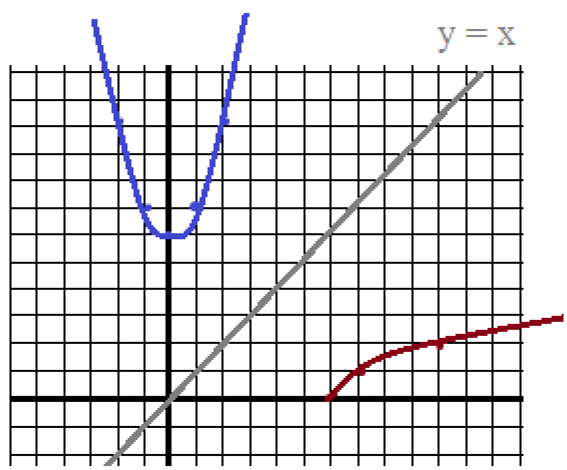
Inverse Functions

Practice questions (with solutions)



Includes graphing, finding inverses, symmetry, cryptography, and more…

Domain, Range, and Inverse Functions

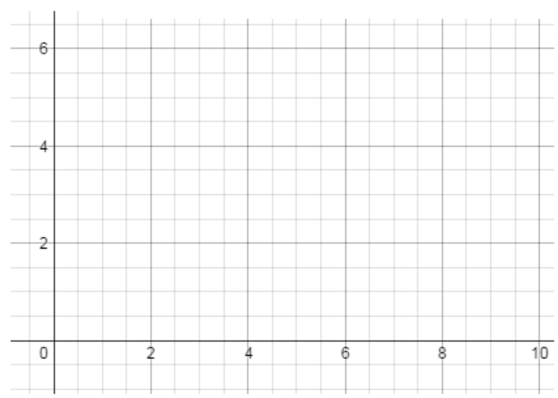
1) For the function $h(x) = \sqrt{3x - 4}$

a) find the inverse $h^{-1}(x)$

b) what is the domain of $h(x)$? the range of $h(x)$?

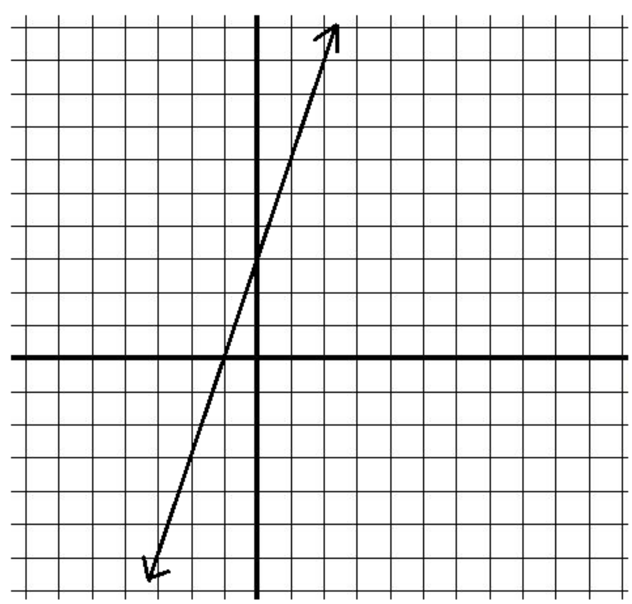
c) what is the domain of $h^{-1}(x)$? the range of $h^{-1}(x)$?

d) Graph the function $h(x)$, the inverse $h^{-1}(x)$, and the line y = x



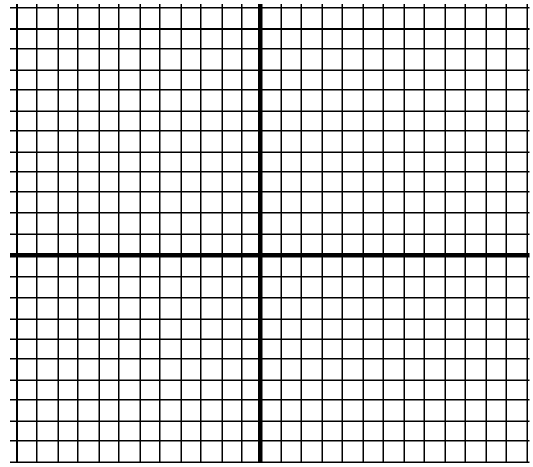
2) Graph the inverse:

Then, verify the results algebraically...



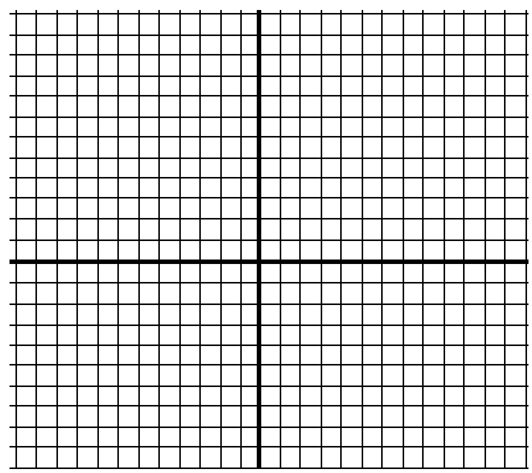
3) $g(x) = \sqrt[3]{(x-1)}$

a) Sketch the function $g(x)$



b) Find the inverse of $g(x)$

c) What is the domain and range of $g^{-1}(x)$?



d) Graph $-(g(x))$

4) If $f(x) = 5 - 2x$, what is $f^{-1}(3)$?

Domain, Range, and Inverse Functions

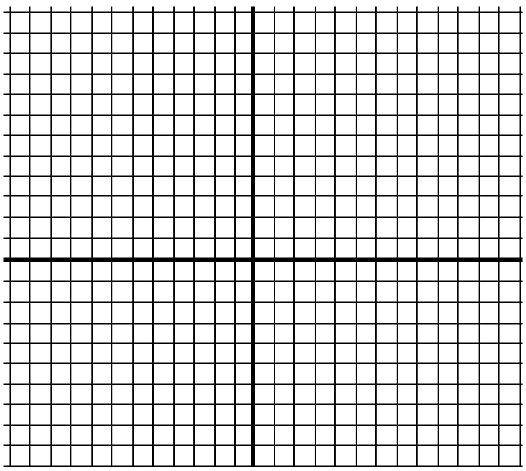
5) $f(x) = x^2 + 6$

a) Find the inverse $f^{-1}(x)$

b) Verify the inverse -- find $f(f^{-1}(x))$ and $f^{-1}(f(x))$

c) What is the domain and range of $f(x)$? Of $f^{-1}(x)$?
Are the "inverses" one-to-one?

d) Graph $f(x)$ and $f^{-1}(x)$



Domain, Range, and Inverse Functions

SOLUTIONS

1) For the function $h(x) = \sqrt{3x - 4}$

a) find the inverse $h^{-1}(x)$

for $y = \sqrt{3x - 4}$ switch the x and y...

$x = \sqrt{3y - 4}$ then, solve for y...

$x^2 = 3y - 4$

$3y = x^2 + 4$

$y = \frac{x^2 + 4}{3}$

$h^{-1}(x) = \frac{x^2 + 4}{3}$

where $x \geq 0$ ("restrict the domain" to make the functions 1 to 1)

b) what is the domain of $h(x)$? the range of $h(x)$?

(no negatives under a radical) domain: $x \geq \frac{4}{3}$ range: $h(x) \geq 0$

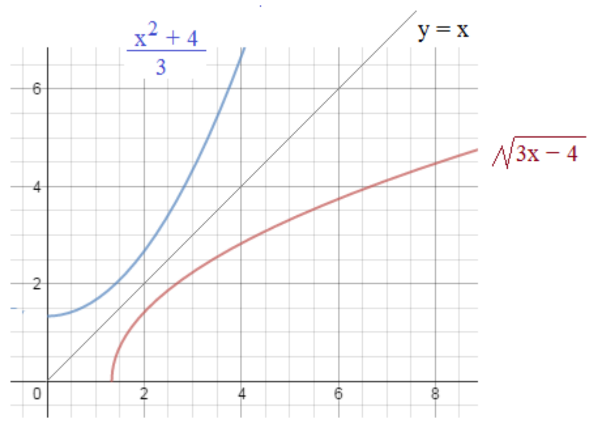
c) what is the domain of $h^{-1}(x)$? the range of $h^{-1}(x)$?

$h^{-1}(x) = \frac{x^2 + 4}{3}$ domain: $h(x) \geq 0$

where $x \geq 0$ range: $x \geq \frac{4}{3}$

Notice: the domain of $h(x)$ is the range of $h^{-1}(x)$
and, the range of $h(x)$ is the domain of $h^{-1}(x)$

d) Graph the function $h(x)$, the inverse $h^{-1}(x)$, and the line y = x



2) Graph the inverse.

Then, verify the results algebraically...

method 1: since it is a line, the inverse will be a line..
therefore, we need just 2 points!
---> pick two points and "flip the coordinates"..

(0, 3) ----> (3, 0)

(-2, -3) ----> (-3, -2)

then, draw a line throught the points...

method 2: the equation of the line is $y = 3x + 3$

find the inverse: $x = 3y + 3$ switch x and y

$3y = x - 3$ solve for y

$y = \frac{x - 3}{3}$

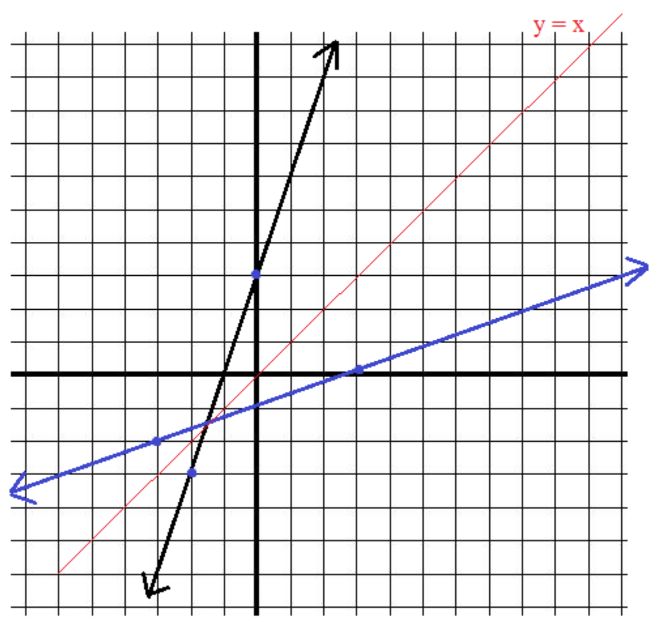
$y = \frac{1}{3}x - 1$

assume line A: $f(x) = 3x + 3$

line B: $g(x) = \frac{1}{3}x - 1$

$f(g(x)) = 3(\frac{1}{3}x - 1) + 3$

$= x - 3 + 3 = x$ ✓



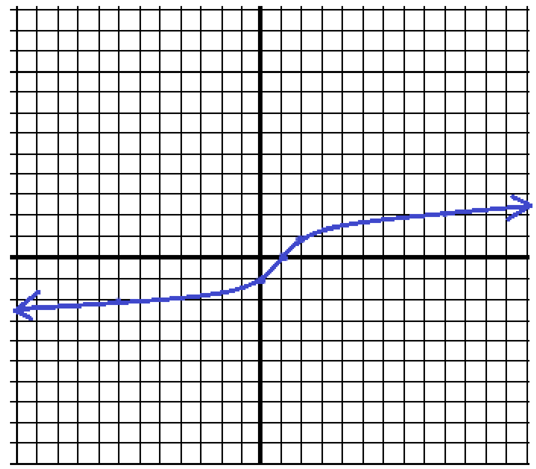
3) $g(x) = \sqrt[3]{(x-1)}$

SOLUTIONS

a) Sketch the function $g(x)$

note: this is $\sqrt[3]{x}$ shifted one unit to the right

x	g(x)
-26	-3
-7	-2
0	-1
1	0
2	1
9	2
28	3



b) Find the inverse of g(x)

$y = (x-1)^{\frac{1}{3}}$ write in exponential form; switch x and y

$x = (y-1)^{\frac{1}{3}}$ solve for y

$x^3 = y - 1$ $y = x^3 + 1$ ⟶ $g^{-1}(x) = x^3 + 1$

c) What is the domain and range of $g^{-1}(x)$?

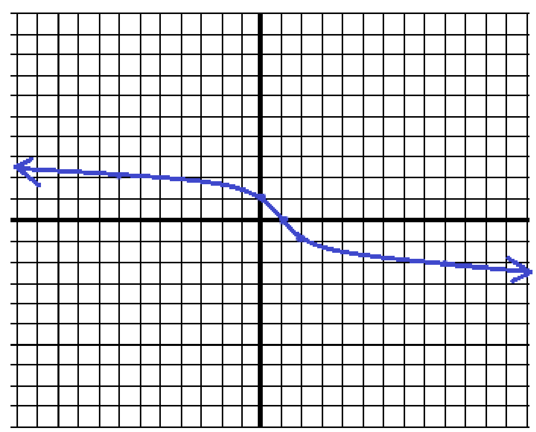
domain and range: all real numbers

d) Graph $-(g(x))$

$-g(x) = -\sqrt[3]{(x-1)}$

note: graph is 'opposite' image of above graph ---
it is *reflected over the x-axis*

x	g(x)	−g(x)
-26	-3	3
-7	-2	2
0	-1	1
1	0	0
2	1	-1
9	2	-2
28	3	-3



4) If $f(x) = 5 - 2x$, what is $f^{-1}(3)$?

$5 - 2x = 3$ $x = 1$

$f(1) = 3$ So, the inverse (reverse the coordinate) is (3, 1)

answer: 1

Domain, Range, and Inverse Functions

SOLUTIONS

5) $f(x) = x^2 + 6$

a) Find the inverse $f^{-1}(x)$

$y = x^2 + 6$ (switch the x and y)

$x = y^2 + 6$ (solve for y)

$y^2 = x - 6$

$y = \sqrt{x - 6}$ ⟶ $f^{-1}(x) = \sqrt{x - 6}$

note: since it is a function, the output is only $+\sqrt{\ }$ (and not $-$)

b) Verify the inverse -- find $f(f^{-1}(x))$ and $f^{-1}(f(x))$

$f(\sqrt{x-6}) = (\sqrt{x-6})^2 + 6$

$= (x - 6) + 6$

$= x$ ✓

$f^{-1}(x^2 + 6) = \sqrt{(x^2 + 6) - 6}$

$= \sqrt{x^2 + 0}$

$= x$ ✓

c) What is the domain and range of $f(x)$? Of $f^{-1}(x)$?
Are the "inverses" one-to-one?

$f(x) = x^2 + 6$

domain: all real numbers
range: $f(x) \geq 6$

$f^{-1}(x) = \sqrt{x - 6}$

domain: $x \geq 6$ (if $x < 6$, then negative under the radical sign)

range: $y = f^{-1}(x) \geq 0$ (the opposites are omitted to preserve the function)

since domain of $f(x)$ and range of $f^{-1}(x)$ are different, functions are not 1-to-1

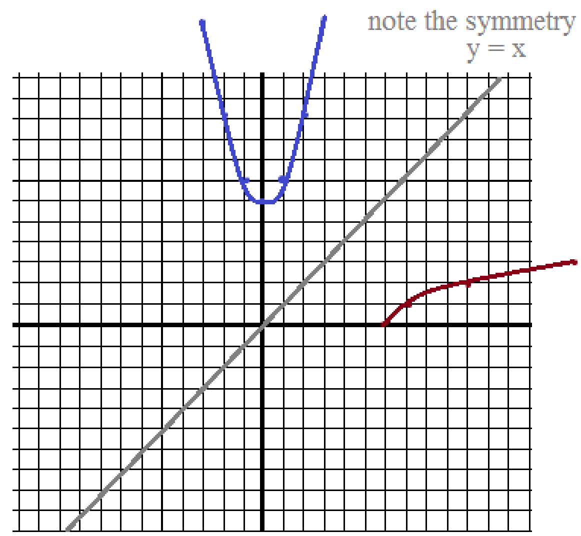
d) Graph $f(x)$ and $f^{-1}(x)$

x	$f(x)$
-3	15
-2	10
-1	7
0	6
1	7
2	10
3	15

x	$f^{-1}(x)$
~~15~~	~~-3~~
~~10~~	~~-2~~
~~7~~	~~-1~~
6	0
7	1
10	2
15	3
22	4

note: the ordered pairs are reversed!

note the symmetry
y = x



Inverses Application: Cryptography

Suppose we want to send a secret message (using an algebraic function/code)

We could establish a 1-1 function for the translation...

Example: $f(x) = 3x + 7$ where x is a number representing a letter in the alphabet...

A = 1
B = 2
C = 3
etc...

If we want to send the letter A, we would find $f(1) = 3(1) + 7 = 10$ and send "10"

Then, how would the receiver decode the message?

The receiver would input the number into the inverse function!

$y = 3x + 7$ Find the inverse: $x = 3y + 7$

$3y = x - 7$

$y = \frac{x-7}{3}$

To decode the message, use $f^{-1}(x) = \frac{x-7}{3}$

$f^{-1}(10) = \frac{10-7}{3} = 1 \longrightarrow$ "A"

Again, this works effectively (accurately), because it's a 1-1 function...

a) If I want to send the message "help", what number sequence would I send?

h ---> 8 $f(8) = 31$

e ---> 5 $f(5) = 22$

l ---> 12 $f(12) = 43$

p ---> 16 $f(16) = 55$

31, 22, 43, 55

$f(x) = 3x + 7$

b) If I received a message with the sequence 46, 10, 67, 31, what would it be?

$f^{-1}(46) = 13$ ---> m

$f^{-1}(10) = 1$ ---> a

$f^{-1}(67) = 20$ ---> t

$f^{-1}(31) = 8$ ---> h

m, a, t, h

$f^{-1}(x) = \frac{x-7}{3}$