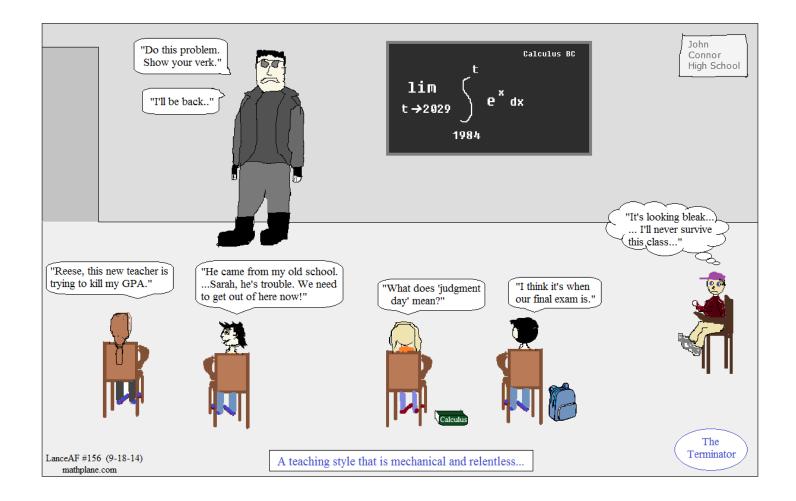
CALCULUS AB:

Multiple Choice Questions 2

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

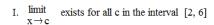
Topics include limits, continuity, differentiation, second derivatives, mean value theorem, implicit differentiation, related rates, and more...



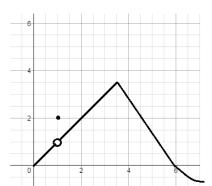
QUESTIONS-→

1)
$$\lim_{x \to \infty} \frac{x^2 - 1}{3 + 4x - x^2}$$

- a) -1
- b) -1/3
- c) -1/4
- d) 0
- e) ∞
- 2) For the function f(x) in the graph,



- II. the function is continuous on the interval [2, 6]
- III. the function is differentiable on the interval [2, 6]
- a) I only
- b) II only
- c) I and II
- d) I and III
- e) I, II, and III



3) What value of c makes this function continuous?

$$\begin{cases} \frac{2x^2 + 7x + 3}{x + 3} & \text{if } x \neq -3 \\ C & \text{if } x = -3 \end{cases}$$

- a) -3
- b) -5
- c) 2
- d) -1/2
- e) 0

4) For the following function
$$s(t) = 2t^3 + t^2 + 8t - 4$$
, where $t = seconds$, what is the displacement over the first 4 seconds?

- a) 35
- b) 36
- c) 88
- d) 140
- e) 144

5) A rational function of the form $y = \frac{ax}{x+b}$ has a vertical asymptote at x = 5

and a horizonal asymptote at y = -3

Calculus Multiple Choice Questions

Which is a possible function?

- a) $\frac{5x}{x-3}$ b) $\frac{3x}{x+5}$ c) $\frac{-3x}{x-5}$ d) $\frac{-5x}{x+3}$ e) $\frac{-3x}{x+5}$
- 6) Let p(x) be a cubic polynomial function, where p(3) < 0, p(7) > 0, and p(9) < 0, Which statements are true?

statement I: there are 3 zeros

statement II: a zero exists at x < 3 OR x > 9

statement III: for p(x) = 0, there are 2 solutions between 3 and 9

- a) I
- b) I and II
- c) I and III
- d) II
- e) I, II, and III
- $\lim_{x \to 3} 9 =$
 - a) 3
 - b) 9
 - c) Does not exist
 - d) 0
 - e) 27
- 8) Find the value of k so g(x) is continuous:

$$g(x) = \begin{cases} k+x & x < 10 \\ xk & x \ge 10 \end{cases}$$

- a) 10
- b) 0
- c) 10/9
- d) 1
- e) no solution

- a) 0
- b) -2, 2
- c) -2, 0, 2
- d) -4,
- e) -4, 4

10) What is the derivative of $x^2 \sin(5x)$?

- a) 2xcos(5x)
- b) 10xcos(5x)
- c) $2x + 5\cos(5x)$
- d) $2x\sin(5x) + x^2\cos(5x)$
- e) $2x\sin(5x) + 5x^2\cos(5x)$

11) Find the slope of the line tangent to the curve $y = x^3 - 3x^2$ at the point of inflection.

- a) -3
- b) -1
- c) 0
- d) 1
- e) 3

12) As x increases to infinity, the function $f(x) = 2e^{-x}$ gets closer to

- a) 0
- b) 1/2
- c) 2
- d) e
- e) infinity

- a) -1
- b) 0
- c) 1
- d) 1T
- e) 21T
- 14) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at (-2, 3)
 - a) increasing, concave up
 - b) increasing, concave down
 - c) decreasing, concave up
 - d) decreasing, concave down
 - e) increasing, point of inflection

- 15) Find the equation of the line tangent to $x^3 + y^3 = 3xy + 4x 5y$ @ (2, 1)
- a) y = 1
- b) 5x + 2y = 12
- c) 2y 5x = -8
- d) 5x y = 9
- e) x = 2

I. Find the average value of the function (on the given interval)

- a) 2
- b) 5/2
- c) 7/3
- d) 14/3
- e) 5

II. Determine the value "c" guaranteed by the 'Mean Value Theorem'

- a) -1.15
- b) -.57
- c) .57
- d) 1.15
- e) 2.3

17) $h(x) = x^3 - 2$ on the interval [-1, 3] Derivative Mean Value Theorem

I. Find the Average Rate of Change (AROC) on the interval

- a) 2
- b) 4
- c) 13/2
- d) 7
- e) 11

II. Find the value "c" to satisfy the 'Mean Value Theorem"

- a) -2.33
- b) -1.32
- c) 1
- d) 1.53
- e) 2.11

Find $\frac{dx}{dt}$

- a) 3
- b) 34/9
- c) -5
- d) 0
- e) $\frac{1}{3}$
- 19) What is the y-intercept of the line that is tangent to $2\sqrt{x} + 4\sqrt{y} = x + y + 3$ at (4, 9)?
 - a) 3
 - b) 6
 - c) 9
 - d) 12
 - e) 15
- 20) If $x^2 y^2 = 16$ find $\frac{d^2y}{dx^2}$
 - a) $\frac{x^2 y^2}{y^2}$
 - b) $\frac{y^2 x^2}{y^3}$
 - c) $\frac{1}{y^2}$
 - d) $\frac{16x}{y^2}$
 - e) $\frac{x^2}{y^2}$

what is the value of g'(6)?

- a) -1/6
- b) 1/6
- c) -6
- d) 6
- e) 121
- 22) g is differentiable and $g(x) = f^{-1}(x)$ for all x

$$f(-4) = 12$$

$$f(9) = -4$$

$$f(-4) = 12$$
 $f(9) = -4$ $f'(4) = -6$ $f'(9) = 3$

$$f'(9) = 3$$

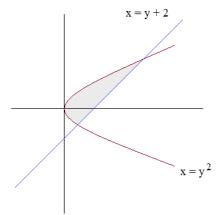
what is g'(-4)?

- a) 1/3
- b) -1/4
- c) 1/9
- d) -1/6
- e) need more information
- 23) Find the area of the region bounded by

$$x = y^2$$

$$x = y + 2$$

- a) 7/2
- b) 4
- c) 9/2
- d) 8
- e) 9



- a) $\ln(3) + 32$
- b) $\ln(4) + 32$
- c) $\ln(4) + 40$
- d) ln(12) 32
- e) ln(12) + 32

25)
$$\int_{-2}^{2} x^7 + k \ dx = 64$$
 What is k?

- a) -16
- b) -4
- c) 0
- d) 8
- e) 16

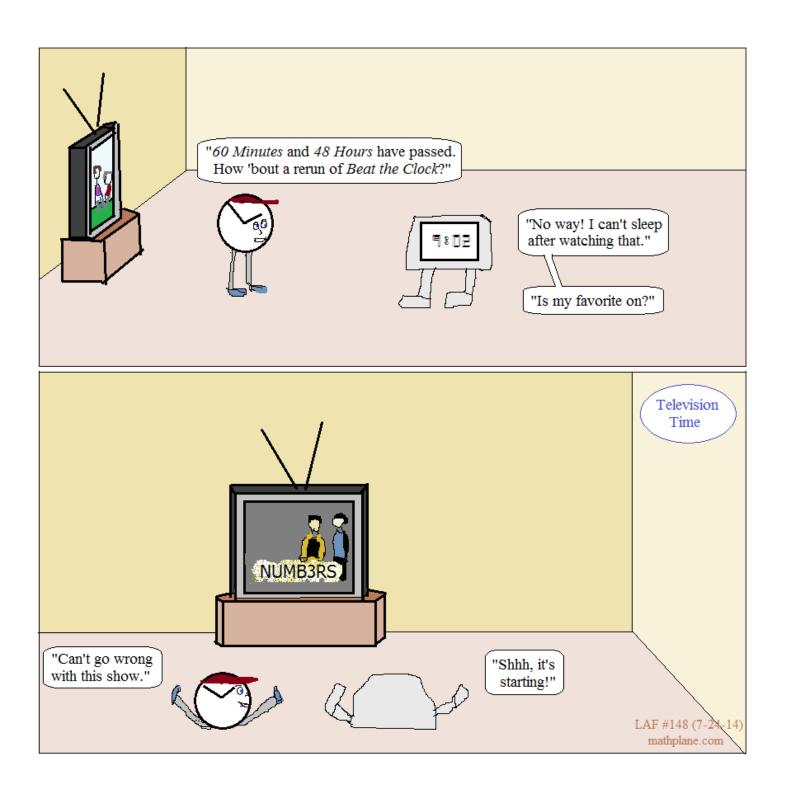
26) Find G'(2) where
$$G(x) = \int_{0}^{x^{2}} \sqrt{t^{3} + 3} dt$$

- a) 11
- b) $\sqrt{67}$
- c) $4\sqrt{67}$
- d) 8
- e) 64

27)
$$\lim_{x \to -5} \frac{g(5) - g(x)}{5 - x} = -.548$$
 The graph of the function g, at the point x = 5, must be

- a) increasing
- b) decreasing
- c) concave up
- d) concave down
- e) undefined

.



SOLUTIONS-→

SOLUTIONS

1)
$$\lim_{x \to \infty} \frac{x^2 - 1}{3 + 4x - x^2}$$

a) -1

b) -1/3

c) -1/4 d) 0

e) ∞

rewrite:

$$\lim_{x \to \infty} \frac{x^2 - 1}{-x^2 + 4x + 3}$$

since degree of numerator (2) and degree of numerator (2) are the same, look at the lead coefficients...

$$\frac{1}{-1} = -1$$

2) For the function f(x) in the graph,

I. limit exists for all c in the interval [2, 6]

II. the function is continuous on the interval [2, 6]

III. the function is differentiable on the interval [2, 6]



b) II only

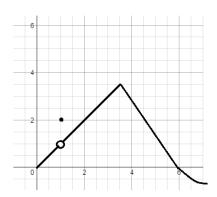
I. limit does exist (in fact, it exists between [0, 6]

c) I and II d) I and III

II. function is continuous on [2, 6] (It is not continuous at x = 1)

e) I, II, and III

III. It is not differentiable at $x = 3 \ 3/4$



3) What value of c makes this function continuous?

$$\begin{cases} \frac{2x^2 + 7x + 3}{x + 3} & \text{if } x \neq -3 \\ C & \text{if } x = -3 \end{cases}$$

the rational expression is a line with a 'hole'

To fill that hole, we find the limit as x approaches -3

a) -3
b) -5
$$x \to -3$$

$$\lim_{x \to -3} \frac{(2x+1)(x+3)}{(x+3)}$$

$$x \rightarrow -3$$

$$\lim_{X \to -3} (2x+1) = -5$$

- b) -5 c) 2
 - d) -1/2
 - e) 0
- 4) For the following function $s(t) = 2t^3 + t^2 + 8t 4$, where t = seconds, what is the displacement over the first 4 seconds?

b) 36

The displacement is the "net change"...

 $\textcircled{a} t = 0, \ \ \$(0) = -4$ a t = 4, s(4) = 140

The displacement/net change is 144 units...

SOLUTIONS

Which is a possible function?

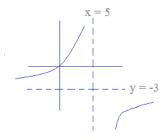
a)
$$5x$$

b)
$$\frac{3x}{x+5}$$

c)
$$\frac{-3x}{x-5}$$

d)
$$\frac{-5x}{x+3}$$

e)
$$\frac{-3x}{x+5}$$



6) Let p(x) be a cubic polynomial function, where p(3) < 0, p(7) > 0, and p(9) < 0, Which statements are true?

statement I: there are 3 zeros

statement II: a zero exists at x < 3 OR x > 9

statement III: for p(x) = 0, there are 2 solutions between 3 and 9

a) I

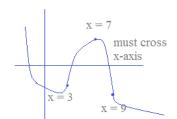
b) I and II

c) I and III

d) II

e) I, II, and III

polynomial function is continuous...



(possible sketch)

$$\begin{array}{ccc}
7) & \lim_{x \to 3} & 9 = & 9
\end{array}$$



c) Does not exist

d) 0

e) 27



8) Find the value of k so g(x) is continuous:

$$g(x) = \begin{cases} k+x & x < 10 \\ xk & x \ge 10 \end{cases}$$

a) 10

b) 0

c) 10/9

d) 1

e) no solution

to be continuous, each part of the piecewise function must meet:

$$k + x = xk$$

at
$$x = 10$$
:

$$10 \pm k = 10k$$

$$10 = 9k$$

$$k = 10/9$$

a) 0

- c) -2, 0, 2
- d) -4,
- e) -4, 4

SOLUTIONS

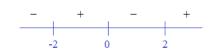
Find derivative: $24x^3 - 96x$

Set equal to zero:
$$24x^3 - 96x = 0$$

(critical values)
$$24x(x^2 - 4) = 0$$

$$x = -2, 0, 2$$

Determine max/min:



decreasing increasing decreasing increasing

-2 and 2 are minimums (0 is a relevant maximum)



- @ x = -2, posittive (concave up) minimum
- @ x = 0, negative (concave down) maximum
- @ x = 2, positive (concave up) minimum



- 10) What is the derivative of $x^2 \sin(5x)$?
 - a) 2xcos(5x)
 - b) 10xcos(5x)
 - c) $2x + 5\cos(5x)$
 - d) $2x\sin(5x) + x^2\cos(5x)$
 - e) $2x\sin(5x) + 5x^2\cos(5x)$

product rule $x^2 \sin(5x)$

$$f'(x)g(x) + g'(x)f(x)$$
 $2x \cdot \sin(5x) + \cos(5x) \cdot 5 \cdot x^2$

$$2x\sin(5x) + 5x^2\cos(5x)$$

- 11) Find the slope of the line tangent to the curve $y = x^3 3x^2$ at the point of inflection.
 - a) -3
 - b) -1
 - c) 0
 - d) 1
 - e) 3

First, where is the point of inflection? Where 2nd derivative equals zero.

$$y' = 3x^2 - 6x$$

$$y'' = 6x - 6$$

$$y'' = 0$$
 when $y = 1$

y'' = 0 when x = 1 Therefore, point of inflection is (1, -2)

$$-2 = (1)^3 - 3(1)^2$$

Now, find the slope at x = 1

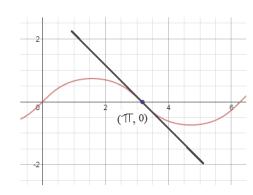
$$y' = 3(1)^2 - 6(1) = -3$$

- 12) As x increases to infinity, the function $f(x) = 2e^{-x}$ gets closer to
 - a) 0 b) 1/2
- rewrite function as $\frac{2}{e^X}$
- c) 2
- d) e
- e) infinity

- as x gets infinitely larger, e^{X} goes to infinity...
 - therefore, $\frac{2}{e^{X}}$ gets smaller and smaller, approaching 0

a) -1

e) 21T



Use implicit differentiation to find the derivative (instantaneous rate of change)

$$1 \cdot \frac{dy}{dx} = \cos(x)\cos(y) + (-\sin(y)\frac{dy}{dx})\sin(x)$$

to find IROC at point, substitute (T, 0)

$$\frac{dy}{dx} = \cos(\text{T})\cos(0) - \sin(0)\frac{dy}{dx} \cdot \sin(\text{T})$$

$$=$$
 (-1)(1) + (0) $\frac{dy}{dx}$ (0) $=$ -1

14) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at (-2, 3)

- b) increasing, concave down
- c) decreasing, concave up
- d) decreasing, concave down
- e) increasing, point of inflection

To determine increasing or decreasing, find first derivative...

$$2x + 4y \frac{dy}{dx} = 0$$

$$4y \frac{dy}{dx} = -2x$$

$$4y \frac{dy}{dx} = -2x$$
 at (-2, 3) $\frac{dy}{dx} = \frac{-(-2)}{2(3)} = \frac{1}{3} > 0$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

To determine concavity, find second derivative...

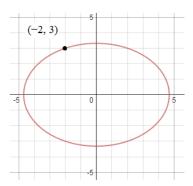
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-x}{2y}$$

$$\frac{d}{dx} \cdot \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2} = \frac{-1(2y) - 2\frac{dy}{dx}(-x)}{(2y)^2} = \frac{-2y + 2x\frac{dy}{dx}}{4y^2}$$

$$\frac{-y + x \frac{dy}{dx}}{2y^2} = \frac{-y + x \left(\frac{-x}{2y}\right)}{2y^2} \quad \text{at (-2, 3)} \quad \frac{d^2y}{dx^2} = \frac{-(3) + (-2) \frac{1}{3}}{2(3)^2}$$
$$= \frac{-11/3}{18} < 0$$

concave down...

Notice, the graph is an ellipse!



Find the equation of the line tangent to $x^3 + y^3 = 3xy + 4x - 5y$ @ (2, 1)

a)
$$y = 1$$

b)
$$5x + 2y = 12$$

c)
$$2y - 5x = -8$$

d)
$$5x - y = 9$$

e)
$$x = 2$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 3\left((1)(y) + (x)(1)\frac{dy}{dx}\right) + 4 - 5\frac{dy}{dx}$$

substitute (2, 1)

$$2\frac{dy}{dx} = -5$$

slope of tangent line at
$$(2, 1) = \frac{-5}{2}$$

$$y-1 = \frac{-5}{2}(x-2)$$

$$y = \frac{-5}{2} x + 6$$

$$5x + 2y = 12$$

I. Find the average value of the function (on the given interval)

To find the average value...

- a) 2
- b) 5/2
- c) 7/3
- d) 14/3
- e) 5

- $\int_{0}^{2} x^{2} + 1 dx = \frac{x^{3}}{3} + x \Big|_{0}^{2} = \frac{8}{3} + 2 (0/3 + 0) = \frac{14}{3} \text{ area under the curve (i.e. total value on interval [0, 2])}$

Average Rate

Of Change

average value:
$$\frac{\frac{14}{3}}{(2-0)} = \frac{7}{3}$$
 average value

II. Determine the value "c" guaranteed by the 'Mean Value Theorem'

- since the function is continuous and closed on the interval, a) -1.15 there must be a value "c" such that f(c) = average value
- b) -.57
- c) .57
- d) 1.15
- e) 2.3
- so, where does the function equal $\frac{7}{3}$?

$$\frac{7}{3} = x^2 + 1$$

$$x = \frac{2\sqrt{3}}{3}$$
 approx. 1.15

 $f(c) = \frac{1}{b-a} \int_{a}^{b} f(x) dx$

We don't include -1.15 (because it is not in the interval)

17) $h(x) = x^3 - 2$ on the interval [-1, 3] Derivative Mean Value Theorem

I. Find the Average Rate of Change (AROC) on the interval

- a) 2
- b) 4
- c) 13/2
- d) 7
- e) 11

II. Find the value "c" to satisfy the 'Mean Value Theorem"

a) -2.33 b) -1.32

c) 1

d) 1.53

e) 2.11

- Rate Of Change

Instantaneous

- $h'(x) = 3x^2 0$
- - at point "c" h'(c) = 7
 - $3c^2 = .7$
 - c = -1.53 or 1.53

If function is continuous and differentiable over interval [a, b] there exists at least one point c where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- instanteous rate of change at c
- average rate of change between a and b

mathplane.com

at t = 1, y = 3 and $\frac{dy}{dt} = 2$

Find $\frac{dx}{dt}$

derivative with respect to t

Using implicit differentiation and related rates....

$$2y \frac{dy}{dt} = (1)\frac{dx}{dt}(y) + (x)(1)\frac{dy}{dt} + 0$$

direct substitution...

Since y = 3,

$$2(3)(2) = (1) \frac{dx}{dt}(3) + (1/3)(1)(2)$$

$$12 = 3 \frac{dx}{dt} + 2/3$$

$$\frac{34}{3} = 3 \frac{dx}{dt}$$

$$\frac{34}{9} = \frac{dx}{dt}$$
Since y = 3,

$$(3)^2 = x(3) + 8$$

$$x = 1/3$$

d) 0

a) 3

e) $\frac{1}{2}$

19) What is the y-intercept of the line that is tangent to $2\sqrt{x} + 4\sqrt{y} = x + y + 3$ at (4, 9)?

- First, to find the tangent line, a) 3 we need the slope and a point...
- b) 6 The point is (4, 9) (the point of tangency)
- the slope is the IROC at (4, 9) c) 9
- d) 12
- e) 15

 $2x^{\frac{1}{2}} + 4v^{\frac{1}{2}} = x + v + 3$

(implicit differentiation to find dy/dx)

$$\frac{-\frac{1}{2}}{x^{2} + 2y^{\frac{-\frac{1}{2}}}} \frac{dy}{dx} = 1 + 1 \frac{dy}{dx} + 0$$

Find dy/dx at (4, 9)

$$(4)^{\frac{-1}{2}} + 2(9)^{\frac{-1}{2}} \frac{dy}{dx} = 1 + 1 \frac{dy}{dx}$$

$$\frac{1}{2} + \frac{2}{3} \frac{\mathrm{dy}}{\mathrm{dx}} = 1 + 1 \frac{\mathrm{dy}}{\mathrm{dx}}$$

$$-\frac{1}{2} = \frac{1}{3} \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{3}{2}$$

slope is -3/2 point is (4, 9) then, equation of the line is

$$y-9 = \frac{-3}{2} (x-4)$$

therefore, the y-intercept is

$$y-9 = \frac{-3}{2} (0-4)$$

20) If
$$x^2 - y^2 = 16$$
 find $\frac{d^2y}{dx^2}$

a)
$$\frac{x^2 - y^2}{y^2}$$

b)
$$\frac{y^2 - x^2}{y^3}$$

c)
$$\frac{1}{y^2}$$

d)
$$\frac{16x}{v^2}$$

e)
$$\frac{x^2}{y^2}$$

first, find $\frac{dy}{dx}$ $2x - 2y \frac{dy}{dx} = 0$ then, find second derivative $\frac{d^2y}{dx^2}$ where $\frac{dy}{dx} = \frac{x}{y}$

$$-2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2 y}{dx^2}$$
 where $\frac{dy}{dx} = \frac{x}{y}$

(quotient rule)

$$\frac{(1)(y) - (1)\frac{dy}{dx}(x)}{y^2}$$

$$\frac{y - x \frac{dy}{dx}}{y^2} = \frac{y - \frac{x^2}{y}}{y^2}$$
$$= \frac{y^2 - x^2}{y^3}$$

what is the value of g'(6)?



$$g'(x) = \frac{1}{f'(y)}$$

 $g'(x) = \frac{1}{f'(y)}$ Slope at (6, 1) is the reciprocal of the slope at (1, 6)

b) 1/6

For
$$y = 6$$

c) -6

$$6 = x^3 + x^2 + x + 3, \qquad x = 1$$

d) 6

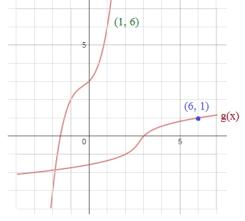
So, the slope at (1, 6) will be the reciprocal of the slope at (6, 1)!

e) 121

$$f'(x) = 3x^2 + 2x + 1 + 0$$

then,
$$f'(1) = 3 + 2 + 1 = 6$$

therefore,
$$g'(6) = \frac{1}{f'(1)} = \frac{1}{6}$$



f(x)

Inverses reflect over the line y = x, and the coordinates are reversed...

22) g is differentiable and $g(x) = f^{-1}(x)$ for all x

$$f(-4) = 12$$

$$f(9) = -4$$

$$f'(4) = -6$$

$$f'(9) = 3$$

what is g'(-4)?

c) 1/9

d) -1/6

g(x) and f(x) are inverses...

So, if f(9) = -4, then g(-4) must equal 9.

Then, f'(9) = 3... therefore, the slope of the inverse g'(-4) = 1/3

23) Find the area of the region bounded by

e) need more information

$$x = v^2$$

$$x = y + 2$$

$$y + 2 = y$$

$$y^2 - y - 2 = 0$$

(1, -1)

$$(y-2)(y+1)=0$$

(4, 2)

b) 4

$$y = -1$$
 and 2

and y-axis

area between line

area between curve and y-axis

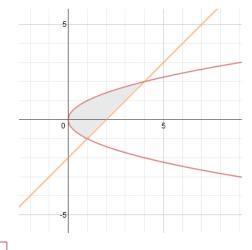
c) 9/2

 $y+2-(y^2) dy$

e) 9

 $\frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{1}^{2}$

$$2 + 4 - \frac{8}{3} - (\frac{1}{2} - 2 + \frac{1}{3}) = \frac{9}{2}$$



24)
$$\int_{2}^{6} \frac{1}{x} + 2x \ dx$$

b)
$$\ln(4) + 32$$

c)
$$\ln(4) + 40$$

d)
$$\ln(12) - 32$$

e)
$$\ln(12) + 32$$

$$\ln(x) + x^2 \Big|_{2}^{6} = \ln(6) + 36 - \left(\ln(2) + 4\right)$$

$$= \ln(6) - \ln(2) + 32$$

$$= \ln \frac{6}{2} + 32$$

$$= \ln(3) + 32$$

25)
$$\int_{-2}^{2} x^7 + k \ dx = 64$$

What is k?

$$\frac{x^8}{8} + kx \Big|_{2}^{2} = \frac{256}{8} + 2k - \left(\frac{256}{8} - 2k\right) = 4k$$

$$4k = 64$$

26) Find G'(2) where $G(x) = \int_{0}^{x^{2}} \sqrt{t^{3} + 3} dt$

Using the fundamental theorem of calculus,

c)
$$4\sqrt{67}$$

$$G'(x) = \sqrt{(x^2)^3 + 3} \cdot (2x)$$

$$G'(2) = \sqrt{(2)^6 + 3} \cdot (2(2))$$

= 4
$$\sqrt{67}$$

27)
$$\lim_{x \to 5} \frac{g(5) - g(x)}{5 - x} = -.548$$

The graph of the function g, at the point x = 5, must be

- a) increasing
- b) decreasing
- c) concave up
- g'(5) = -.548 which is < 0 so, decreasing
- d) concave down

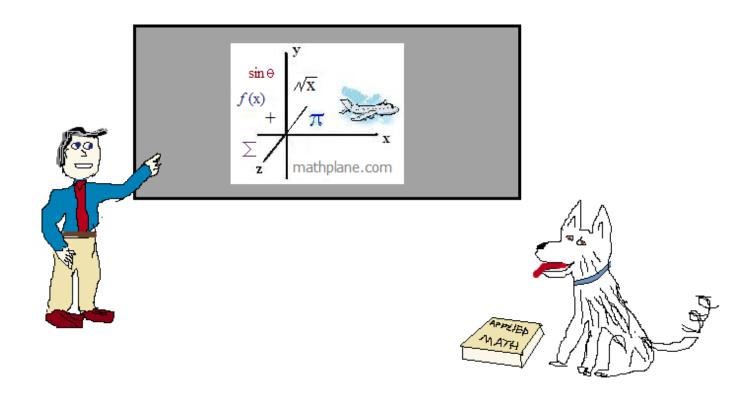
(it might be concave up or concave down. But, it MUST be decreasing...)

e) undefined

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, TES, Google+, TeachersPayTeachers, and Pinterest

And, Mathplane Express for mobile at mathplane.ORG