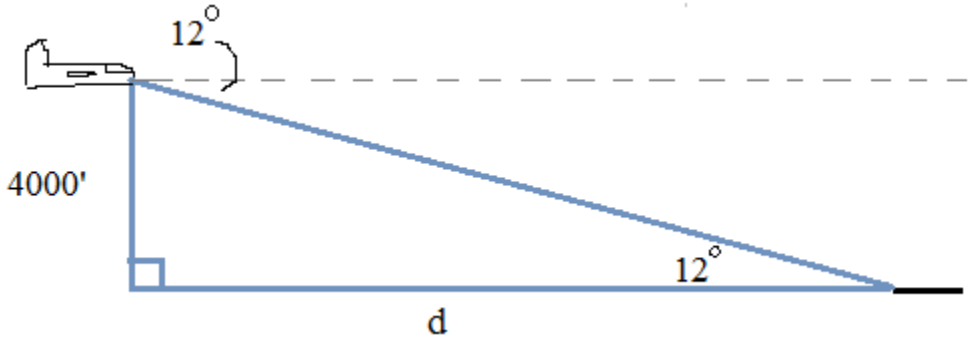
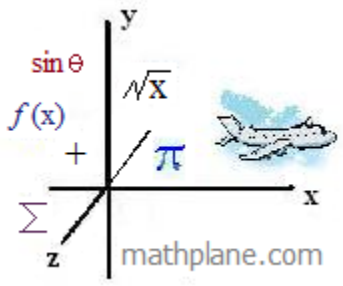
Trigonometry Word Problems

Applications of Right Triangles and Trig Functions



Includes angle of elevation and depression, examples, step-by-step solutions, and more…



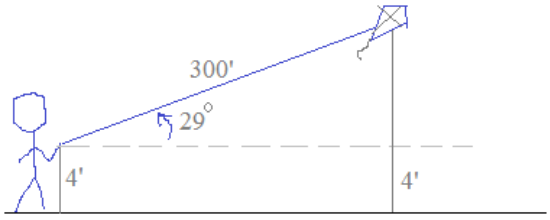
Trigonometry Word Problems

Basic Steps:

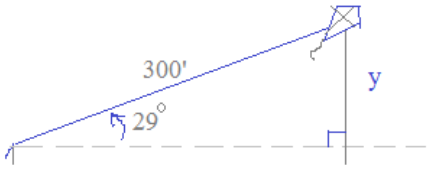
1) Draw a picture
2) Label the parts
3) Isolate the triangle
4) Solve
5) Answer the question

Example: You fly a kite 4 feet off the ground with 300 feet of string. There is a 40 mile per hour wind, and the kite forms a 29° angle from the ground.
How high is the kite (from the ground)?

Draw a picture and label the parts



Isolate the triangle and Solve



Since we have a right triangle -- with an angle and hypotenuse -- we can use the sine function to find the "opposite" side.

$$\sin(29^\circ) = \frac{y}{300'}$$

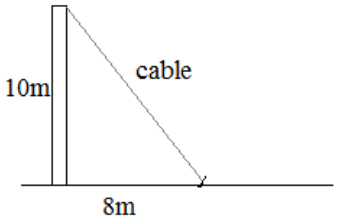
$$y = 300'(.485) = 145.4'$$

Answer the question! Since the triangle is 4 feet off the ground, we need to add 4' to determine the height of the kite *from the ground*.

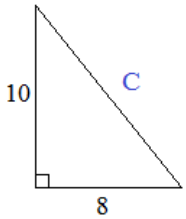
Therefore, the kite is approximately 149.4' from the ground.

Example: A cable is attached to a pole 10 meters high. If the other end is attached to the ground 8 meters from the base of the pole. How long is the cable?

Draw a picture and label the parts



Isolate the triangle and solve



Since it is a right triangle, Pythagorean theorem will determine the length of the cable...

$$8^2 + 10^2 = C^2$$

$$C^2 = 164$$

$$\text{Length of Cable} = \sqrt{164}$$

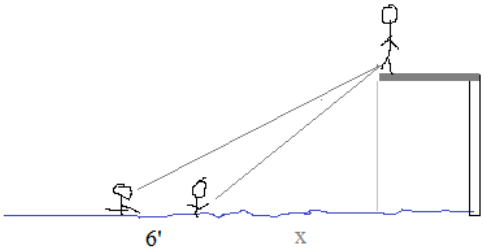
$$= 2\sqrt{41}$$

approx. 12.8 meters

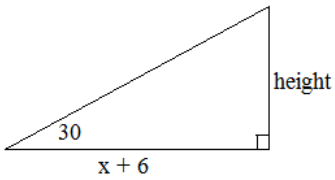
Trigonometry Word Problems

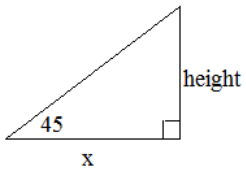
Example: A diver stands on a diving board above 2 swimmers.
The angle of depression from the diver to each swimmer
is 30 and 45 degrees.
If the swimmers are 6 feet apart, how high is the diving board?

Step 1: Draw a picture



Step 2: Determine the right triangle(s)





Step 3: Solve

$\tan(45) = \frac{(\text{height})}{x}$ $\quad h = x\tan(45)$

$\tan(30) = \frac{(\text{height})}{x+6}$ $\quad h = (x+6)\tan(30)$

$x\tan(45) = (x+6)\tan(30)$

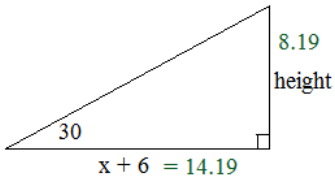
$x(1) = (x+6)(.577)$

$x = .577x + 3.464$

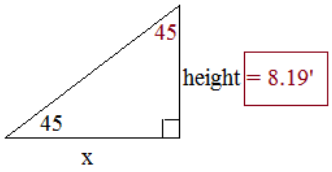
$.423x = 3.464$

$x = 8.19$ feet

Step 4: Check



Then, if x = 8.18....

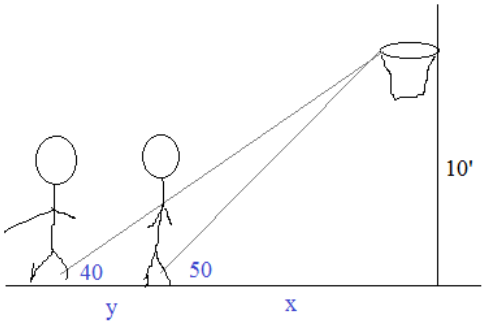


30-60-90 triangle...

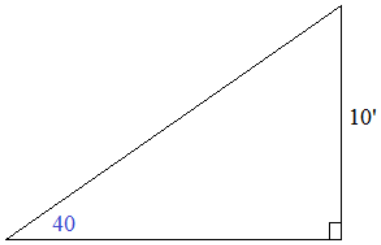
8.18 x $\sqrt{3}$ = 14.184 ✓

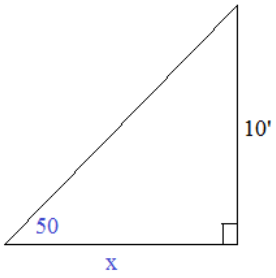
Example: Two players stand on a basketball court.
The angles of elevation from the foot of each player to the 10' high basket are
40 and 50 degrees.
How far apart are the players from each other?

Step 1: Sketch



Step 2: Identify triangles and label





Step 3: Solve

$\tan(50) = \frac{10'}{x}$ $\qquad \tan(40) = \frac{10'}{x+y}$

$x = \frac{10'}{1.19} = 8.39$ $\qquad x + y = \frac{10'}{.84} = 11.91$

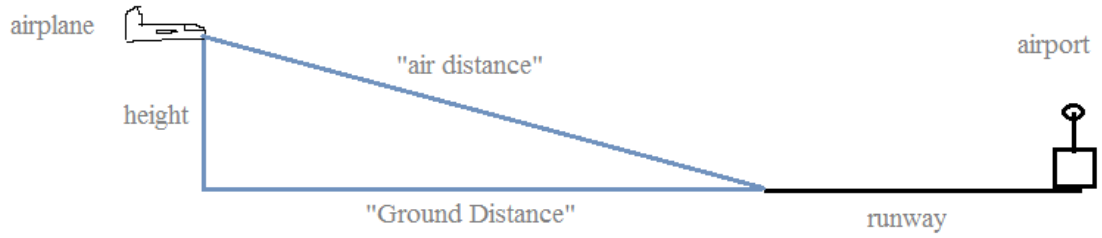
therefore, y = 3.52 feet

Angle of Elevation vs. Angle of Depression

Recognizing and identifying angle of elevation or angle of depression can be confusing.
Perhaps, this example will clarify the differences...

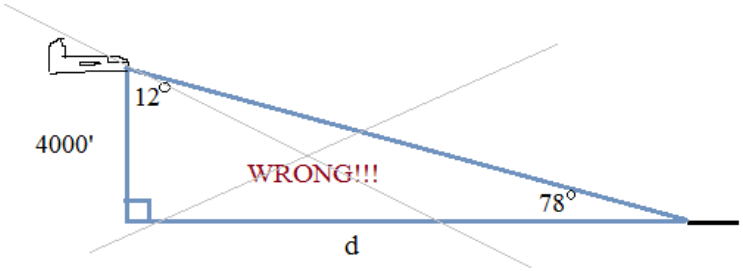
Example: An airplane is flying 4000 feet above the ground. If the angle of depression to the airport runway is 12 degrees, how far is the airplane from the runway? (what is the "ground distance?")

Step 1: Draw a picture



Step 2: Label the parts

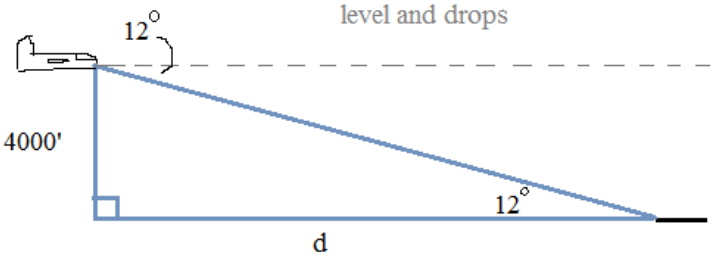
Step 3: Isolate the triangle



The *angle of depression* (from the plane to the runway) is 12 degrees..
Therefore the *angle of elevation* (from the runway to the plane) is also 12 degrees!

NOTE: From geometry theorems, "if parallel lines cut by a transversal, then *alternate interior angles* are congruent"...

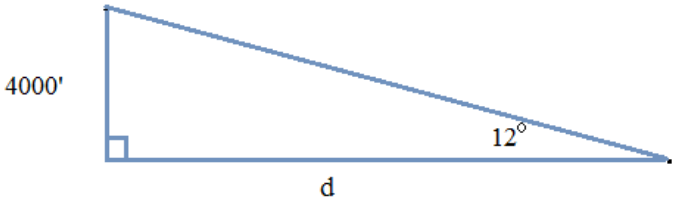
Notice, the angle of depression begins at the airplane's horizon level and drops



Step 4: Solve

$$\tan(12^\circ) = \frac{4000'}{d}$$

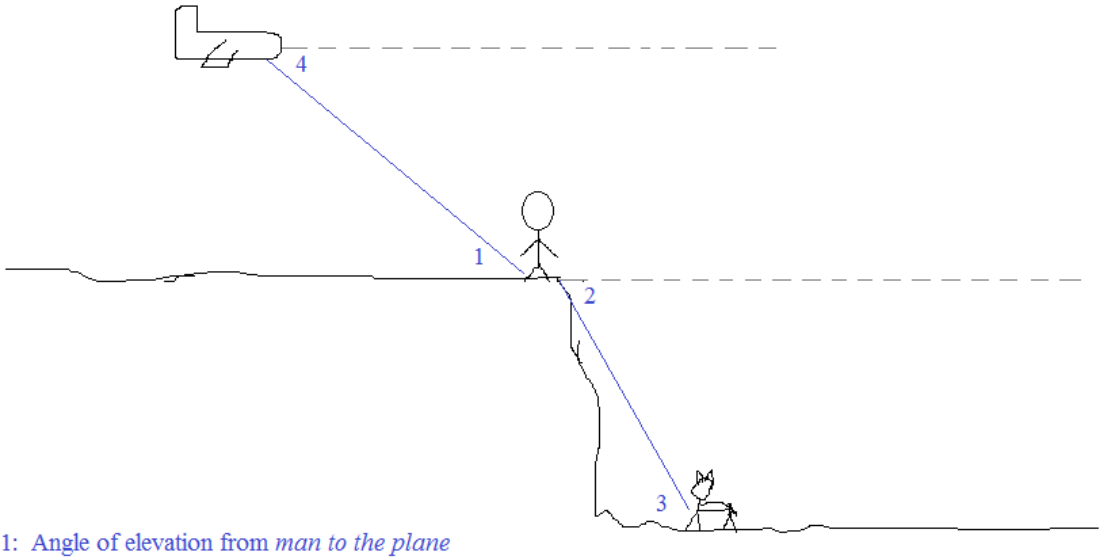
$$d = \frac{4000'}{\tan(12^\circ)} = 18{,}818 \text{ feet}$$



Step 5: Answer the question

The "ground distance" of the plane to the runway is approximately 18,818 feet

Example: Describe the angles as they relate to the objects in the diagram



Angle 1: Angle of elevation from *man to the plane*

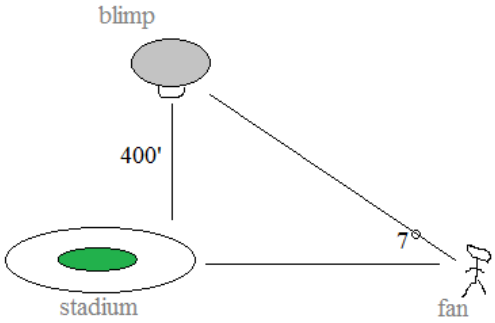
Angle 2: Angle of depression from *man to animal*

Angle 3: Angle of elevation from *animal to man*

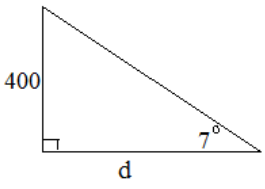
Angle 4: Angle of depression from *plane to man*

Example: A blimp hovers 400 feet above a stadium. A fan is standing outside the stadium. To directly view the blimp, the fan looks up at an angle (of elevation) of 7 degrees. Approximately, how far from the stadium is the fan standing?

Step 1: Draw a diagram



Step 2: Extract the right triangle

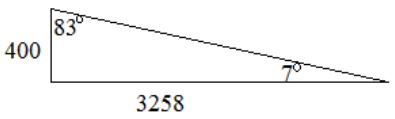


$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

Step 3: Solve

$$\tan(7^\circ) = \frac{400}{d} \qquad \frac{400}{.1228} \approx 3258 \text{ feet}$$

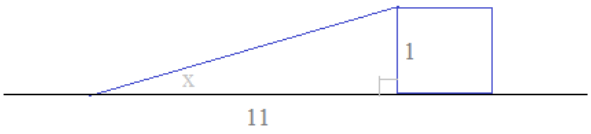
Step 4: Check for reasonableness



the side opposite the 83 degree angle is significantly larger than the side opposite the 7 degree angle...

Trigonometry Word Problems

Example: The slope of a ramp is $\frac{1}{11}$. What is the angle of elevation of the ramp?

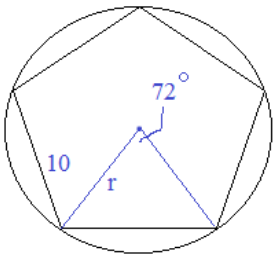


Slope is $\frac{\text{"rise"}}{\text{"run"}}$ $\tan(x) = \frac{1}{11}$ $\frac{\text{opposite}}{\text{adjacent}}$

$x = \tan^{-1}\left\langle \frac{1}{11} \right\rangle = 5.19^\circ$

Example: A regular pentagon inscribed in a circle has a perimeter 50.
What is the radius of the circle?

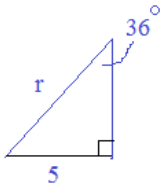
Step 1: Draw a sketch and label



Perimeter = 50, so each side is 10

5 triangles -- 360 degrees.. each part is 72°

Step 2: Extract the right triangle



To find r, we use the right triangle...

Step 3: Solve

$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin(36^\circ) = \frac{5}{r}$

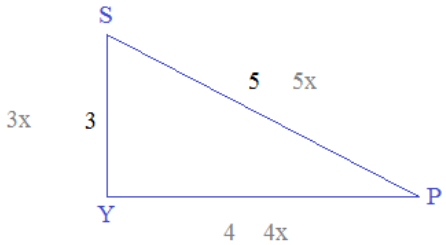
$r = \frac{5}{\sin(36^\circ)} = 8.51$ (approximately)

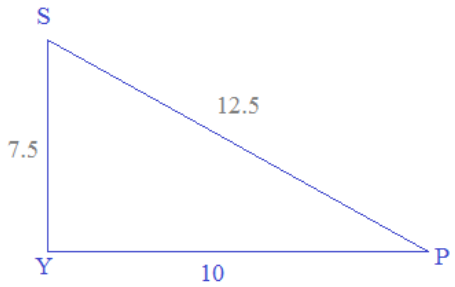
Geometry/Trigonometry

Example: In right triangle SPY,

$\sin \angle P = \frac{3}{5}$ If $\overline{PY} = 10$, what is $\overline{SP}$?

Looking at angle P, the *ratio* of the opposite side to the hypotenuse is 3/5.





Then, we set up a proportion to find the actual lengths of the sides....

$\frac{SY}{10} = \frac{3}{4}$ $SY = 7.5$

$\frac{SP}{5} = \frac{10}{4}$ $SP = 12.5$

Therefore, the adjacent side must be 4. (Pythagorean Triple 3-4-5)

Trigonometry Word Problems

1) One diagonal of a rhombus makes an angle of 29° with a side of the rhombus. If each side of the rhombus has a length of 7.2", find the lengths of the diagonals.

2) An observer on a cliff 1200 feet above sea level sights two ships due East. The angles of depression to the ships are 48° and 33°. What is the distance between the ships?

3) I'm standing on a 50 foot cliff, looking at my two dogs sitting on the beach below. If my line of sight is 6' above the ground and the angles of depression are 51° 37°, how far apart are the dogs?

4) Suppose a tree 40' tall casts a shadow of length 60'. What is the angle of elevation (with respect to the ground) from the end of the shadow to the top of the tree?

Trigonometry Word Problems

5) Two boats leave a dock at the same time. Boat A goes due North 500 feet and stops.
Boat B goes due East 400 feet, stops and turns toward Boat A.
What angle must B turn to face and proceed directly to Boat A?

6) The *angle of elevation* from the top of a small building to the top of a nearby tall building is 47 degrees.
And, the *angle of depression* from the top of the small building to the bottom of the tall building is 15 degrees.
If the smaller building is 30 feet high, determine the height of the tall building.

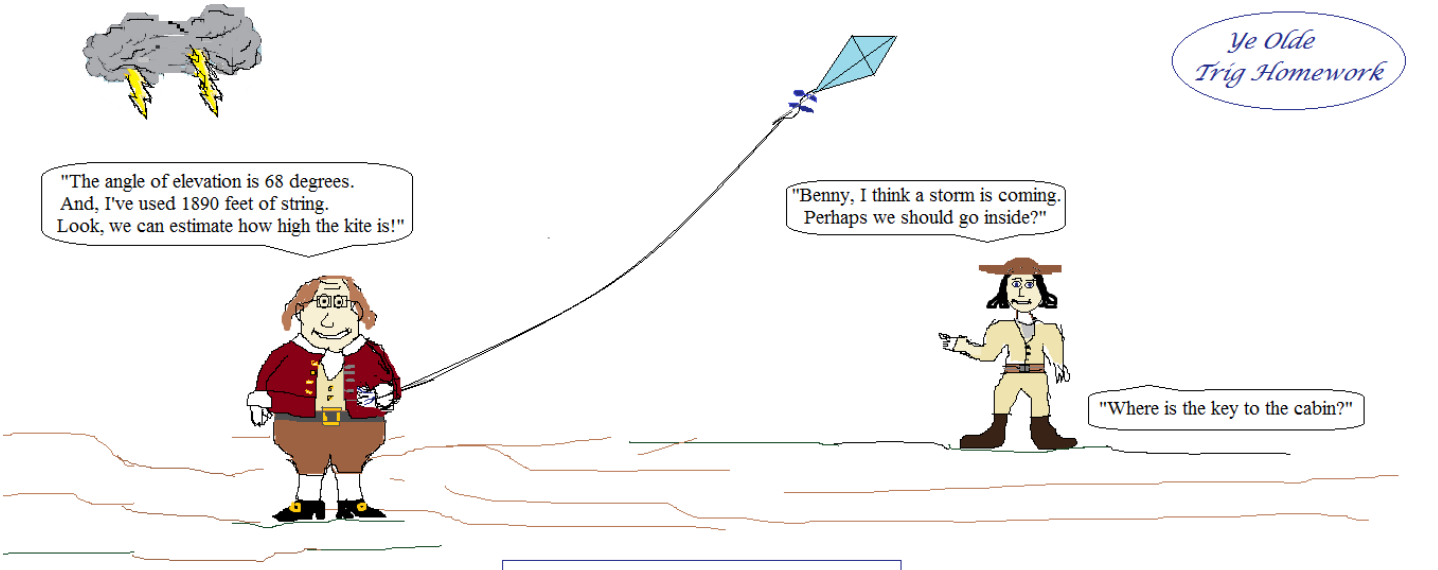
7) The distance from the bottom of a ramp to the back of a moving truck is 11 feet.
If the angle between ramp and the ground is $21^{\circ}20'$, how high is the back of the truck off the ground?

Trigonometry Word Problems

8) Standing in a lighthouse, 150 feet above the shore, I spot a boat at an angle of depression of 11 degrees. How far away is the boat from shore?

9) Looking out from a balcony, the angle of elevation to the top of the next building is approximately 22°. And, the angle of depression to the bottom of the building is approximately 29°. If the building is 200 feet away, how tall is it?

10) A hiker approximates an angle of elevation to the top of a hill to be 22 degrees. After walking 700 feet closer, the hiker estimates the angle of elevation *increased* by 16 degrees. Approximately, how high is the hill?


Ye Olde
Trig Homework
"The angle of elevation is 68 degrees.
And, I've used 1890 feet of string.
Look, we can estimate how high the kite is!"
"Benny, I think a storm is coming.
Perhaps we should go inside?"
"Where is the key to the cabin?"
During his math assignment,
Franklin makes another shocking discovery!
What is the approximate height of the kite?
Hint: It's the year of Ben Franklin's
famous kite experiment!
LanceAF #33 5-20-12
www.mathplane.com

SOLUTIONS -→

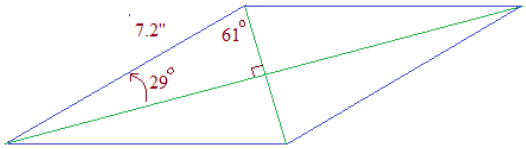
Trigonometry Word Problems (Solutions)

1) One diagonal of a rhombus makes an angle of 29° with a side of the rhombus. If each side of the rhombus has a length of 7.2", find the lengths of the diagonals.

Draw a sketch:

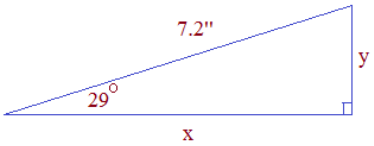
Since it is a rhombus, we know all the sides are 7.2".
-- The opposite angles are congruent
-- The adjacent angles are supplementary.
-- The diagonals are perpendicular.

Label the rest:



If we isolate the triangle, we have a hypotenuse length (7.2") and measure of the angles.
Using trig functions, we can find the lengths of the legs!

Solve:



$\sin 29^\circ = \frac{y}{7.2}$ $(.4848) \times 7.2 = 3.49$

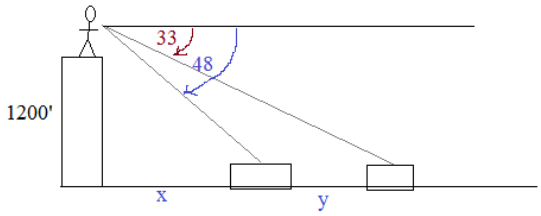
Therefore, the minor diagonal is approximately 6.98 inches.

$\cos 29^\circ = \frac{x}{7.2}$ $(.8746) \times 7.2 = 6.30$

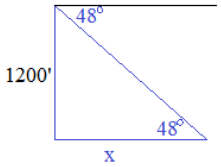
Therefore, the major diagonal is approximately 12.60 inches.

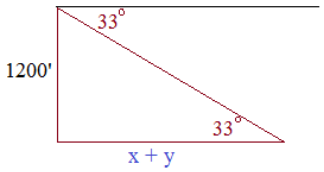
2) An observer on a cliff 1200 feet above sea level sights two ships due East. The angles of depression to the ships are 48° and 33°. What is the distance between the ships?

Draw a sketch:



"Isolate triangles":





Solve:

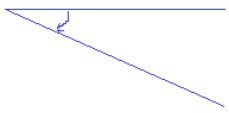
$\tan 48 = \frac{1200}{x}$ $\tan 33 = \frac{1200}{(x+y)}$

$1.1106 = \frac{1200}{x}$ y = 768' $.6494 = \frac{1200}{(x+y)}$

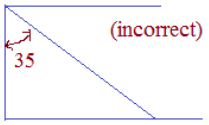
$x = 1080'$ $(x + y) = 1848'$

Note: Angle of depression is measured "going down".

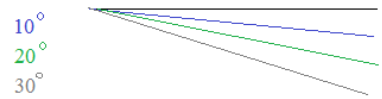
35° is

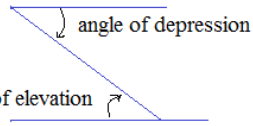


NOT



angle of depression is increasing as the lines go down





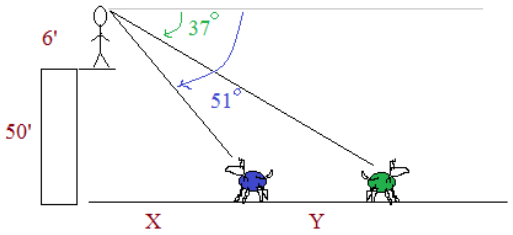
angle of elevation = angle of depression

(geometry theorem: if parallel lines are cut by a transveral, then the opposite interior angles are congruent.)

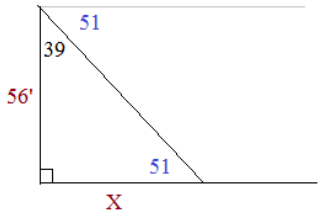
3) I'm standing on a 50 foot cliff, looking at my two dogs sitting on the beach below. If my line of sight is 6' above the ground and the angles of depression are 51° 37°, how far apart are the dogs?

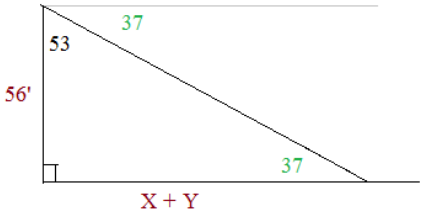
(Solutions)

Draw a Sketch:



"Isolate Triangles"





Solve:

$\tan 51 = \frac{56}{X}$ $\quad 1.235 = \frac{56}{X}$ $\quad X = 45.3'$

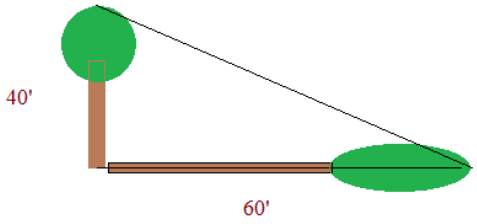
$\tan 37 = \frac{56}{(X+Y)}$ $\quad .754 = \frac{56}{(X+Y)}$ $\quad X + Y = 74.3'$

$Y = 29'$

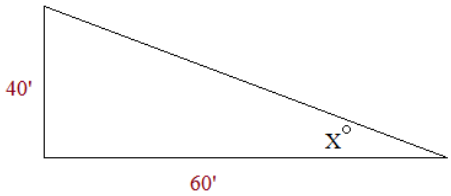
(The dogs are 29 feet apart)

4) Suppose a tree 40' tall casts a shadow of length 60'. What is the angle of elevation (with respect to the ground) from the end of the shadow to the top of the tree?

Draw a sketch:

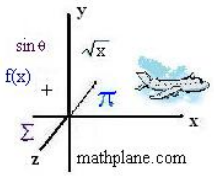


"Isolate Triangle" and Solve:



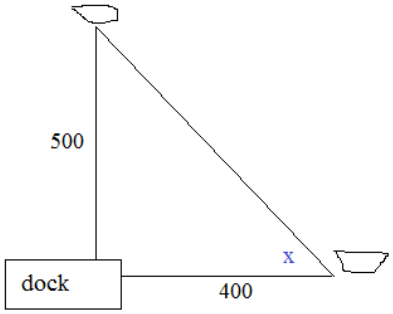
$\text{Tan } X = \frac{40'}{60'}$

ArcTan .667 = $33.7°$



5) Two boats leave a dock at the same time. Boat A goes due North 500 feet and stops.
Boat B goes due East 400 feet, stops and turns toward Boat A.
What angle must B turn to face and proceed directly to Boat A?

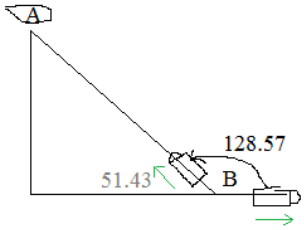
ANSWERS



First, use trig functions to find angle x...

$$\tan(x) = \frac{500}{400} = 1.25$$

$$x = \tan^{-1}(1.25) = 51.43^\circ$$

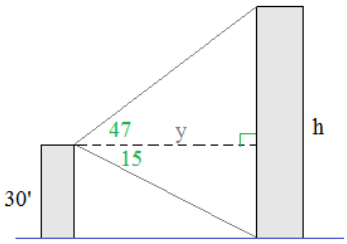


Then, answer the question...

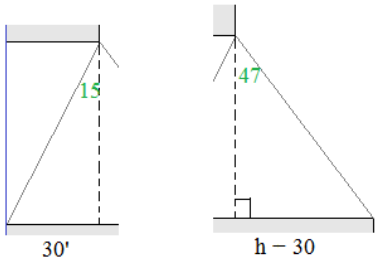
Since Boat B is facing due East, it must turn 128.57° to face Boat A

6) The *angle of elevation* from the top of a small building to the top of a nearby tall building is 47 degrees.
And, the *angle of depression* from the top of the small building to the bottom of the tall building is 15 degrees.
If the smaller building is 30 feet high, determine the height of the tall building.

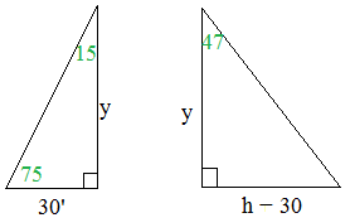
Step 1: Draw a diagram



Step 2: Extract the right triangles



Step 3: Use trig functions to solve

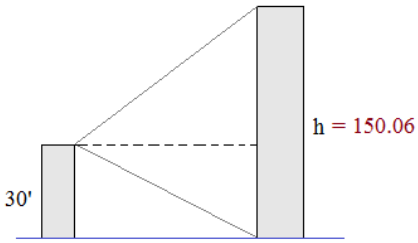


$$\tan(15^\circ) = \frac{30'}{y} \qquad \tan(47^\circ) = \frac{h-30}{y}$$

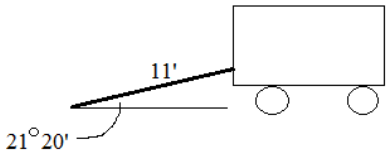
$$y = 111.96 \qquad \tan(47^\circ) = \frac{h-30}{111.96}$$

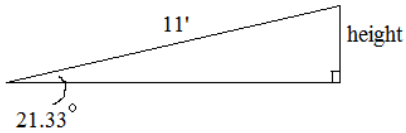
$$h - 30 = 120.06$$

$$h = 150.06$$



7) The distance from the bottom of a ramp to the back of a moving truck is 11 feet.
If the angle between ramp and the ground is 21° 20' , how high is the back of the truck off the ground?





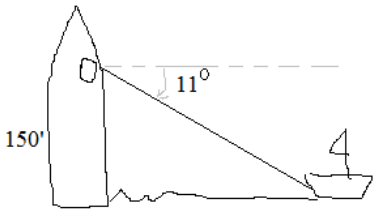
$$\sin(21.33) = \frac{(\text{height})}{11'}$$

height = 4.00 feet

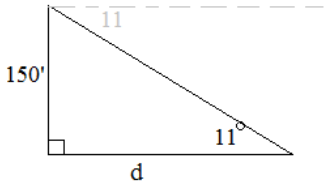
Trigonometry Word Problems

8) Standing in a lighthouse, 150 feet above the shore, I spot a boat at an angle of depression of 11 degrees. How far away is the boat from shore?

Step 1: Draw a diagram



Step 2: Extract the right triangle



Step 3: Solve

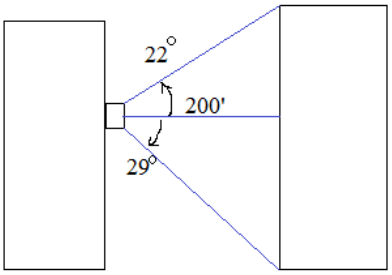
SOLUTIONS

$$\tan(11^\circ) = \frac{150'}{d}$$

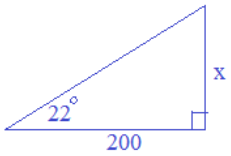
$$d = \frac{150'}{1.943} \approx \boxed{772 \text{ feet}}$$

check for reasonableness:
772 feet is opposite the 79 degree angle
and, 150 feet is opposite the 11 degree angle.

9) Looking out from a balcony, the angle of elevation to the top of the next building is approximately 22°. And, the angle of depression to the bottom of the building is approximately 29°. If the building is 200 feet away, how tall is it?



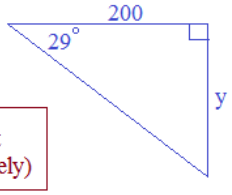
Upper part:



$$\tan(22^\circ) = \frac{x}{200}$$

$$x = 80.8 \text{ feet}$$

Lower part

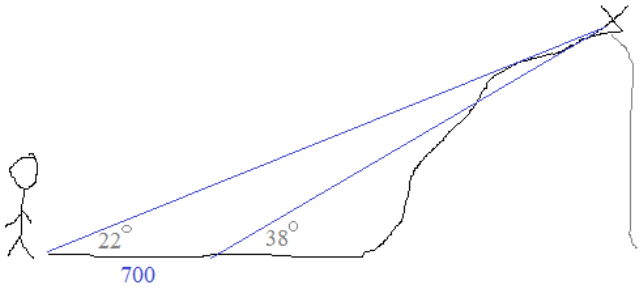


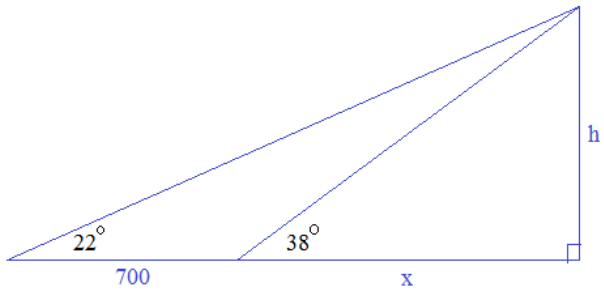
$$\tan(29^\circ) = \frac{y}{200}$$

$$y = 110.9 \text{ feet}$$

total height: 191.7 feet (approximately)

10) A hiker approximates an angle of elevation to the top of a hill to be 22 degrees. After walking 700 feet closer, the hiker estimates the angle of elevation *increased* by 16 degrees. Approximately, how high is the hill?





$$\tan(38^\circ) = \frac{h}{x}$$

$$\tan(22^\circ) = \frac{h}{(700 + x)}$$

$$(x)(\tan 38^\circ) = (700 + x)(\tan 22^\circ)$$

$$.7813x = 282.818 + .4040x$$

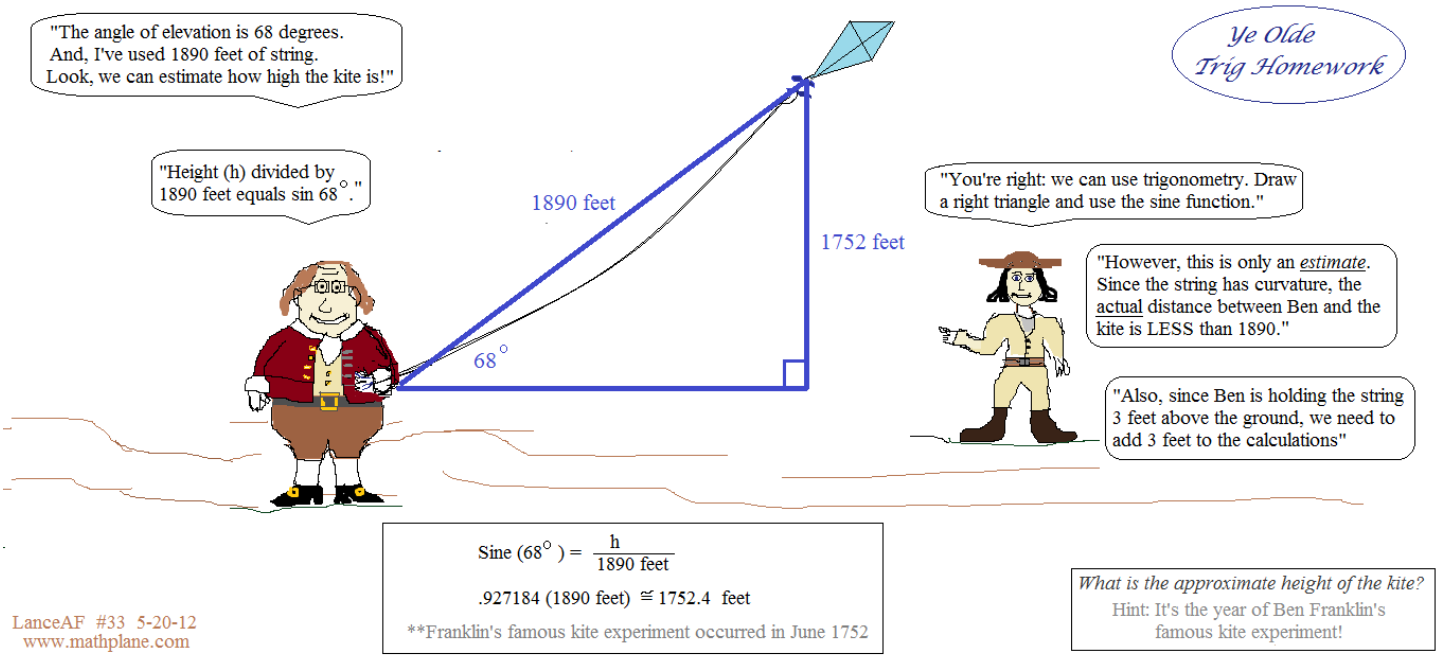
$x = 749.58$ (approximately)

Then, find h, using x and trig function:

$$\tan(38^\circ) = \frac{h}{749.58}$$

$$\boxed{h = 585.6 \text{ (approximately)}}$$

$$\tan(22) = \frac{585.6}{1449.58}$$

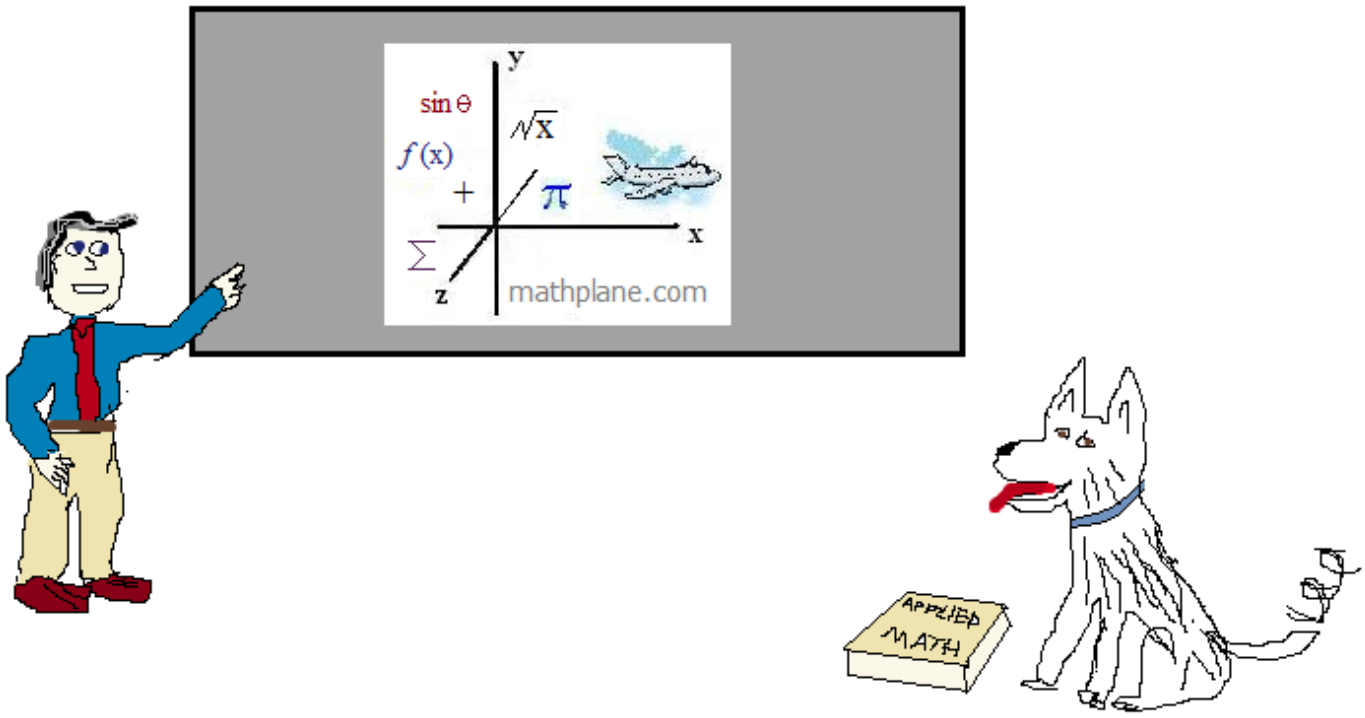


Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Also, find more advanced trig word problems throughout the site.

Good luck!



Also, at Facebook, Google+, Pinterest, and TeachersPayTeachers