

Solving Rational Equations

Notes, Examples, and practice (with solutions)

Topics include cross multiplying, word problems, factoring, inequalities, extraneous answers, and more.

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Solving Rational Equalities/Equations

Method 1: Combine terms (using least common denominator)

Example: $\frac{x}{2} + \frac{x}{5} = 14$

(Combine terms on left)

$$\frac{5x}{10} + \frac{2x}{10} = 14$$

Find least common multiple of 2 and 5

$$\frac{7x}{10} = 14$$

(Solve)

$$7x = 140$$

$$x = 20$$

Check solution: $\frac{20}{2} + \frac{20}{5} = 14$

$$10 + 4 = 14 \quad \checkmark$$

Method 2: Multiply entire equation by least common denominator

Example: $\frac{5n}{2} + \frac{1}{4} = 4$

(multiply entire equation by LCD to get rid of the fractions)

$$4 \cdot \left(\frac{5n}{2} + \frac{1}{4} = 4 \right)$$

Denominators are 2, 4, and 1...
LCM of 1, 2 and 4 is 4...

$$10n + 1 = 16$$

Solve

$$10n = 15$$

$$n = \frac{3}{2}$$

Check solution: $\frac{5 \cdot \frac{3}{2}}{2} + \frac{1}{4} = 4$

$$\frac{15}{2} + \frac{1}{4} = 4$$

$$\frac{15}{4} + \frac{1}{4} = 4 \quad \checkmark$$

2 rational expressions in an equality? Try cross multiplying...

Example: $\frac{1}{x^2 + 1} = \frac{3}{2x + 4}$

$$(2x + 4)(1) = (3)(x^2 + 1)$$

$$2x + 4 = 3x^2 + 3$$

$$0 = 3x^2 - 2x - 1$$

$$(3x + 1)(x - 1) = 0$$

$$x = -1/3 \text{ or } 1$$

Check solutions: $\frac{1}{(1)^2 + 1} = \frac{3}{2(1) + 4}$

$$\frac{1}{2} = \frac{3}{6} \quad \checkmark$$

$$\frac{1}{(-1/3)^2 + 1} = \frac{3}{2(-1/3) + 4}$$

$$\frac{1}{\frac{10}{9}} = \frac{3}{\frac{-2}{3} + \frac{12}{3}} = \frac{9}{10} \quad \checkmark$$

Example: $\frac{x}{5} = \frac{x+3}{8}$ Cross Multiply

$$8x = 5(x+3)$$

$$8x = 5x + 15$$

$$3x = 15$$

$$x = 5$$

Check: $\frac{5}{5} = \frac{5+3}{8}$

$$1 = 1 \quad \checkmark$$

Important: Check your answers! Sometimes, math techniques produce *extraneous* solutions

Example: $\frac{1}{x+1} = \frac{3}{x^2-1}$ Cross Multiply

$$1(x^2-1) = 3(x+1)$$

$$x^2-1 = 3x+3$$

$$x^2-3x-4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4, -1$$

Check solutions: 4:
(substitute in the original equation)

$$\frac{1}{4+1} = \frac{3}{4^2-1}$$

$$\frac{1}{5} = \frac{3}{15} \quad \checkmark$$

$$\frac{1}{-1+1} = \frac{3}{(-1)^2-1}$$

$$\frac{1}{0} = \frac{3}{0} \quad \times$$

$x = -1$ is an
extraneous solution...

Pick the approach that you prefer...

Example: $\frac{3x+2}{12} - \frac{1}{6} = \frac{1}{3}$

method 1: combine terms and cross multiply

$$\frac{3x+2}{12} - \frac{2}{12} = \frac{1}{3}$$

$$\frac{3x+2-2}{12} = \frac{1}{3}$$

$$\frac{3x}{12} = \frac{1}{3}$$

$$3(3x) = 12(1)$$

$$9x = 12$$

$$x = \frac{4}{3}$$

method 2: get rid of denominators and solve

Least common multiple of 12, 6, and 3 is 12

--> multiply entire equation by 12

$$12 \left(\frac{3x+2}{12} - \frac{1}{6} = \frac{1}{3} \right)$$

$$3x + 2 - 2 = 4$$

$$3x = 4$$

$$x = \frac{4}{3}$$

Example: Underground pipes can fill a swimming pool in 4 hours.
 A regular garden hose can fill the pool in 15 hours.
 If both are used at the same time, how long will it take to fill the pool?

Step 1: Write Formulas and Variables

distance = rate x time
 In this case, filling pool = (rate)(time)

therefore, rate = $\frac{\text{filling one pool}}{\text{time}}$

$$\text{rate}_p = \frac{1 \text{ pool}}{4 \text{ hours}} \quad (\text{filling rate of pipes})$$

$$\text{rate}_h = \frac{1 \text{ pool}}{15 \text{ hours}} \quad (\text{filling rate of hose})$$

Step 2: Establish equation and solve

We want to fill a pool with BOTH hose and pipes...

hose water + pipes water = total water

$$r_h(t) + r_p(t) = \text{total}$$

$$\frac{1 \text{ pool}}{15 \text{ hours}} t + \frac{1 \text{ pool}}{4 \text{ hours}} t = 1 \text{ pool}$$

(multiply by 60 hours; divide out the pool)

$$4t + 15t = 60 \text{ hours}$$

$$t = \frac{60}{19} \text{ hours or } 3.158 \text{ hours (approx)}$$

Step 3: Check Answer!

If time is 3.158 hours, the pipes will add

$$3.158 \text{ hrs} \times \frac{1 \text{ pool}}{4 \text{ hours}}$$

$$.79 \text{ pools...}$$

the hose will add

$$3.158 \times \frac{1 \text{ pool}}{15 \text{ hours}}$$

$$.21 \text{ pools...}$$

Example: Tom can paint a fence in 5 hours. Huck can paint a fence in 4 hours.
 On Saturday, they have to paint 3 houses. Huck begins at 9:00am, and Tom shows up late (and begins at noon).. Working together, how long did it take to paint the 3 houses?

Step 1: Write formulas and variables

'task' = rate x time
 In this case, painting the house = (rate)(time)

therefore, rate = $\frac{\text{painting 1 house}}{\text{time}}$

$$\text{rate}_H = \frac{1 \text{ house}}{4 \text{ hours}} \quad (\text{Huck's painting rate})$$

$$\text{rate}_T = \frac{1 \text{ house}}{5 \text{ hours}} \quad (\text{Tom's painting rate})$$

Step 2: Establish equation and solve

We want to paint THREE houses, and we have to adjust the time each spends painting...

Tom paint + Huck paint = 3 houses

$$\text{rate}_T(t_T) + \text{rate}_H(t_H) = 3 \text{ houses}$$

$$\frac{1 \text{ house}}{5 \text{ hours}}(t - 3 \text{ hrs}) + \frac{1 \text{ house}}{4 \text{ hours}}(t) = 3 \text{ houses}$$

(multiply by 20 hours; divide out the 'houses')

$$4(t - 3 \text{ hrs}) + 5(t) = 60 \text{ hours}$$

$$4t - 12 \text{ hrs} + 5t = 60 \text{ hours}$$

$$t = 8 \text{ hours}$$

(note: Tom's painting time is 3 less than Huck's)

Step 3: Check your answer!

Since t = 8 hours, Huck painted for 8 hours and Tom painted for 5 hours..

$$\text{Huck paint} = (8 \text{ hours}) \frac{1 \text{ house}}{4 \text{ hours}} = 2 \text{ houses}$$

$$\text{Tom paint} = (5 \text{ hours}) \frac{1 \text{ house}}{5 \text{ hours}} = 1 \text{ house}$$

Here are a few more examples to examine....

Example: $9x^{-1} + 4(6x + 3)^{-1} = 2(6x + 3)^{-1}$

$$\frac{9}{x} + \frac{4}{(6x + 3)} = \frac{2}{(6x + 3)} \quad \text{change to positive exponents}$$

$$\frac{9}{x} = \frac{2}{(6x + 3)} - \frac{4}{(6x + 3)} \quad \text{collect 'like' terms}$$

$$\frac{9}{x} = \frac{-2}{(6x + 3)} \quad \text{cross multiply}$$

$$54x + 27 = -2x$$

$$56x = -27$$

$$x = \frac{-27}{56}$$

Example: $\frac{1}{x+2} + \frac{3}{x+7} = \frac{5}{x^2+9x+14}$

$$\frac{(x+7)1}{(x+7)(x+2)} + \frac{3(x+2)}{(x+7)(x+2)} = \frac{5}{(x+7)(x+2)} \quad \text{Common denominators}$$

$$x + 7 + 3x + 6 = 5 \quad \text{Simplify}$$

$$4x = -8$$

$$x = -2$$

since $\frac{1}{x+2}$ is undefined at $x = -2$, there is **NO SOLUTION!**

Example: $\frac{1}{x+6} - \frac{3}{x^2-36} = \frac{x}{x-6}$

Since the least common multiple of the denominators is $x^2 - 36$

$$(x^2 - 36) \left(\frac{1}{x+6} - \frac{3}{x^2-36} = \frac{x}{x-6} \right) = \frac{1(x^2-36)}{x+6} - \frac{3(x^2-36)}{x^2-36} = \frac{x(x^2-36)}{x-6}$$

$$\frac{1(x+6)(x-6)}{x+6} - \frac{3(x^2-36)}{x^2-36} = \frac{x(x+6)(x-6)}{x-6}$$

$$1(x-6) - 3 = x(x+6)$$

$$x - 9 = x^2 + 6x$$

$$0 = x^2 + 5x + 9$$

No real solutions

$$x = \frac{-5 \pm i\sqrt{11}}{2}$$

Solving Rational Expressions/Equations

Example: Solve $\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$

Factor the terms to reveal factors...

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2(x-1)}{(x+3)(x-4)}$$

"Reverse the middle term" for convenience...

$$(x+3)(x-4) \left(\frac{2}{x+3} - \frac{-3}{x-4} = \frac{2(x-1)}{(x+3)(x-4)} \right)$$

Multiply by least common denominator... (this removes the denominators!)

$$2(x-4) - -3(x+3) = 2(x-1)$$

$$2x - 8 + 3x + 9 = 2x - 2$$

$$3x = -3$$

$$\boxed{x = -1}$$

Quick check: $\frac{2}{(-1)+3} - \frac{3}{4-(-1)} = \frac{2(-1)-2}{(-1)^2 - (-1) - 12}$

$$\frac{2}{2} - \frac{3}{5} = \frac{-4}{-10}$$

$$\frac{2}{5} = \frac{2}{5} \checkmark$$

Example: Solve $\frac{4}{3x+3} = \frac{12}{x^2-1}$ Cross-multiply

$$36x + 36 = 4x^2 - 4$$
 Divide by 4

$$9x + 9 = x^2 - 1$$
 Collect terms to one side

$$x^2 - 9x - 10 = 0$$

$$(x-10)(x+1) = 0$$

$$x = 10, -1$$

Quick check: If $x = -1$, the right equation is undefined! ~~X~~

If $x = 10$, $\frac{4}{33} = \frac{12}{99} \checkmark$

$$\boxed{x = 10}$$

Example: $\frac{x+2}{6} = \frac{x+2}{x-1}$

Obviously, $x = 7$ is a solution (because the numerators are the same and $6 = (7) - 1$)

Let's go through the steps and cross-multiply...

$$(x+2)(x-1) = (x+2)(6)$$

$$x^2 + x - 2 = 6x + 12$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$\boxed{x = 7 \text{ and } -2}$$

$$0 = 0 \checkmark$$

and

$$3/2 = 3/2 \checkmark$$

Example: Solve and graph $\frac{x+3}{x-1} \geq 2$

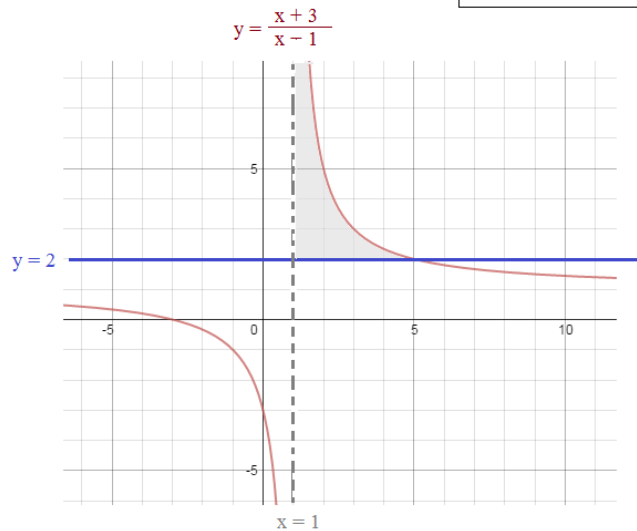
Step 1: Look for "critical values"

$$\frac{x+3}{x-1} = \frac{2}{1}$$

$$x+3 = 2x-2$$

$$x = 5$$

AND, there is a vertical asymptote at $x = 1$



Step 2: Set up number line (with open and closed intervals)



Step 3; Test regions

If $x = 0$: $\frac{0+3}{0-1} = -3 \not\geq 2$

If $x = 3$: $\frac{3+3}{3-1} = 3 \geq 2$

If $x = 7$: $\frac{7+3}{7-1} = 5/3 \not\geq 2$



Example: Solve $\frac{x+2}{x+3} \leq \frac{x-1}{x-2}$

Method 1: Cross-Multiply, Solve, and Check Regions

$$(x+2)(x-2) = (x+3)(x-1)$$

$$x^2 - 4 = x^2 + 2x - 3$$

$$-1 = 2x$$

$$x = -1/2$$

Also, the equations are undefined at $x = -3$ or $x = 2$



asymptotes/undefined points are always open circles. since inequality is less than or equal, it's a closed circle

test -5: $\frac{-5+2}{-5+3} \leq \frac{-5-1}{-5-2} \quad 3/2 \leq 6/7 \quad \text{NO}$

test -2: $\frac{-2+2}{-2+3} \leq \frac{-2-1}{-2-2} \quad 0 \leq 3/4 \quad \text{YES}$

test 0: $\frac{0+2}{0+3} \leq \frac{0-1}{0-2} \quad 2/3 \leq 1/2 \quad \text{NO}$

test 5: $\frac{5+2}{5+3} \leq \frac{5-1}{5-2} \quad 7/8 \leq 4/3 \quad \text{YES}$



Method 2: Combine Terms on one-side and Check Regions

$$\frac{x+2}{x+3} - \frac{x-1}{x-2} \leq 0$$

$$\frac{x+2}{x+3} \cdot \frac{(x-2)}{(x-2)} - \frac{x-1}{x-2} \cdot \frac{(x+3)}{(x+3)} \leq 0$$

$$\frac{x^2-4 - (x^2+2x-3)}{(x+3)(x-2)} \leq 0$$

$$\frac{-2x-1}{(x+3)(x-2)} \leq 0$$

	-3	-1/2	2	
-2x-1	+	+	-	-
x+3	-	+	+	+
x-2	-	-	-	+
All multiplied together	+	-	+	-

Less than zero

$(-3, -1/2] \cup (2, \infty)$

"... if y-y-y-ou invert and multiply... then, c-c-cross cancel... Well, That's "L" Folks! "



Assume $y = s$, *rational expressions*

$$\frac{LOONe^y}{TUNE^s} = \frac{O^2}{UT}$$



Mel Blanc Hall



"What's 'ut', Doc?"

"U, T is the denominator, varmint!"



"That's a goofy looking problem!"

Professor Porky Pig

Despite a slight stutter and bizarre math examples, many enjoyed his Saturday morning Algebra lessons.

Practice Questions ->

Solving Rational Equations Quiz

1) $\frac{x}{2} + \frac{x}{10} = 6$

2) $\frac{3y}{2} + \frac{7}{4} = 1$

3) $\frac{3}{x+1} = \frac{1}{x^2-1}$

4) $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$

- 5) If Jim can paint a fence in 6 hours, and Tim can paint a fence in 4 hours.
How long will it take them to paint a fence *working together*?

Solving Rational Expressions Quiz

$$6) \frac{\frac{4}{x^2 - 25} + \frac{2}{x - 5}}{\frac{1}{x + 5} + \frac{1}{x - 5}} = 1$$

$$7) \frac{2}{x + 1} + \frac{x}{x - 1} = \frac{2}{x^2 - 1}$$

$$8) \frac{6x}{x + 4} + 4 = \frac{2x + 2}{x - 1}$$

$$9) \frac{3}{x + 1} = \frac{27}{x^2 - 1}$$

$$10) \frac{x}{5} + \frac{3x}{x+1} = \frac{36}{5x+5}$$

$$11) \frac{7}{n-3} = \frac{2n+1}{21}$$

$$12) \frac{5}{z+1} + \frac{z}{z+3} = \frac{10z+15}{z^2+4z+3}$$

$$13) \frac{x}{x+3} - \frac{2}{x+6} = \frac{-18}{2(x^2+9x+18)}$$

Rational Expressions Quiz

Simplify:

1) $\frac{x}{3} + \frac{x}{5}$

2) $\frac{1}{a+4} + \frac{3}{a+4}$

3) $\frac{4}{x^2+4x+3} - \frac{1}{x+3}$

4) $\frac{3}{2x+6} + \frac{4}{6x+18}$

5) $\frac{7}{2d} - \frac{3}{2d}$

6) $\frac{2x}{x^2-1} - \frac{3}{x+1}$

7) $\frac{k-10}{20-2k}$

Solve:

1) $\frac{x}{x+2} + \frac{4}{x-2} = 1$

2) $\frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$

3) $\frac{1}{3s} = \frac{s}{2} - \frac{1}{6s}$

4) $\frac{x+2}{x+8} = \frac{x-2}{x+4}$

5) $1 - \frac{3}{z} = \frac{4}{z^2}$

6) $\frac{d}{3} + \frac{1}{2} = \frac{1}{3d}$

7) $\frac{1}{c-3} = \frac{c}{4}$

8) $\frac{5}{x-2} = \frac{5x+10}{x^2}$

Solving Rational Equations Quiz

SOLUTIONS

1) $\frac{x}{2} + \frac{x}{10} = 6$

$\frac{10x}{20} + \frac{2x}{20} = 6$

$\frac{12x}{20} = 6$

$12x = 120$

$x = 10$

Quick Check:

$\frac{(10)}{2} + \frac{(10)}{10} = 6$

$5 + 1 = 6 \checkmark$

2) $\frac{3y}{2} + \frac{7}{4} = 1$ (multiply entire equation by 4)

$\frac{12y}{2} + \frac{28}{4} = 4$

$6y = -3$

$y = \frac{-1}{2}$

Quick Check:

$\frac{3(-1/2)}{2} + \frac{7}{4} = 1$

$\frac{-3}{4} + \frac{7}{4} = 1 \checkmark$

3) $\frac{3}{x+1} = \frac{1}{x^2-1}$ Cross Multiply

$3x^2 - 3 = x + 1$

$3x^2 - x - 4 = 0$

$(3x - 4)(x + 1) = 0$

$x = \frac{4}{3}$ or ~~$x = -1$~~

Check solutions:

-1: EXTRANEIOUS

$\frac{3}{(-1)+1} = \frac{3}{0}$

4/3:

$\frac{3}{(4/3)+1} = \frac{1}{(4/3)^2 - 1}$

$\frac{9}{7} = \frac{9}{7} \checkmark$

4) $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$ combine terms on left side of equation

$\frac{11}{3x} - \frac{x}{3x} = \frac{-4}{x^2}$

$\frac{11-x}{3x} = \frac{-4}{x^2}$ cross multiply

$-12x = -x^3 + 11x^2$

$x^3 - 11x^2 - 12x = 0$ factor/solve

$x(x-12)(x+1) = 0$

$x = \cancel{0}, 12, -1$

check solutions!!

0: extraneous (because undefined) ~~X~~

12: $\frac{11}{36} - \frac{1}{3} = \frac{-4}{144}$

$\frac{-1}{36} = \frac{-1}{36} \checkmark$

-1: $\frac{11}{-3} - \frac{1}{3} = \frac{-4}{1} \checkmark$

5) If Jim can paint a fence in 6 hours, and Tim can paint a fence in 4 hours. How long will it take them to paint a fence *working together*?

amount of fence = (rate)(time)

therefore, $\text{rate} = \frac{\text{fence}}{\text{time}}$

Jim's rate: $\text{rate}_J = \frac{1 \text{ fence}}{6 \text{ hours}}$

Tim's rate: $\text{rate}_T = \frac{1 \text{ fence}}{4 \text{ hours}}$

Working together:

Jim's amount + Tim's amount = 1 fence

$\text{rate}_J(t) + \text{rate}_T(t) = 1 \text{ fence}$

$\frac{1 \text{ fence}}{6 \text{ hours}}(t) + \frac{1 \text{ fence}}{4 \text{ hours}}(t) = 1 \text{ fence}$

Solve (multiply by 24 hours; divide by fence)

$4t + 6t = 24 \text{ hours}$

$t = 12/5 \text{ hours (2 hours, 24 minutes)}$

Check:

Jim: $(1/6)(12/5) = 2/5$

Tim: $(1/4)(12/5) = 3/5$

total: $5/5 \checkmark$

Solving Rational Expressions Quiz

SOLUTIONS

$$6) \frac{\frac{4}{x^2-25} + \frac{2}{x-5}}{\frac{1}{x+5} + \frac{1}{x-5}} = 1$$

numerator $\longrightarrow \frac{4}{(x-5)(x+5)} + \frac{2(x+5)}{(x-5)(x+5)} = \frac{2x+14}{(x-5)(x+5)}$

denominator $\longrightarrow \frac{1 \cdot (x-5)}{(x+5)(x-5)} + \frac{1 \cdot (x+5)}{(x-5)(x+5)} = \frac{2x}{(x-5)(x+5)}$

Strategy: Simplify numerator and denominator separately...

Then, divide!

To divide fractions, we can 'invert and multiply':

$$\frac{2x+14}{(x-5)(x+5)} \cdot \frac{(x-5)(x+5)}{2x} = \frac{2x+14}{2x} = \frac{x+7}{x}$$

$$\frac{x+7}{x} = 1$$

NO SOLUTIONS!

$$7) \frac{2}{x+1} + \frac{x}{x-1} = \frac{2}{x^2-1}$$

Least Common Denominator of the 3 rational expressions is

$$x^2-1$$

multiply entire equation by LCD

$$2(x-1) + x(x+1) = 2$$

$$2x-2 + x^2 + x = 2$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4, \cancel{x=1}$$

Check Solutions:

If $x = 1$, then there are 0's in the denominators of 2 of the expressions...

therefore, it is extraneous... ~~X~~

If $x = -4$, then $\frac{2}{(-4)+1} + \frac{(-4)}{(-4)-1} = \frac{2}{(-4)^2-1}$

$$\frac{2}{-3} + \frac{-4}{-5} = \frac{2}{15}$$

$$\frac{-10}{15} + \frac{12}{15} = \frac{2}{15}$$

x = -4

$$8) \frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$$

Approach: combine terms on left side.. then, cross-multiply..

$$\frac{6x + 4(x+4)}{(x+4)} = \frac{2x+2}{x-1}$$

$$\frac{10x+16}{(x+4)} = \frac{2x+2}{x-1}$$

cross multiply: FOIL

$$10x^2 + 16x - 10x - 16 = 2x^2 + 8x + 2x + 8$$

$$8x^2 - 4x - 24 = 0$$

$$4(2x^2 - x - 6) = 0$$

$$4(2x+3)(x-2) = 0$$

$$x = -3/2, 2$$

Test answers: If $x = 2$, $\frac{6(2)}{(2)+4} + 4 = \frac{2(2)+2}{(2)-1}$
 $2 + 4 = 6$ ✓

If $x = -3/2$,

$$\frac{6(-3/2)}{(-3/2)+4} + 4 = \frac{2(-3/2)+2}{(-3/2)-1}$$

$$\frac{-9}{(5/2)} + 4 = \frac{-1}{(-5/2)}$$

$$\frac{-18}{5} + \frac{20}{5} = \frac{2}{5}$$
 ✓

$$9) \frac{3}{x+1} = \frac{27}{x^2-1}$$

Cross multiply: $3x^2 - 3 = 27x + 27$

Collect terms, factor, and solve: $3x^2 - 27x - 30 = 0$

$$3(x^2 - 9x - 10) = 0$$

$$3(x+1)(x-10) = 0$$

$$x = \cancel{-1}, 10$$

Check solutions: $x = -1$:

$$\frac{3}{(-1)+1} = \frac{27}{(-1)^2-1}$$

$$\frac{3}{0} \text{ is undefined;}$$

-1 is extraneous!

x = 10

$$\frac{3}{(10)+1} = \frac{27}{(10)^2-1}$$

$$\frac{3}{11} = \frac{27}{99}$$
 ✓

$$10) \frac{x}{5} + \frac{3x}{x+1} = \frac{36}{5x+5}$$

Since least common denominator / least common multiple is $5x+5$,

$$5x+5 \left(\frac{x}{5} + \frac{3x}{x+1} = \frac{36}{5x+5} \right) = x(x+1) + 3x(5) = 36$$

$$= x^2 + x + 15x - 36 = 0$$

$$= (x-2)(x+18) = 0$$

SOLUTIONS

$$x = 2, -18$$

Check: $\frac{(2)}{5} + \frac{3(2)}{(2)+1} = \frac{36}{5(2)+5}$

$$2 \frac{2}{5} = \frac{36}{15} \checkmark$$

$$\frac{(-18)}{5} + \frac{3(-18)}{(-18)+1} = \frac{36}{5(-18)+5}$$

$$\frac{-18}{5} + \frac{-54}{-17} = \frac{36}{-85} \quad \frac{306}{-85} + \frac{-270}{-85} = \frac{36}{-85} \checkmark$$

$$11) \frac{7}{n-3} = \frac{2n+1}{21}$$

Cross multiply...

$$(n-3)(2n+1) = (7)(21)$$

$$2n^2 - 5n - 3 = 147$$

$$2n^2 - 5n - 150 = 0$$

$$(2n+15)(n-10) = 0$$

$$n = -15/2, 10$$

Check: $\frac{7}{(-7.5)-3} = \frac{2(-7.5)+1}{21}$

$$\frac{7}{-10.5} = \frac{-14}{21} \checkmark$$

$$\frac{7}{(10)-3} = \frac{2(10)+1}{21}$$

$$\frac{7}{7} = \frac{21}{21} \checkmark$$

$$12) \frac{5}{z+1} + \frac{z}{z+3} = \frac{10z+15}{z^2+4z+3}$$

Since $(z+3)(z+1)$ is common denominator,

$$(z+3)(z+1)$$

$$\frac{5(z+3)}{(z+1)(z+3)} + \frac{z(z+1)}{(z+3)(z+1)} = \frac{10z+15}{(z+3)(z+1)}$$

Eliminate all the denominators and collect terms...

$$5z + 15 + z^2 + z = 10z + 15$$

$$z^2 - 4z = 0$$

$$z(z-4) = 0$$

$$z = 0, 4$$

Check: $\frac{5}{(0)+1} + \frac{(0)}{(0)+3} = \frac{10(0)+15}{(0)+4(0)+3}$

$$5 + 0 = 5 \checkmark$$

$$\frac{5}{(4)+1} + \frac{(4)}{(4)+3} = \frac{10(4)+15}{(4)^2+4(4)+3}$$

$$1 \frac{4}{7} = \frac{55}{35} = \frac{11}{7} \checkmark$$

$$13) \frac{x}{x+3} - \frac{2}{x+6} = \frac{-18}{2(x^2+9x+18)}$$

$$\frac{2x(x+6)}{2(x+3)(x+6)} - \frac{2(2)(x+3)}{2(x+6)(x+3)} = \frac{-18}{2(x+3)(x+6)}$$

$$2x^2 + 12x - (4x + 12) = -18$$

$$2x^2 + 8x + 6 = 0$$

$$2(x+1)(x+3) = 0$$

$$x = -1, -3$$

Check: if $x = -3$, then $\frac{x}{x+3}$ is undefined..

if $x = -1$, then

$$\frac{(-1)}{(-1)+3} - \frac{2}{(-1)+6} = \frac{-18}{2(10)}$$

$$-1/2 - (2/5) = -9/10 \checkmark$$

-3 is extraneous...

Rational Expressions Quiz

SOLUTIONS

Simplify:

$$1) \frac{x}{3} + \frac{x}{5}$$

$$\frac{5x}{15} + \frac{3x}{15} =$$

$$\frac{8x}{15}$$

$$2) \frac{1}{a+4} + \frac{3}{a+4}$$

$$\frac{4}{a+4}$$

$$3) \frac{4}{x^2+4x+3} - \frac{1}{x+3}$$

$$\frac{4}{(x+1)(x+3)} - \frac{1}{(x+3)} =$$

$$\frac{4}{(x+1)(x+3)} - \frac{(x+1)}{(x+1)(x+3)} =$$

$$\frac{3-x}{(x+1)(x+3)}$$

$$4) \frac{3}{2x+6} + \frac{4}{6x+18}$$

$$\frac{3}{2(x+3)} + \frac{4}{6(x+3)} =$$

$$\frac{9}{6(x+3)} + \frac{4}{6(x+3)} =$$

$$\frac{13}{6(x+3)}$$

$$5) \frac{7}{2d} - \frac{3}{2d}$$

$$\frac{4}{2d} =$$

$$\frac{2}{d}$$

$$6) \frac{2x}{x^2-1} - \frac{3}{x+1}$$

$$\frac{2x}{(x+1)(x-1)} - \frac{3}{(x+1)} =$$

$$\frac{2x}{(x+1)(x-1)} - \frac{3(x-1)}{(x+1)(x-1)} =$$

$$\frac{-x+3}{(x+1)(x-1)} = \frac{3-x}{x^2-1}$$

$$7) \frac{k-10}{20-2k}$$

$$\frac{k-10}{-2(k-10)} =$$

$$-\frac{1}{2}$$

Solve: (Plug in to check solutions)

$$1) \frac{x}{x+2} + \frac{4}{x-2} = 1$$

$$\frac{x(x-2)}{(x+2)(x-2)} + \frac{4(x+2)}{(x-2)(x+2)} = 1$$

$$\frac{x^2-2x+4x+8}{x^2-4} = \frac{1}{1}$$

$$\frac{x^2-2x+4x+8}{x^2-4} = \frac{(-6)}{(-6)+2} + \frac{4}{(-6)-2}$$

$$x^2-4 = x^2+2x+8$$

$$\frac{3}{2} + \frac{4}{-8} = 1 \checkmark$$

$$x = -6$$

$$2) \frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$$

Use common denominator

$$\frac{6}{6x} + \frac{3}{6x} = \frac{x}{6x}$$

$$\frac{9}{6x} = \frac{x}{6x}$$

$$\frac{1}{(9)} + \frac{1}{2(9)} = \frac{1}{6}$$

$$\frac{2}{18} + \frac{1}{18} = \frac{3}{18} \checkmark$$

$$x = 9$$

$$3) \frac{1}{3s} = \frac{s}{2} - \frac{1}{6s}$$

multiply by 6s

$$2 = 3s^2 - 1$$

$$3s^2 = 3$$

$$s = 1, -1$$

$$\frac{1}{3(1)} = \frac{(1)}{2} - \frac{1}{6(1)}$$

$$\frac{1}{3} = \frac{1}{2} - \frac{1}{6} \checkmark$$

$$\frac{1}{3(-1)} = \frac{(-1)}{2} - \frac{1}{6(-1)}$$

$$-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6} \checkmark$$

$$4) \frac{x+2}{x+8} = \frac{x-2}{x+4}$$

cross multiply

$$x^2+2x+4x+8 = x^2+8x-2x-16$$

$$6x+8 = 6x-16$$

$$\text{No solution!}$$

$$5) 1 - \frac{3}{z} = \frac{4}{z^2}$$

multiply by z^2

$$z^2 - 3z = 4$$

$$z^2 - 3z - 4 = 0$$

$$(z-4)(z+1) = 0$$

$$z = 4, -1$$

$$1 - \frac{3}{(4)} = \frac{4}{(4)^2}$$

$$1 - 3/4 = 1/4 \checkmark$$

$$1 - \frac{3}{(-1)} = \frac{4}{(-1)^2}$$

$$1 - (-3) = 4/1 \checkmark$$

$$6) \frac{d}{3} + \frac{1}{2} = \frac{1}{3d}$$

multiply by 6d

$$2d^2 + 3d = 2$$

$$2d^2 + 3d - 2 = 0$$

$$(2d-1)(d+2) = 0$$

$$d = 1/2, -2$$

$$\frac{(-2)}{3} + \frac{1}{2} = \frac{1}{3(-2)}$$

$$-\frac{4}{6} + \frac{3}{6} = \frac{1}{-6} \checkmark$$

$$\frac{(1/2)}{3} + \frac{1}{2} = \frac{1}{3(1/2)}$$

$$\frac{1}{6} + \frac{3}{6} = \frac{2}{3} \checkmark$$

$$7) \frac{1}{c-3} = \frac{c}{4}$$

cross multiply

$$c^2 - 3c = 4$$

$$c^2 - 3c - 4 = 0$$

$$(c-4)(c+1) = 0$$

$$c = 4, -1$$

$$\frac{1}{(4)-3} = \frac{(4)}{4} \checkmark$$

$$\frac{1}{(-1)-3} = \frac{(-1)}{4} \checkmark$$

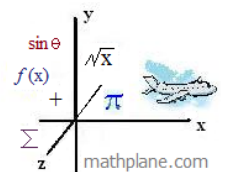
$$8) \frac{5}{x-2} = \frac{5x+10}{x^2}$$

cross multiply

$$5x^2 = 5x^2 - 10x + 10x - 20$$

$$5x^2 = 5x^2 - 20$$

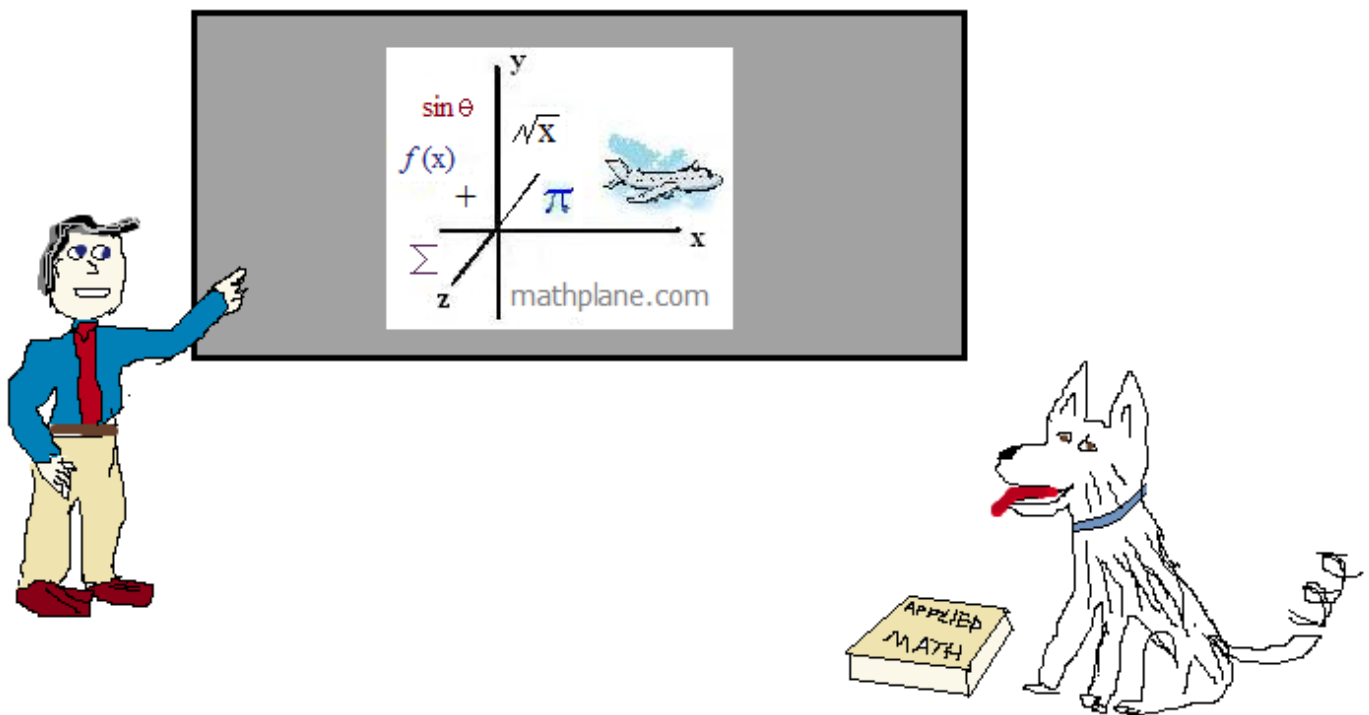
$$\text{No Solution!}$$



Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, Google+, Teacherspayteachers, TES, and Pinterest

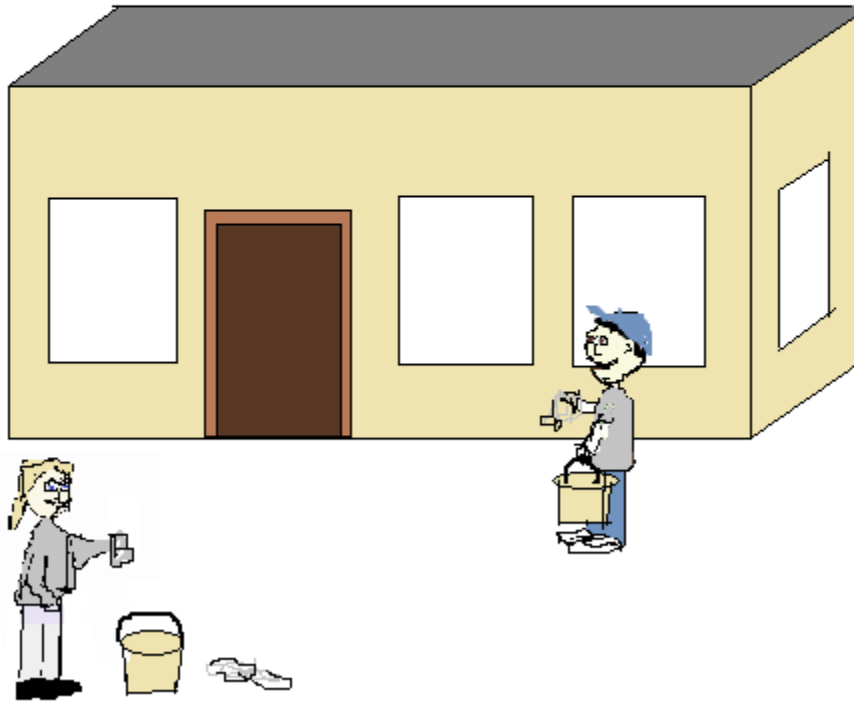
One more....

Work - Rate Question

A girl can wash 10 windows in 45 minutes...

Working with her brother, together, they can wash 5 windows in 10 minutes.

Working by himself, how fast can the brother wash 15 windows?



ANSWER on the next page....



A girl can wash 10 windows in 45 minutes...
 Working with her brother, together, they can wash 5 windows in 10 minutes.
 Working by himself, how fast can the brother wash 15 windows?

$$\text{output} = \text{rate} \cdot \text{time}$$

(set up a work-rate equation)

$$\frac{5 \text{ windows}}{10 \text{ minutes}} (\text{time}) = \frac{10 \text{ windows}}{45 \text{ minutes}} (\text{time}) + \frac{15 \text{ windows}}{x \text{ minutes}} (\text{time})$$

together output
girl's output
brother's output

Solve for x

common denominator of 90 minutes; divide by 'windows'

$$\frac{45(t)}{90 \text{ minutes}} = \frac{20(t)}{90 \text{ minutes}} + \frac{15(t)}{x \text{ minutes}}$$

multiply by 'minutes'; combine terms

$$\frac{25t}{90} = \frac{15t}{x}$$

cross multiply ; divide by t

$$1350 = 25x$$

$$x = 54$$

The brother can wash
15 windows in 54 minutes...