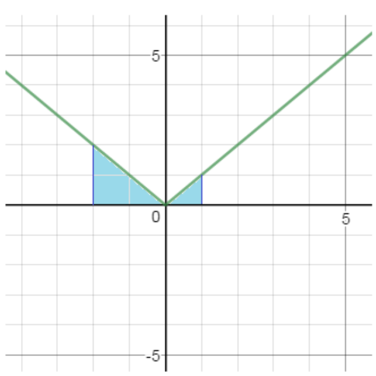
$\boxed{t = 2 \text{ and } 4}$

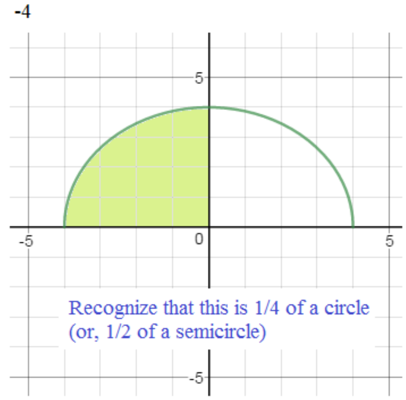
4) Graph and Evaluate:

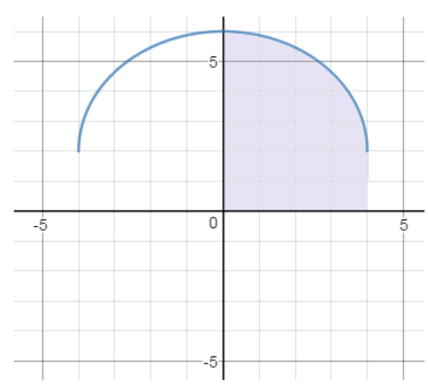
a) $\int_{-2}^{1} |x|\, dx$ area of triangles: $2 + 1/2 = \boxed{5/2}$

b) $\int_{-4}^{0} \sqrt{16 - x^2}\, dx$ $\frac{1}{4}\pi(4)^2 = \boxed{4\pi}$

c) $\int_{0}^{4} 2 + \sqrt{16 - x^2}\, dx$ $\boxed{8 + 4\pi}$

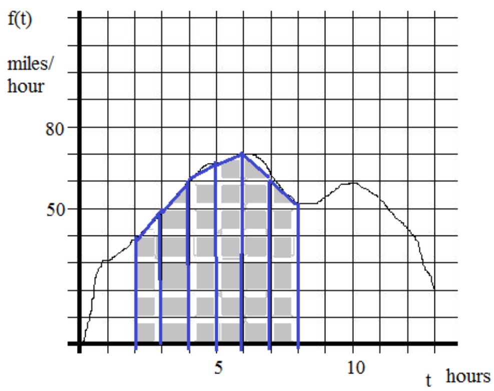




SOLUTIONS

5) The following graph shows the speed of a car traveling cross country...
Estimate how far the car traveled between hour 2 and hour 8.
(***Use trapezoid rule with 6 intervals to approximate)




$$\int_2^8 f(t)\,dt$$

(Using integrals to find distance traveled)

NOTE: the y-axis is "miles/hour" and the x-axis is "hours"...

so, if you multiply the x by the y, you get "miles"! Each square unit is miles..

6 intervals: $f(2) = 40$
$f(3) = 50$
$f(4) = 60$
$f(5) = 67$
$f(6) = 70$
$f(7) = 63$
$f(8) = 52$

$dt = 1$

$$\frac{1}{2}(1)\,[40 + 100 + 120 + 134 + 140 + 126 + 52]$$

$\boxed{\text{approx. } 356 \text{ miles...}}$

6) The acceleration of a particle can be modeled by the function $a(t) = 2t - 8$ where t is time in seconds

If the velocity of the particle at 1 second is 5 ft/sec,

a) find the velocity function v(t).

$$v(t) = \int a(t)$$
$$= \int 2t - 8\,dt$$
$$= t^2 - 8t + C$$

$v(1) = 5$

so, $5 = (1)^2 - 8(1) + C$

$C = 12$

$\boxed{v(t) = t^2 - 8t + 12}$

b) Find the *displacement* of the particle over the first 6 seconds.

$$\int_a^b v(t) = \qquad \int_0^6 t^2 - 8t + 12\,dt \qquad \left.\frac{t^3}{3} - 4t^2 + 12t\right|_0^6 = \frac{216}{3} - 144 + 72 = \boxed{0}$$

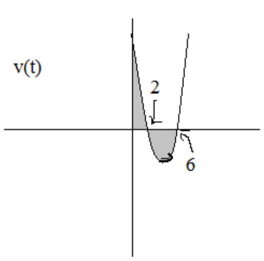
c) Find the *distance traveled* by the particle over the first 6 seconds.

Find positive intervals and negative intervals...

$t^2 - 8t + 12 = 0$

$(t - 2)(t - 6) = 0$ $\qquad t = 2$ and $t = 6$ the particle changes direction!

$$\left|\int_0^2 t^2 - 8t + 12\,dt\right| + \left|\int_2^6 t^2 - 8t + 12\,dt\right| = \left.\frac{t^3}{3} - 4t^2 + 12t\right|_0^2 = \frac{8}{3} - 16 + 24 - (0 - 0 + 0) = \frac{32}{3}$$



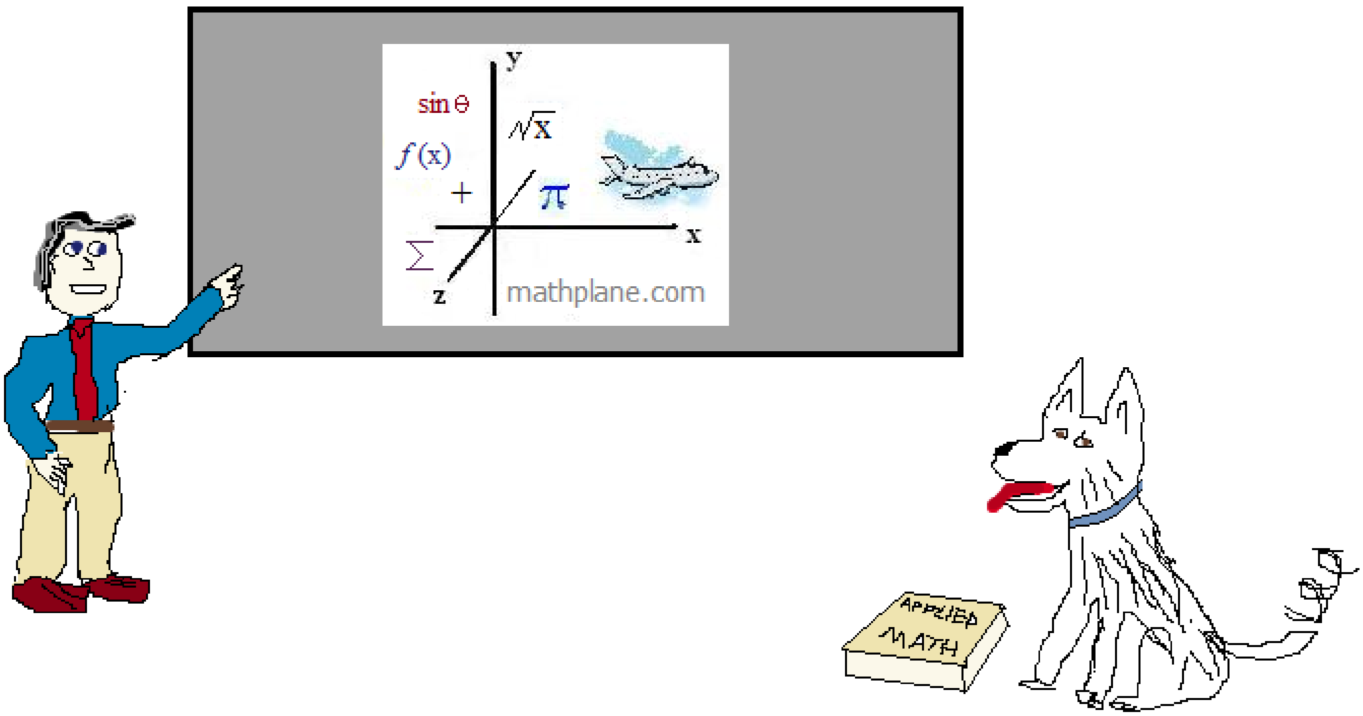

$$\left|\left.\frac{t^3}{3} - 4t^2 + 12t\right|_2^6\right| = \left|\frac{216}{3} - 144 + 72 - \left(\frac{8}{3} - 16 + 24\right)\right| = \left|\frac{-32}{3}\right|$$

distance traveled: $\boxed{\frac{64}{3}}$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers.

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