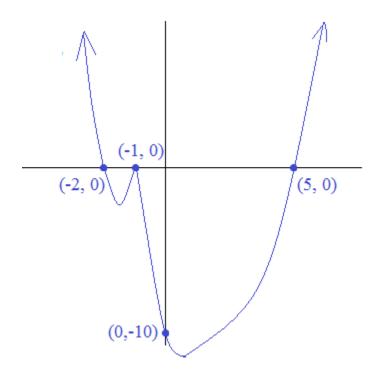
Algebra II Semester Finals Review

Practice Test and Solutions



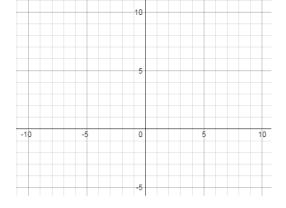
Topics include polynomials, exponents, matrices, imaginary numbers, piecewise functions, and more.

line A: 3x + 5y = 15 (standard form)

- a) What is the x-intercept? y-intercept?
- b) Graph line A
- c) Line B is parallel to A and goes through (-4, 10). What is the equation of Line B

in point slope form:

in slope intercept form:



II. Quadratic

parabola A: $x^2 + 8x - 20$ (standard form)

- a) Write the equation of A in vertex form
- b) Write the equation of A in factored (intercept) form
- c) Graph the curve. Label the axis of symmetry, vertex, any x-intercepts, and y-intercept

Miscellaneous:

- 1) What is the vertex of $3x^2 + 6x 90$
- 2) Describe the solutions of the following: real, imaginary, rational, irrational.....

$$x^2 + 5x + 12$$

$$3x^2 + 12x + 9$$

$$x^2 - 14x + 49$$

$$-2(x^2+4)-11$$

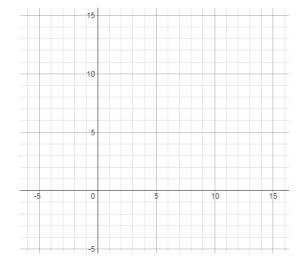
$$-x^2 + 4x + 15$$

$$(x + 4)(x - 2)$$

$$y = -3|x - 5| + 12$$

$$y = -3|x - 5| + 12$$

a) Identify the vertex, axis of symmetry, y-intercept, and any x-intercepts



- b) Graph the equation
- c) What is the domain? Range?

IV: Polynomials

Describe the end behavior of the following:

$$f(x) = (x+3)^2(x-13)$$

$$g(x) = 4x - 2x^4 + 5x^2$$

$$h(x) = -(x^6 + 7x^3 + 2)$$

Find a polynomial of degree 4 with integer coefficients and zeros 3i and 4, where 4 is a double zero.

Graph $(x - 5)(x + 1)^2(x + 2)$

Label the zeros and y-intercept

Graph
$$x^3 + 6x^2 - 3x - 18$$

Identify the linear factors

	-	

$$f(x) = x^2 + 4$$

Find the following:

$$g(x) = 5x + 1$$

5)
$$f(h(6))$$

$$h(x) = 7$$

6)
$$h(g(5))$$

7)
$$(g \circ g)(x)$$

8)
$$f(g(x))$$

VI. Exponents and Roots

Evaluate:

$$\frac{1}{64^{3}}$$

$$\frac{2}{32}$$

$$\left(\frac{9}{25}\right)^{-\frac{1}{2}}$$

Simplify:

$$(3x^3 y^{-4})^2$$

$$\left(\frac{12x^2y^{-3}}{3x^4yz}\right)^{-1}$$

$$\frac{4a^2b^5}{7bc^{-1}} \cdot (2ac^3)^{-3}$$

VII. Factoring

Simplify:
$$3x^2 - 75$$
 $x^4 + 3x^2 - 4$

$$v^4 + 3v^2 - 4$$

$$2x^3 + 54$$

$$x^3 - 7x^2 - 2x + 14$$

Solve:

$$2x^2 + 10x + 8 = 0$$
 $2x^2 - 5x = -2$ $x^2 + 5x + 19 = 0$

$$2x^2 - 5x = -2$$

$$x^2 + 5x + 19 = 0$$

- 1) What are the *possible* rational roots of the polynomial $2x^5 + 3x^2 + 7x 6$
- 2) What is the remainder of $g(x) = x^{100} + x^{40} + 2x^{25} \div (x-1)$?
- 3) Using synthetic division, find $x^4 3x^3 + 7x + 8 \div (x + 2)$
- 4) Find $x^4 + 5x^3 11x^2 + 2x + 1 \div (x^2 + 2)$

IX. Matrices

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 6 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

- 1) Identify the following elements: from A: a_{21} from B: b_{12} from C: c_{12}
- 2) What are the dimesions of each matrix? A: B: C:
- 3) Find the following:

$$A + 2C = AB = BA = AC =$$

X. Word Problems

Algebra II Review Questions/Topics

A company invested \$100,000 in 3 funds.
 Last year, the growth fund earned 12%, the income fund earned 8%, and the money market fund yielded 5%. So, after one year, the company had \$109,000.

 If the company invested twice as much in the income fund as the money market fund, how much did it invest in each fund?

2) A math test consists of number problems and graphing problems. Number problems are worth 6 points each, and graphing problems are worth 10 points each. You can accurately solve a number problem in 2 minutes and a graphing problem in 4 minutes. Assuming you have 40 minutes and may choose no more than 12 problems, to answer, how many of each type should you solve to get the highest score? What is that highest score?

XI. Imaginary and Complex Numbers

1)
$$(5i-6)^2 =$$

2)
$$2i^{25} + 4i^{11} + 2i^{20} =$$

$$\frac{3i+4}{4i-9} =$$

XII. Piecewise Functions

$$h(t) = \begin{cases} \sqrt{(-t)} , & \text{if } t < -3 \\ 5 , & \text{if } 0 \le t < 5 \\ -2t , & \text{if } 5 \le t \end{cases}$$

$$h(-4) =$$

$$h(5) =$$

$$h(10) =$$



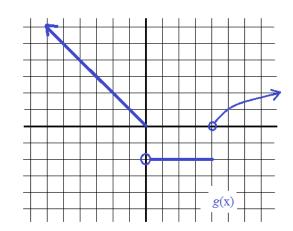
What is the domain and range of each function? ?

h(x): Domain -

Range -

g(x): Domain -

Range -



$$g(-3) =$$

$$g(4) =$$

$$g(-20) =$$

XIII. Three more questions

Change to vertex form
 (i.e complete the square)

$$y = 2x^2 + 8x + 11$$

2) Solve and graph on number line

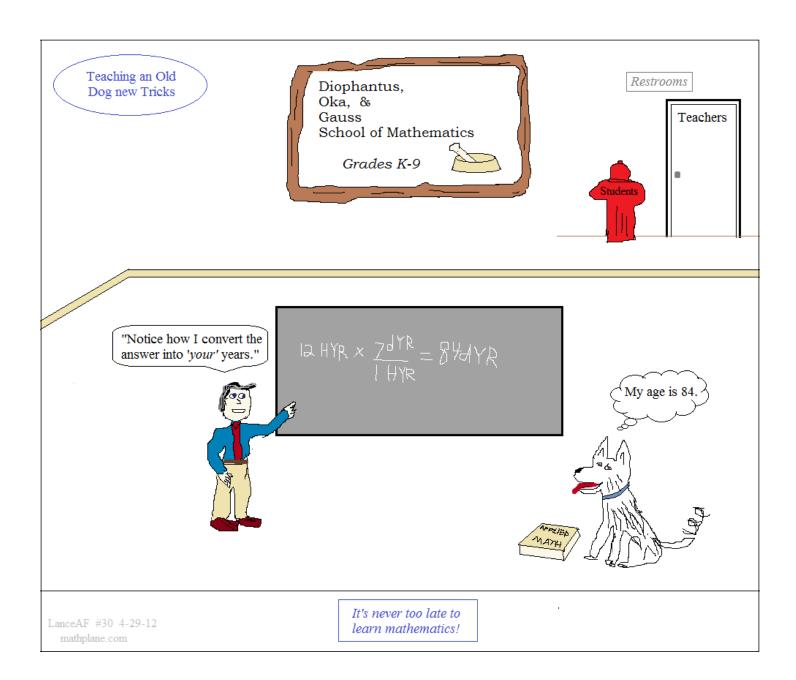
$$3|x-5|-7>8$$



3) Circle the irrational numbers

.243
$$\sqrt{7}$$
 3*i* .23 $\overline{23}$

Good luck on your final exam!



SOLUTIONS -→

point: (-4, 10) slope: parallel to line A

> slope of line A is 5y = -3x + 15 y = (-3/5)x + 15

line A: 3x + 5y = 15 (standard form)

a) What is the x-intercept? y-intercept?

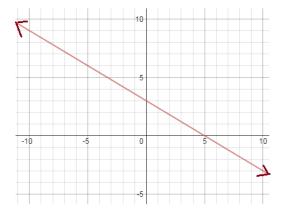
b) Graph line A
$$\begin{array}{c} x\text{-intercept: where }y=0 & y\text{-intercept: where }x=0 \\ 3x+5(0)=15 & 3(0)+5y=15 \\ x=5 & y=3 \\ \hline (5,0) & (0,3) \\ \end{array}$$

c) Line B is parallel to A and goes through (-4, 10). What is the equation of Line B

what is the equation of Line B For line, we need a point and the slope in point slope form:
$$(y - 10) = \frac{-3}{5}(x + 4)$$
 point: (-4, 10) slope: parallel to line A

in slope intercept form: $y - 10 = \frac{-3x}{5} - \frac{12}{5}$

$$y = \frac{-3}{5}x + \frac{38}{5}$$



II. Quadratic

parabola A:
$$x^2 + 8x - 20$$
 (standard form)

a) Write the equation of A in vertex form

Complete the square! separate the x's:
$$x^2 + 8x - 20$$

complete the square: $(x^2 + 8x + 16) - 20 - 16$

factor and collect terms: $(x + 4)^2 - 36$

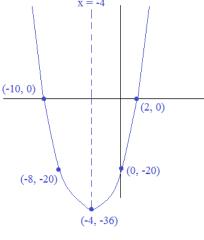
b) Write the equation of A in factored (intercept) form

$$(x + 10)(x - 2)$$

c) Graph the curve. Label the axis of symmetry, vertex, any x-intercepts, and y-intercept

vertex:
$$(-4, -36)$$

x-intercepts: $(-10, 0)$ $(2, 0)$ axis of symmetry: $x = -4$
y-intercept: $(0, -20)$



Miscellaneous:

1) What is the vertex of $3x^2 + 6x - 90$ $\frac{-b}{2a} = \frac{-6}{2(3)} = -1$ (axis of symmetry) vertex is on axis of symmetry!

$$3(-1)^2 + 6(-1) - 90 = -93$$
 (-1, -93)

2) Describe the solutions of the following: real, imaginary, rational, irrational.....

$$x^2 + 5x + 12$$
 use discriminant: $b^2 - 4ac$ ----> $(5)^2 - 4(1)(12) = -23$ ----> since discriminant < 0, 2 IMAGINARY

 $3x^2 + 12x + 9$ use discriminant: $b^2 - 4ac$ ----> $(12)^2 - 4(3)(9) = 36$ ----> since discriminant > 0, 2 REAL since discriminant is perfect square, 2 RATIONAL

 $x^2 - 14x + 49$ discriminant: $(-14)^2 - 4(1)(49) = 0$ ----> since discriminant = 0, 1 REAL solution

 $-2(x^2 + 4) - 11$ vertex is $(-4, -11)$; curve faces DOWN ----> no x-intercepts! 2 IMAGINARY

 $-x^2 + 4x + 15$ discriminant: $(4)^2 - 4(-1)(15) = 76$ ----> since discriminant > 0, 2 REAL since 76 is not a perfect square, 2 IRRATIONAL

 $(x + 4)(x - 2)$ there are 2 zeros! ---> 2 REAL solutions

(9, 0)

$$y = -3|x - 5| + 12$$

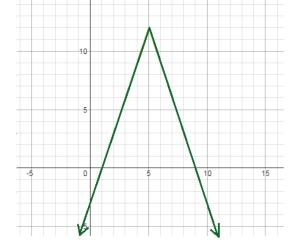
a) Identify the vertex, axis of symmetry, y-intercept, and any x-intercepts

y = a|x - h| + kvertex (h, k): (5, 12) axis of symmetry: x = 5 y-intercept: when x = 0, y = -3|0 - 5| + 12 = -3(0, -3)

x-intercept: when y = 00 = -3|x - 5| + 12-12 = -3|x - 5|

4 = |x - 5|(1, 0)

4 = x - 5 -4 = x - 5



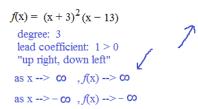
c) What is the domain? Range?

b) Graph the equation

domain: all real (-∞ , ∞ range: $y \le 12$ ($-\infty$, 12]

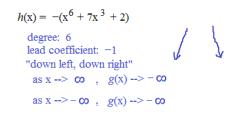
IV: Polynomials

Describe the end behavior of the following:



$$g(x) = 4x - 2x^4 + 5x^2$$

degree: 4
lead coefficient: -2 ''down left, down right''
as x --> ∞ , $g(x)$ --> $-\infty$



Find a polynomial of degree 4 with integer coefficients and zeros 3i and 4, where 4 is a double zero.

zero w/multiplicity 2)

since 3i is a zero, -3i must be a zero ("conjugate theorem")

$$(x-3i)(x--3i)$$
 \longrightarrow $(x-3i)(x+3i) = (x^2+9)$

since 4 is a double zero \longrightarrow (x-4)(x-4)

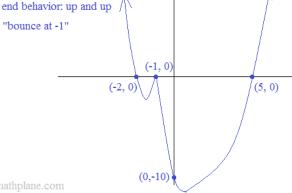
$$y = (x^2 + 9)(x - 4)^2$$
 another solution: $y = 3(x^2 + 9)(x - 4)^2$

Graph $(x-5)(x+1)^2(x+2)$

zeros: 5, -1, -1, -2 Label the zeros and y-intercept (-1 is 'double zero' or

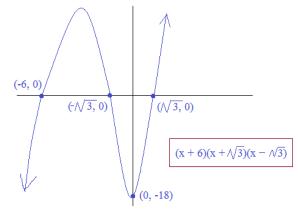
y-intercept: $(0-5)(0+1)^2 (0+2) = -10$ (0, -10)

end behavior: up and up



Graph $x^3 + 6x^2 - 3x - 18$ Identify the linear factors

to find intercepts: factor by grouping $x^3 + 6x^2 - 3x - 18$ $x^2 (x+6) -3(x+6)$ $(x^2-3)(x+6)$



$$f(x) = x^2 + 4$$

Find the following:

$$g(x) = 5x + 1$$

1)
$$f(2)$$
 $(2)^2 + 4 = 8$

$$h(x) = 7$$

2)
$$g(-3)$$
 $5(-3) + 1 = -14$

3)
$$h(4)$$
 every input of $h = 7$

4)
$$f(w+1) (w+1)^2 + 4$$

 $(w+1)(w+1) + 4$
 $w^2 + 2x + 5$

5)
$$f(h(6))$$
 $h(6) = 7$ then, $f(7) = 53$

6)
$$h(g(5))$$
 $g(5) = 26$ then, $h(26) = 7$

7)
$$(g \circ g)(x)$$
 $g(g(x))$ ---> $g(x) = 5x + 1$
then, $g(5x + 1) = 5(5x + 1) + 1$

8)
$$f(g(x))$$
 $f(5x + 1) = \frac{g(5x + 1) + 5}{25x + 6}$
 $(5x + 1)^2 + 4 = (5x + 1)(5x + 1) + 4$
 $25x^2 + 10x + 5$

VI. Exponents and Roots

Evaluate:

$$(32^{\frac{1}{5}})^2$$

$$\left\langle \frac{25}{9} \right\rangle^{\frac{1}{2}} = \frac{\sqrt{25}}{\sqrt{9}} = \boxed{\frac{5}{3}}$$

Simplify:

$$(3x^3 y^{-4})$$

 $9x^6 y^{-8}$

 $\frac{\left(3x^{3} \ y^{-4}\right)^{2}}{9x^{6} \ y^{-8}} \qquad \frac{\left(\frac{12x^{2} \ y^{-3}}{3x^{4} \ yz}\right)^{-1}}{3x^{4} \ yz}$ "flip the fraction" $\frac{3x^{4} \ yz}{12x^{2} \ y^{-3}}$ combine "like" terms $1x^2 y^4 z$

 $\frac{4a^2b^5}{7bc^{-1}} \cdot (2ac^3)^{-3}$

$$\frac{4a^2b^5c^1}{7b} \cdot \frac{1}{(2ac^3)^3}$$

$$\frac{4a^2b^4c^1}{7} \cdot \frac{1}{8a^3c^9} = \frac{b^4}{14ac^8}$$

VII. Factoring

Simplify:
$$3x^2 - 75$$

$$x^4 + 3x^2 - 4$$

$$2x^3 + 54$$

$$x^3 - 7x^2 - 2x + 14$$

GCF: $3(x^2 - 25)$ difference

$$3(x+5)(x-5)$$

multiplies to -4 adds to +3

(x + 4)(x - 1)

GCF: $2(x^3 + 27)$

factor by grouping: $x^3 - 7x^2 - 2x + 14$

Solve:

of squares:

$$2x^2 + 10x + 8 = 0$$

Divide by 2 (GCF)
$$x^2 + 5x + 4 = 0$$

property)

$$(x+1)(x+4)=0$$

$$(x+1)(x+4) = 0$$

$$2x^2 - 5x = -2$$
$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$x^2 + 5x + 19 = 0$$

Use Quadratic Formula

$$x^2 + 5x + 19 = 0$$

the discriminant: $b^2 - 4ac = (5)^2 - 4(1)(19)$

$$(2x - 1)(x - 2) = 0$$

Solve x = -1, -4(zero product

$$x = 1/2, 2$$

$$x = \frac{-5 \pm i \sqrt{51}}{2a} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{-51}}{2}$$

1) What are the *possible* rational roots of the polynomial $2x^5 + 3x^2 + 7x - 6$

factors of constant ('p'): 1, 2, 3, 6

2) What is the remainder of $g(x) = x^{100} + x^{40} + 2x^{25} \div (x-1)$?

can find g(1) to get the remainder!

Taking advantage of the *remainder theorem*, we can find
$$g(1)$$
 to get the remainder! $g(1) = 1^{100} + 1^{40} + 2(1)^{25} = 4$

3) Using synthetic division, find $x^4 - 3x^3 + 7x + 8 \div (x + 2)$

 $1x^3 - 5x^2 + 10x - 13$ Remainder 34

4) Find $x^4 + 5x^3 - 11x^2 + 2x + 1 \div (x^2 + 2)$

Use Polynomial Long Division:

 $x^{2} + 2 \overline{\smash)x^{4} + 5x^{3} - 11x^{2} + 2x + 1}$ $-\underline{(x^{4} + 2x^{2})}$ $5x^{3} - 13x^{2}$ $-\underline{(5x^{3} + 10x)}$ $-13x^{2} - 8x$

 $x^2 + 5x - 13 + \frac{-8x + 27}{x^2 + 2}$

IX. Matrices

 $A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 6 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

- 1) Identify the following elements: from A: a_{21} from B: b_{12} from C: c_{12}

- row2, column1: 0 row1, column2: 0 row1, column2: 3

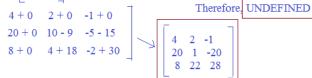
- 2) What are the dimesions of each matrix? $A: _{2 \times 3}$
- B: 3 x 2 C: 2 x 3
- row x column

3) Find the following:

$$A + 2C = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 1 \\ 2 & 7 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$$

 $A + 2C = AB = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 6 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} AC = 2 \times 3 \cdot 2 \times 3$ insides must be same.. $\begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 4 + 10 - 2 & 0 - 6 - 6 \\ 0 + 15 + 10 & 0 - 9 + 30 \end{bmatrix}$ $\begin{bmatrix} 8 & 8 & 1 \\ 2 & 7 & 11 \end{bmatrix}$ $\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$ $\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$ $\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$ $\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$ $\begin{bmatrix} 12 & -12 \\ 25 & 21 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 5 & -3 \\ 2 & 6 \end{bmatrix}$ $\begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix}$ $\begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix}$ $\begin{bmatrix} 4 & 2 & -1 \\ 20 & 1 & -20 \\ 8 & 22 & 28 \end{bmatrix}$ $\begin{bmatrix} 4 & 2 & -1 \\ 20 & 1 & -20 \\ 8 & 22 & 28 \end{bmatrix}$



1) A company invested \$100,000 in 3 funds.

Last year, the growth fund earned 12%, the income fund earned 8%, and the money market fund yielded 5%. So, after one year, the company had \$109,000.

If the company invested twice as much in the income fund as the money market fund, how much did it invest in each fund?

Let G = Growth Fund I = Income Fund M = Money Market Fund

3 equations with 3 unknowns
$$\begin{cases} G+I+M = 100,000 \\ (1.12)G+(1.08)I+(1.05)M = 109,000 \\ 2M=I & \text{or} \\ I-2M=0 \end{cases}$$

Solve using substitution: I = 2M, so

$$G + (2M) + M = 100,000$$

 $(.12)G + (.08)(2M) + (.05)M = 9,000$
 $G + 3M = 100,000$ $-7G - 21M = -700,000$
 $12G + 21M = 900,000$ $12G + 21M = 900,000$
 $5G = 200,000$

One method: solve using augemented matrix

$$\operatorname{rref} \left[\begin{array}{cccc}
1 & 1 & 1 & 100000 \\
.12 & .08 & .05 & 9000 \\
0 & 1 & -2 & 0
\end{array} \right] = \left[\begin{array}{ccccc}
1 & 0 & 0 & 40000 \\
0 & 1 & 0 & 40000 \\
0 & 0 & 1 & 20000
\end{array} \right] =$$

using reduced row echelon form method:

Growth: 40,000 Income: 40,000 Money Market: 20,000

$$G = 40,000$$

 $G + 3M = 100,000$
so, $M = 20,000$
 $G + I + M = 100,000$
so, $I = 40,000$

- 2) A math test consists of number problems and graphing problems. Number problems are worth 6 points each, and graphing problems are worth 10 points each. You can accurately solve a number problem in 2 minutes and a graphing problem in 4 minutes. Assuming you have 40 minutes and may choose no more than 12 problems, to answer, how many of each type should you solve to get the highest score? What is that highest score?
 - 1) Identify and label variables: N = # of number problems G = # of graphing problems
 - 2) Determine the objective function: "how many to get highest score?"

$$6N + 10G = Score$$

- 3) List and graph the constraints: (time) $2N+4G \le 40$ (problems) $N+G \le 12$
- 4) Test the corner points of the feasibility region

$$(0, 0)$$
: $6(0) + 10(0) = 0$

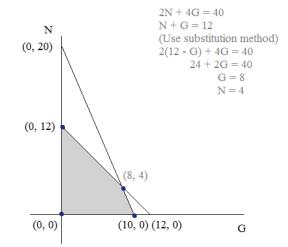
$$(0, 12): 6(12) + 10(0) = 72$$

$$(8, 4): 6(4) + 10(8) = 104$$

$$(10, 0): 6(0) + 10(10) = 100$$

Under the test constraints, answering 8 graphing problems and 4 number problems would get the best score!

The maximum score would be 104..



XI. Imaginary and Complex Numbers

SOLUTIONS

1)
$$(5i - 6)^2 =$$

 $(5i - 6)(5i - 6) =$

$$2) \quad 2i^{25} + 4i^{11} + 2i^{20} =$$

$$2i^{24}i + 4i^{8}i^{3} + 2i^{20}$$

$$25i^2 - 30i - 30i + 36 =$$

$$2(1)i + 4(1)(-i) + 2(1)$$

$$-25 - 60i + 36 =$$

$$11 - 60i$$

$$2i - 4i + 2$$

$$-2i + 2$$

3)
$$\frac{3i+4}{4i-9} = \frac{3i+4}{4i-9} \cdot \frac{4i+9}{4i+9} = \frac{3i+4}{4i+9} = \frac{3i+4}{4$$

$$\frac{12i^2 + 16i + 27i + 36}{16i^2 - 81} =$$

$$\frac{24 + 43i}{-97} = \frac{-24}{97} + \frac{-43}{97}i$$

XII. Piecewise Functions

$$h(t) = \begin{cases} \sqrt{(-t)} &, & \text{if } t < -3 \\ 5 &, & \text{if } 0 \le t < 5 \\ -2t &, & \text{if } 5 \le t \end{cases}$$

$$h(-4) = \sqrt{(-(-4))} = 2$$

$$h(5) = -2(5) = -10$$

$$h(10) = -2(10) = -20$$

(**challenge)

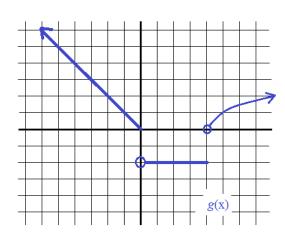
What is the domain and range of each function?

$$h(x)$$
: Domain - $(-\infty, -3) \cup (-3, \infty)$

Range -
$$(-\infty, -10]$$
 U $(\sqrt{3}, \infty)$

$$g(x)$$
: Domain - $(-\infty, \infty)$

Range -
$$g(x) = -2$$
 or $g(x) \ge 0$

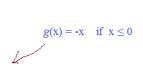


$$g(-3) = 3$$

$$g(4) = -2$$

$$g(5) = 1$$

$$g(-20) = 20$$



XIII. Three more questions

1) Change to vertex form (i.e complete the square)

separate the $y = 2x^2 + 8x + 11$ x's from rest

factor out 2
$$y = 2(x^2 + 4x) + 11$$

complete the $y = 2(x^2 + 4x + 4) + 11 - 8$ square $y = 2(x + 2)^2 + 3$

$$y = 2(x+2)^2 + 3$$

2) Solve and graph on number line

$$3|x-5|-7>8$$

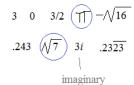


|x - 5| > 5

$$x - 5 > 5$$
 $x - 5 < -5$

$$x > 10$$
 or $x < 0$

3) Circle the irrational numbers

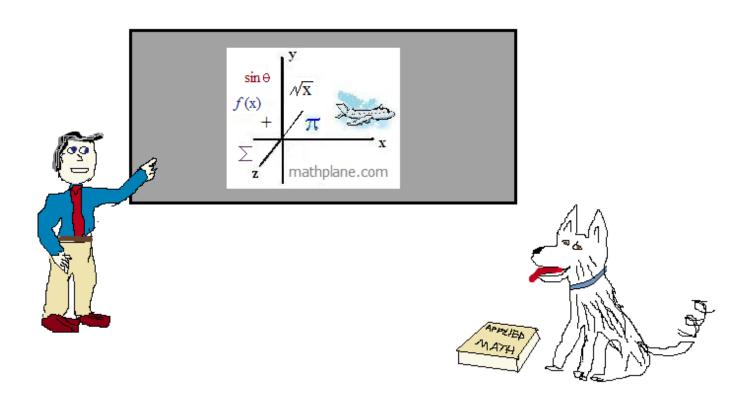


Good luck on your final exam!

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Good luck!



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Thanks for your support. Any proceeds go to the mathplane site (and treats for Oscar the dog!)

