Trigonometry Word Problems

Applications of Right Triangles and Trig Functions

Includes angle of elevation and depression, examples, step-by-step solutions, and more...
Trigonometry Word Problems

**Example:** You fly a kite 4 feet off the ground with 300 feet of string. There is a 40 mile per hour wind, and the kite forms a 29° angle from the ground. How high is the kite (from the ground)?

Draw a picture
and
label the parts

<table>
<thead>
<tr>
<th>300'</th>
<th>29°</th>
</tr>
</thead>
<tbody>
<tr>
<td>4'</td>
<td>4'</td>
</tr>
</tbody>
</table>

Isolate the triangle and
Solve

Since we have a right triangle -- with an angle and hypotenuse -- we can use the sine function to find the "opposite" side.

\[
\sin(29°) = \frac{y}{300'}
\]

\[
y = 300'(0.485) = 145.4'
\]

Answer the question! Since the triangle is 4 feet off the ground, we need to add 4' to determine the height of the kite from the ground.

Therefore, the kite is approximately 149.4' from the ground.

**Example:** A cable is attached to a pole 10 meters high. If the other end is attached to the ground 8 meters from the base of the pole. How long is the cable?

Draw a picture and label the parts

\[
\begin{align*}
10m & \quad \text{cable} \\
8m &
\end{align*}
\]

Isolate the triangle and solve

Since it is a right triangle, Pythagorean theorem will determine the length of the cable...

\[
C^2 = 164
\]

Length of Cable = \( \sqrt{164} \)

\[
= 2 \sqrt{41}
\]

approx. 12.8 meters
Trigonometry Word Problems

Example: A diver stands on a diving board above 2 swimmers. The angle of depression from the diver to each swimmer is 30 and 45 degrees. If the swimmers are 6 feet apart, how high is the diving board?

Step 1: Draw a picture

Step 2: Determine the right triangle(s)

Step 3: Solve
\[ \tan(45) = \frac{\text{height}}{x} \]
\[ \tan(30) = \frac{\text{height}}{x + 6} \]

\[ x\tan(45) = (x + 6)\tan(30) \]
\[ x = (x + 6)(0.577) \]
\[ x = 0.577x + 3.464 \]
\[ 0.423x = 3.464 \]
\[ x = 8.19 \text{ feet} \]

Step 4: Check

Then, if \( x = 8.18 \ldots \)

\[ 30-60-90 \text{ triangle...} \]
\[ 8.18 \times \frac{1}{\sqrt{3}} = 14.184 \]

Example: Two players stand on a basketball court. The angles of elevation from the foot of each player to the 10’ high basket are 40 and 50 degrees. How far apart are the players from each other?

Step 1: Sketch

Step 2: Identify triangles and label

Step 3: Solve
\[ \tan(50) = \frac{10’}{x} \]
\[ \tan(40) = \frac{10’}{x + y} \]

\[ x = \frac{10’}{0.119} = 8.39 \]
\[ x + y = \frac{10’}{0.84} = 11.91 \]

therefore, \( y = 3.52 \text{ feet} \)
Angle of Elevation vs. Angle of Depression

Recognizing and identifying angle of elevation or angle of depression can be confusing. Perhaps, this example will clarify the differences...

Example: An airplane is flying 4000 feet above the ground. If the angle of depression to the airport runway is 12 degrees, how far is the airplane from the runway? (what is the "ground distance"?)

Step 1: Draw a picture

Step 2: Label the parts

Step 3: Isolate the triangle

The angle of depression (from the plane to the runway) is 12 degrees. Therefore the angle of elevation (from the runway to the plane) is also 12 degrees!

NOTE: From geometry theorems, "if parallel lines cut by a transversal, then alternate interior angles are congruent"...

Step 4: Solve

$$\tan(12^\circ) = \frac{4000'}{d}$$

$$d = \frac{4000'}{\tan(12^\circ)} = 18,818 \text{ feet}$$

Step 5: Answer the question

The "ground distance" of the plane to the runway is approximately 18,818 feet

Notice, the angle of depression begins at the airplane's horizon level and drops
1) One diagonal of a rhombus makes an angle of $29^\circ$ with a side of the rhombus. If each side of the rhombus has a length of 7.2", find the lengths of the diagonals.

2) An observer on a cliff 1200 feet above sea level sights two ships due East. The angles of depression to the ships are $48^\circ$ and $33^\circ$. What is the distance between the ships?

3) I'm standing on a 50 foot cliff, looking at my two dogs sitting on the beach below. If my line of sight is 6' above the ground and the angles of depression are $51^\circ$ and $37^\circ$, how far apart are the dogs?

4) Suppose a tree 40' tall casts a shadow of length 60'. What is the angle of elevation (with respect to the ground) from the end of the shadow to the top of the tree?
5) Two boats leave a dock at the same time. Boat A goes due North 500 feet and stops. Boat B goes due East 400 feet, stops and turns toward Boat A. What angle must B turn to face and proceed directly to Boat A?

6) The *angle of elevation* from the top of a small building to the top of a nearby tall building is 47 degrees. And, the *angle of depression* from the top of the small building to the bottom of the tall building is 15 degrees. If the smaller building is 30 feet high, determine the height of the tall building.

7) The distance from the bottom of a ramp to the back of a moving truck is 11 feet. If the angle between ramp and the ground is 21° 20', how high is the back of the truck off the ground?
"The angle of elevation is 68 degrees. And, I've used 1890 feet of string. Look, we can estimate how high the kite is!"

"Benny, I think a storm is coming. Perhaps we should go inside?"

"Where is the key to the cabin?"

During his math assignment, Franklin makes another shocking discovery!

What is the approximate height of the kite?
Hint: It's the year of Ben Franklin's famous kite experiment!

SOLUTIONS →
1) One diagonal of a rhombus makes an angle of 20° with a side of the rhombus. If each side of the rhombus has a length of 7.2", find the lengths of the diagonals.

Draw a sketch:

Since it is a rhombus, we know all the sides are 7.2".
- The opposite angles are congruent
- The adjacent angles are supplementary.
- The diagonals are perpendicular.

Label the rest:

If we isolate the triangle, we have a hypotenuse length (7.2") and measure of the angles. Using trig functions, we can find the lengths of the legs!

Solve:

\[
sin 20° = \frac{y}{7.2} \quad (0.3445) \times 7.2 = 3.01
\]

Therefore, the minor diagonal is approximately 6.09 inches.

\[
cos 20° = \frac{x}{7.2} \quad (0.9397) \times 7.2 = 6.55
\]

Therefore, the major diagonal is approximately 12.60 inches.

2) An observer on a cliff 1200 feet above sea level sights two ships due East. The angles of depression to the ships are 48° and 33°. What is the distance between the ships?

Draw a sketch:

"Isolate triangles":

Solve:

\[
tan 48° = \frac{1200}{x}
\]

1.1106 = \frac{1200}{x} \quad y = 768'

\[
x = 1080'
\]

\[
tan 33° = \frac{1200}{x+y}
\]

0.6494 = \frac{1200}{x+y} \quad (x+y) = 1848'

Note: Angle of depression is measured "going down".

angle of depression is increasing as the lines go down

angle of elevation = angle of depression

(geometry theorem: if parallel lines are cut by a transversal, then the opposite interior angles are congruent.)
3) I'm standing on a 50 foot cliff, looking at my two dogs sitting on the beach below. If my line of sight is 6° above the ground and the angles of depression are 51° and 37°, how far apart are the dogs?

**Draw a Sketch:**

```
50'  
6°    37°  51°  
X       Y
```

"Isolate Triangles"

```
\begin{align*}
\text{Solve:} & \\
\tan 51° &= \frac{56}{X} & \tan 37° &= \frac{56}{(X + Y)} \\
& 1.235 = \frac{56}{X} & \quad .754 = \frac{56}{(X + Y)} \\
& X = 45.3' & \quad X + Y = 74.3' \\
\text{(The dogs are 29' apart)}
\end{align*}
```

4) Suppose a tree 40' tall casts a shadow of length 60'. What is the angle of elevation (with respect to the ground) from the end of the shadow to the top of the tree?

**Draw a sketch:**

```
\begin{align*}
\text{"Isolate Triangle" and Solve:} & \\
\tan X &= \frac{40'}{60'} \\
\text{ArcTan}.667 &= 33.7°
\end{align*}
```
5) Two boats leave a dock at the same time. Boat A goes due North 500 feet and stops. Boat B goes due East 400 feet, stops and turns toward Boat A. What angle must B turn to face and proceed directly to Boat A?

First, use trig functions to find angle x...

\[ \tan(x) = \frac{500}{400} = 1.25 \]

\[ x = \tan^{-1}(1.25) = 51.43^\circ \]

Then, answer the question...

Since Boat B is facing due East, it must turn \(128.57^\circ\) to face Boat A.

6) The angle of elevation from the top of a small building to the top of a nearby tall building is 47 degrees. And, the angle of depression from the top of the small building to the bottom of the tall building is 15 degrees. If the smaller building is 30 feet high, determine the height of the tall building.

Step 1: Draw a diagram

Step 2: Extract the right triangles

Step 3: Use trig functions to solve

7) The distance from the bottom of a ramp to the back of a moving truck is 11 feet. If the angle between ramp and the ground is 21°20', how high is the back of the truck off the ground?

\[ \sin(21.33^\circ) = \frac{\text{height}}{11'} \]

height = 4.00 feet
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Also, find more advanced trig word problems throughout the site.

Good luck!
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