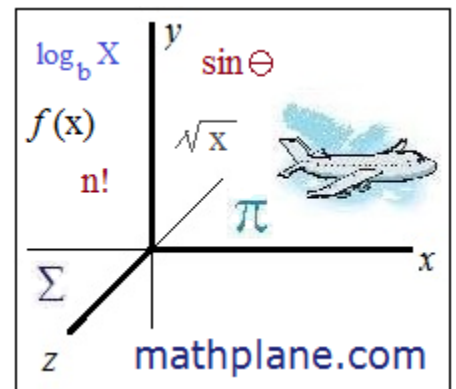


# % Percentages %

Notes, Examples, and Quiz (w/Solutions)



Percentages: Notes & Examples

A percentage expresses "a ratio out of 100"

Per - Cent ---> Per 100 "portion out of 100"

(In latin, centum means 100)  
century  
centipede  
centimeter....

Example: 37 percent

37%     .37      $\frac{37}{100}$      37 out of every 100

A percentage shows a "portion of something"

25% represents 25 out of 100     or, 50 out of 200  
or, 1 out of 4  
or, any other equivalent fraction or ratio

Finding X% of a given number:

Set up ratios. Then, solve...

Example: What is 25% of 80?

Set up the ratios:  $\frac{25}{100} = \frac{X}{80}$      Part  
Whole

Solve: (cross multiply)  
 $100X = (25)(80)$   
 $X = 20$

("20 out of 80 is the same as 25 out of 100")

Shortcut: Change percentage to a decimal. Then, multiply...

Example: What is 35% of 200?

35% ----> .35  
.35 x 200 = 70

Finding the percentage of one number out of another:

Set up the ratios. Solve.

*Example:* What percentage of 30 is 12? Or, "12 is what percentage of 30?"

Set up the ratios:  $\frac{X}{100} = \frac{12}{30}$  Part/Portion  
Whole

Solve for X:  $\frac{X}{100} = \frac{2}{5}$  Reduce the fraction

$5X = 2(100)$  Cross multiply

$X = 40$  40%

Alternate method: Convert the ratio into a decimal. Then, convert the decimal into a percentage.  
 (i.e. move 2 decimal places and add % symbol)

*Example:* 23 is what percentage of 58? Or, "what percent of 58 is 23?"

Convert the ratio into a decimal:  $\frac{23}{58}$  is approximately .39655

Convert into a percentage:  $.39655 \xrightarrow{\text{2 decimal places}} 39.655\%$

Quick check:  
 20 out of 50 is 40%  
 23 out of 58 is close to 40%

X% of what number is another number:

Set up ratios. Solve.

*Example:* 30% of what number is 42?

Set up ratios:  $\frac{30}{100} = \frac{42}{X}$  Part that's taken  
Whole

Solve:  $\frac{3}{10} = \frac{42}{X}$

$3X = 420$  X = 140

Shortcut: Set up decimal equation. Solve.

*Example:* 28% of what number is 20?

$.28X = 20$

$X = \frac{20}{.28} \approx 71.43$  (i.e. 28% of 71.43 is approx. 20...)

Percentages: Notes & Examples

Steps to determine percentage increase/decrease:

Percentage Increase/Decrease

- 1) Find Change
- 2) Establish "starting point"
- 3) Solve
- 4) Check for "reasonableness"

Example: What is the percentage increase from 20 to 25?

Find change: the increase is 5

"starting point": the starting point is 20

solve: what percentage is 5 out of 20?  $\frac{5}{20} = .25$  therefore, 25%

Example: What is the percentage decrease from 25 to 20?

Find change: the decrease is 5 (i.e. -5)

"starting point" (basis): the starting point is 25

solve: what percentage is 5 out of 25?  $\frac{5}{25} = .20$  therefore, 20%

check: 10% of 25 is 2.5... So, 20% of 25 is 5...

\*\*\*NOTE: In the above examples, the numbers are 20 and 25... And, the change is 5...  
But, the percentages are different! (percentage change depends on the starting point!)

Example: After a 30% increase, the total is 100. What was the original amount?

Suppose you simply decreased 30% from 100. The result is 70. But, that is NOT correct!  
IF you added 30% to 70, the answer is not 100... It is 91...

SOLUTION: original amount + increase = final amount

$$X + (30\% \text{ of } X) = 100$$

$$X + .30X = 100$$

$$1.3X = 100$$

$$X = 76.9 \text{ (approximately)}$$

check:

30% of 76.9 is approx. 23.1

76.9 + 23.1 = 100 ✓

Example: A \$34 shirt is on sale for 20% off. What is the price of the shirt?

original amount + increase/decrease = final amount

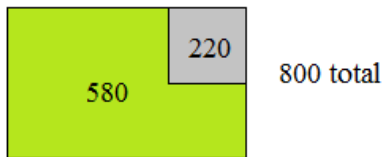
$$\$34 + (-.20)(\$34) = \text{final price}$$

$$\$34 - \$6.8 = \$27.20$$

Word problems:

Like most word problems, a solid strategy is to draw a picture and/or label variables. Then, construct the formula and solve.

*Example:* Joe's entire backyard is 800 square feet. If the garden is 220 square feet, what percentage of Joe's backyard is the garden?



$$\text{percentage} = \frac{X \text{ (percent)}}{100} = \frac{220}{800}$$

"portion" (garden)  
whole (backyard)

$$\frac{X}{100} = \frac{11}{40}$$

$$40X = 1100 \quad X = 27.5$$

27.5%

("check for reasonableness": 25% or 1/4 of 800 is 200... So, 220 out of 800 should be a little bit more than 25%)

*Example:* Sam enjoyed the dinner and service at his favorite restaurant. The final bill was \$48. If he left an 18% tip, how much did he spend at the restaurant?

$$\text{Total cost} = \text{dinner bill} + \text{tip}$$

$$.18 \times \$48 = \$8.64$$

$$\text{Cost} = \$48 + (18\% \text{ of } \$48)$$

$$\text{or, } \frac{18}{100} = \frac{X}{48} \quad X = 8.64$$

$$\$48 + \$8.64 = \boxed{\$56.64}$$

*Example:* After a 35% discount, the cost of a shirt \$19.50. What is the original price of the shirt (without the discount)?

$$\text{Original price} - \text{discount} = \text{Cost of shirt}$$

35% of X

$$X - (35\% \text{ of } X) = \$19.50$$

$$\frac{35}{100} = \frac{\text{discount}}{X}$$

$$X - .35X = \$19.50$$

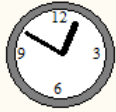
$$35X = 100(\text{discount})$$

$$.65X = \$19.50$$

$$\text{discount} = \frac{35X}{100} = .35X$$

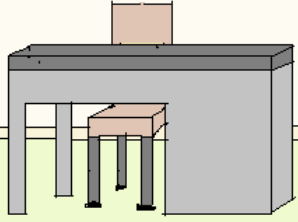
$$\boxed{X = \$30}$$

"75% of this class passed the test --  
-- which is 50% more than last  
year's class...."



24 students

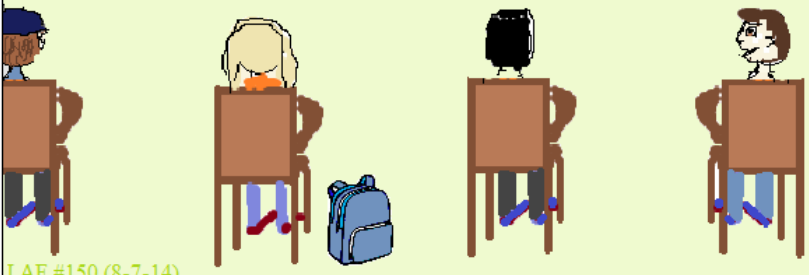
A	-	5
B	-	8
C	-	



Okay, 75% of 24  
is 18.. And,---

"25% last year!  
What a bunch of idiots!!"

And, there's a 100%  
chance *he'll* be in  
next year's class...



Pre-Algebra

Questions: Assuming each class has 24 students, how  
many passed last year? what percentage passed?  
(Answers will come in time!)

# Practice Test (And, Solutions) ->

Percentages Quiz

I. Find X in each expression:

1) 35% of 200 is X

2) 4% of 20 is X

3) 22% of X is 11

4) 55% of X is 100

5) X% of 48 is 6

6) X% of 6 is 48

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6

2) 90 to 108

3) 30 to 70

4) 41 to 38

5) 26.50 to 21.50

6) 9 to 0

Find the result:

7) cut 13% from 200

8) decrease 88 by 25%

9) increase 34 by 20%

10) increase 25 by 500%

## III. Word problems/applications

- 1) The price of the lunch special is \$9. If sales tax is 7%, what is the total cost?
- 2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?
- 3) The price of gas has gone up from \$2.75 to \$4.35. What is the percentage increase?
- 4) If you borrow \$500 at an annual interest rate of 6%, how much will you owe after 1 year?
- 5) After a 20% discount, the price of a men's suit is \$185. What was the *original* price of the suit?

\*\*\*Challenge: Last year, an investment lost 30% of its value.  
What *percentage* increase is necessary this year to recover the lost value?



Percentages Quiz

SOLUTIONS

I. Find X in each expression:

1) 35% of 200 is X  $\frac{35}{100} = \frac{X}{200}$   
 $.35 \times 200 = 70$  OR  $100X = 35(200)$   
 $X = 70$

2) 4% of 20 is X note: 40% of 20 is 8  
 then, 4% of 20 is .8  
 $.04 \times 20 = .8$

3) 22% of X is 11  
 $\frac{22}{100} = \frac{11}{X}$  ← part  
 whole  
 $22X = 1100$   
 $X = 50$

4) 55% of X is 100  
 $\frac{55}{100} = \frac{100}{X}$   
 $55X = 10000$   
 $X \approx 181.82$

5) X% of 48 is 6 "6 out of 48 is what %?"  
 $\frac{X}{100} = \frac{6}{48}$   
 $\frac{X}{100} = \frac{1}{8}$   
 $X = 12.5$

6) X% of 6 is 48  
 $\frac{X}{100} = \frac{48}{6}$   
 $X = 800$

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6 increase amount: 2  
 starting amount: 4  $\frac{2}{4}$  is 50%

2) 90 to 108 increase: 18 starting basis: 90  
 $\frac{18}{90} = .20$  20%

3) 30 to 70 increase: 40  
 starting amount: 30  $\frac{40}{30} = 1.3\bar{3}$   
 133.33%

4) 41 to 38 decrease: 3  
 starting point: 41  $\frac{3}{41} \approx .0732$   
 approx. 7.32%

5) 26.50 to 21.50  
 decrease: 5  
 start: 26.50  $\frac{5}{26.5} \approx .189$   
 approx. 18.9%

6) 9 to 0  
 taking all away → 100%  
 decrease: 9 starting basis: 9  
 $\frac{9}{9} = 1$  (move 2 decimal places)

Find the result:

7) cut 13% from 200  
 $.13 \times 200 = 26$   $200 - 26 = 174$

8) decrease 88 by 25%  
 25% of 88 is 22...  $88 - 22 = 66$   $\frac{25}{100} = \frac{22}{88}$

9) increase 34 by 20%  
 $.20 \times 34 = 6.8$  then,  $34 + 6.8 = 40.8$

10) increase 25 by 500%  
 increase 25 by 100% is +25  
 so, increase 25 by 500% is  $5 \times 25 = 125$   
 $25 + 125 = 150$

III. Word problems/applications

SOLUTIONS

- 1) The price of the lunch special is \$9. If sales tax is 7%, what is the total cost?

$$\begin{aligned} \text{Cost} &= \text{lunch price} + \text{sales tax} \\ &= \$9 + (.07)(\$9) = \boxed{\$9.63} \end{aligned}$$

- 2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?

total population:  $357 + 395 = 752$

$$\frac{X}{100} = \frac{357}{752} \begin{array}{l} \text{"portion"} \\ \text{"whole"} \end{array} \quad 752X = 100(357)$$

$$X = 47.47$$

- 3) The price of gas has gone up from \$2.75 to \$4.35. What is the percentage increase?

The increase is \$1.60  
The "starting point" (basis) is \$2.75

$$\frac{1.6}{2.75} \text{ is approx. } .582$$

$$\boxed{\text{price has gone up about } 58.2\%}$$

- 4) If you borrow \$500 at an annual interest rate of 6%, how much will you owe after 1 year?

After 1 year, you will owe  $\$500 + \text{interest} =$

$$\$500 + .06(\$500) = \boxed{\$530}$$

- 5) After a 20% discount, the price of a men's suit is \$185. What was the *original* price of the suit?

Notice, we are looking for the *original* price. So, we need to construct the appropriate formula and variables....

$$\begin{aligned} \text{Original price} - \text{discount} &= \text{final price} \\ X - X(.20) &= \$185 \end{aligned}$$

Quick check:  $231.25$   
20% of  $231.25$  is  $46.25$   
 $231.25 - 46.25 = 185$  ✓

$$.80X = \$185$$

$$X = \boxed{\$231.25}$$

\*\*\*Challenge: Last year, an investment lost 30% of its value. What *percentage* increase is necessary this year to recover the lost value?

Suppose the investment were \$100...

After 1 year, the investment would be worth \$70..

$$100 - .30(100) = 70$$

So, what *percentage* increase is necessary to increase 70 to 100?

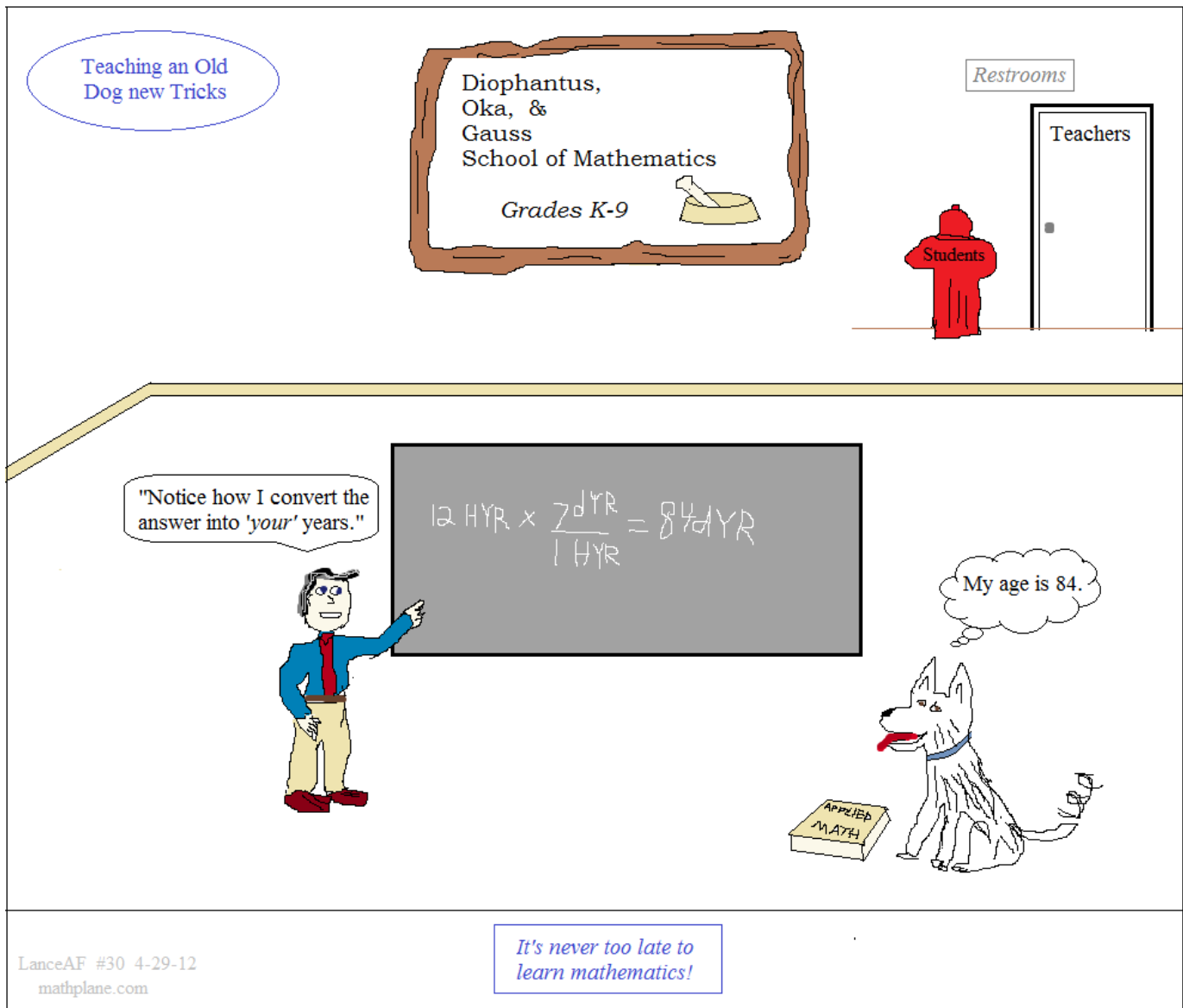
increase is 30; starting basis is 70

$$\frac{30}{70} \cong .4286 \quad \boxed{\text{approximately } 42.86\%}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know!

Cheers



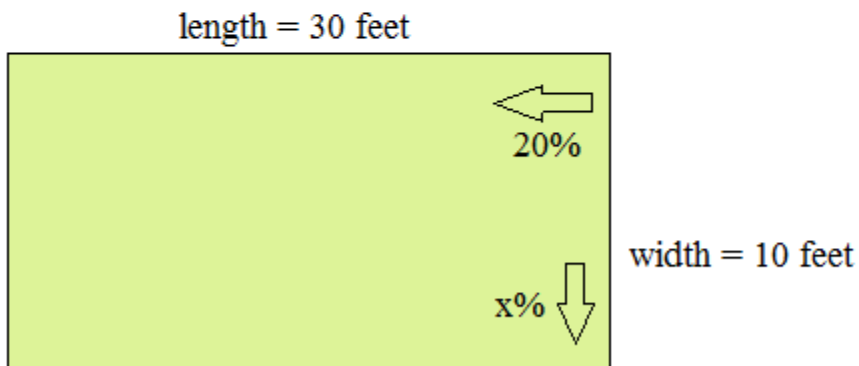
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## TWO MORE QUESTIONS....

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width

- a) to maintain the same area?
- b) to maintain the same perimeter?

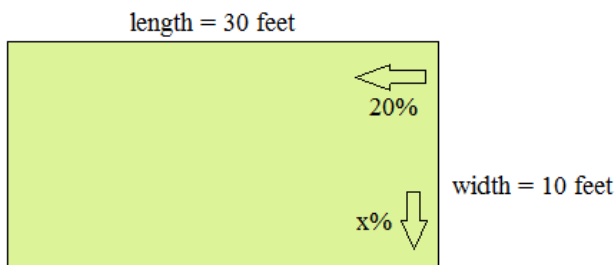


**ANSWERS on next page....**

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width

- a) to maintain the same area?
- b) to maintain the same perimeter?



**ANSWERS**

a) Area....

Original area is 300 square feet...

New length is  $30' - (.20 \times 30') = 24'$

$24' \times (\text{new width}) = 300 \text{ square feet}$

new width = 12.5'

10 feet ---> 12.5 feet (increase of 2.5 feet)

percentage increase:  $\frac{2.5 \text{ feet}}{10 \text{ feet}} = 25\%$

b) Perimeter....

Original perimeter is 80 feet....

New length is 24'...

$(2 \times 24') + (2 \times \text{new width}) = 80 \text{ feet}$

$2 \times \text{new width} = 32 \text{ feet}$

new width = 16 feet

10 feet ---> 16 feet (increase of 6 feet)

percentage increase:  $\frac{6 \text{ feet}}{10 \text{ feet}} = 60\%$