Triangle Characteristics
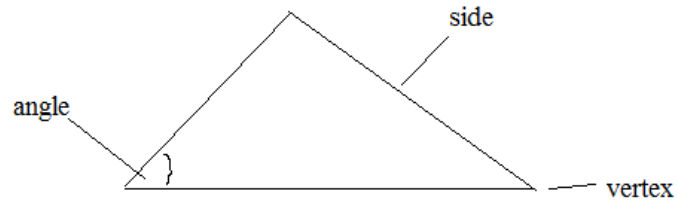
Notes, Illustrations, and practice quiz (& Solutions)

Side

Angle

180 degrees

Vertex
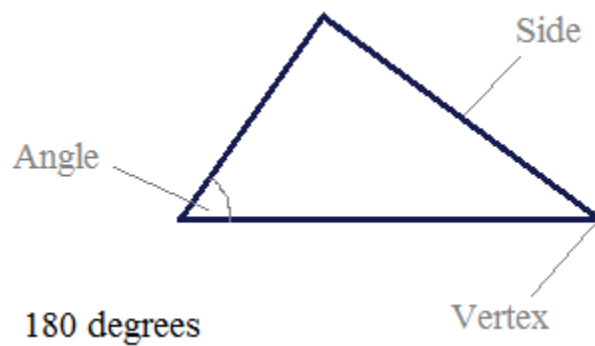
Topics include classification of triangles, polygons, inequality theorem, restrictions, perimeter, angle measurements, and more.

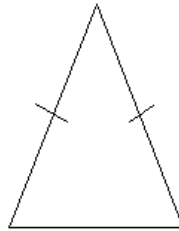What is it?   A 2-dimensional, enclosed figure containing 3 line segments linked end to end (at the vertices).
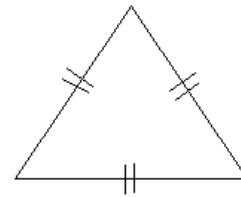
side

angle

}

vertex

Classification (by sides)

| Scalene | Isosceles | Equilateral |

(no equal sides)

(2 equal sides)

(3 equal sides)

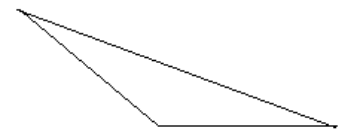Classification (by angles)

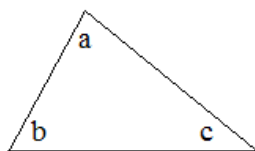| Right | Acute | Obtuse |
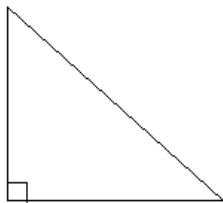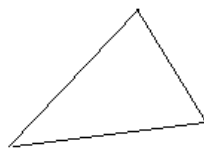
(1 angle = 90 degrees)

(all angles < 90 degrees)

(1 angle > 90 degrees)

All Triangles are 180°

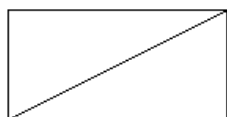The sum of the interior angles of a triangle is 180 degrees

$$a^{\circ} + b^{\circ} + c^{\circ} = 180^{\circ}$$

observation: for any polygon, the sum of the interior angles is $(n - 2) \times 180^{\circ}$ where n is the number of sides
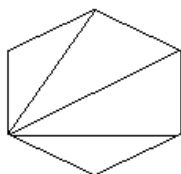
Why? Because, polygons can be cut into triangles.

Examples:

n = 4 (sides)

$(n - 2) \longrightarrow$ 2 triangles

$(n - 2) \times 180^{\circ} = 360^{\circ}$

Interior angles of a quadrilateral add up to 360 degrees...

n = 6 (sides)

$(n - 2) \longrightarrow$ 4 triangles

$(n - 2) \times 180^{\circ} = 720^{\circ}$

The sum of the interior angles of a hexagon is 720 degrees.

observation: for any *regular* polygon, each interior angle is

$$\frac{(n - 2) \times 180^{\circ}}{n}$$

Example:
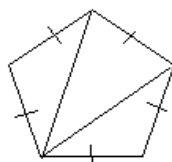
n = 5

$(n - 2) \times 180^{\circ} = 540^{\circ}$

then, $\dfrac{180(n - 2)}{n} = 108^{\circ}$

Each interior angle of a *regular* pentagon is 108 degrees.

Triangle Inequality Theorem

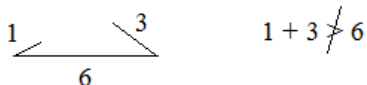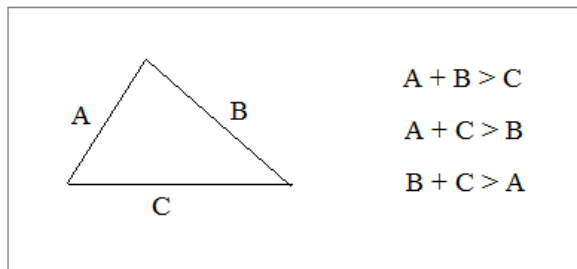Definition: The sum of the lengths of <u>any 2 sides of a triangle</u> is always <u>greater than</u> the length of <u>the 3rd side.</u>

Why? Because, if a 3rd side is too long, then
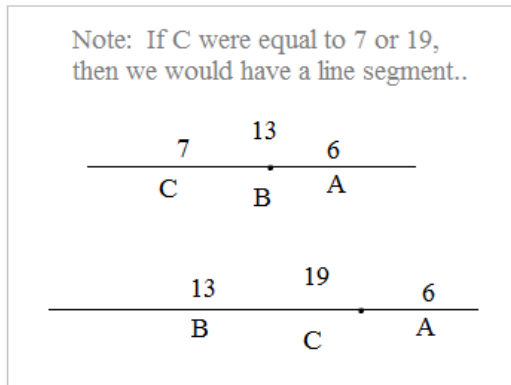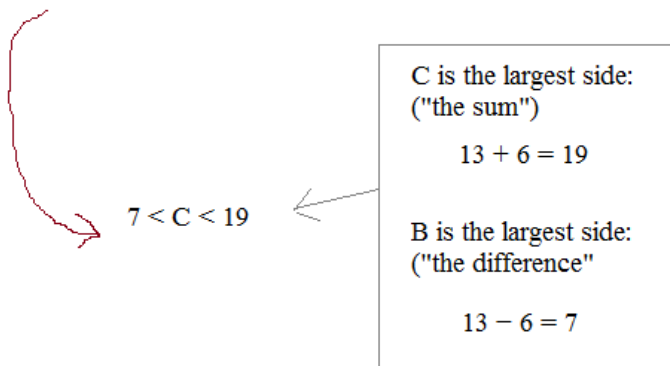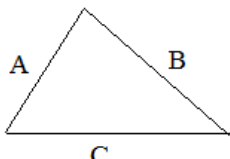   the others can't reach!

Example:

1 _____ 3 $1 + 3 \not> 6$
      6
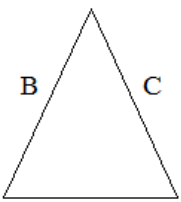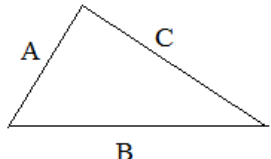
This cannot be a triangle...

$A + B > C$

$A + C > B$

$B + C > A$

Example: If A = 6 and B = 13, what are the <u>possible</u> lengths of side C?

The get the 3rd side, "find the sum & the difference"....

C is the largest side:
("the sum")

$13 + 6 = 19$

$7 < C < 19$

B is the largest side:
("the difference"

$13 - 6 = 7$

Note: If C were equal to 7 or 19,
then we would have a line segment..

7   13   6
C   B   A

13   19   6
B   C   A

Case 1: C is largest side

A = 6   B = 13

$13 < C < 19$

If C > 19, then
A and B won't touch!

Case 2: isosceles

A = 6
B = C = 13

Case 3: B is the largest side

A = 6   B = 13

$7 < C < 13$

If C < 7, then
A and C won't touch!

Patients quickly lose patience with this specialist -- who held a doctorate in geometry...

Practice Exercises-→

Sides: Equilateral, Isosceles, Scalene

Angles: Right, Acute, Obtuse

I. Identify the following:

sides: _____

angles: _____

sides: _____

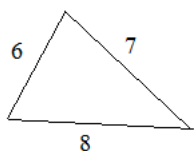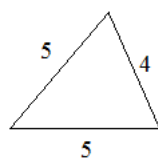angles: _____

$\triangle$ABC where   m $\angle$A = 25°
                          m $\angle$B = 35°

sides: _____

angles: _____

II. Classify the following triangles:

"30-60-90 triangle":

    sides: _____

    angles: _____

"45-45-90 triangle":

    sides: _____

    angles: _____

"60-60-60 equiangular triangle":

    sides: _____

    angles: _____

III. Always, Sometimes, or Never?

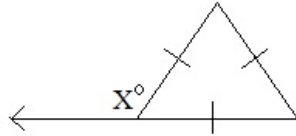   1) An equilateral triangle is obtuse.

    2) A right triangle is isosceles.

    3) The sum of the interior angles of an obtuse triangle is 180° .

Triangle Characteristics: Applications Quiz

1) If the perimeter of an equilateral triangle is 18 feet, then what are the
   lengths of each side?

2) What is the measure of exterior angle X?

   X°

3) If △ABC is isosceles and right, what are the measures of each angle?

4) In diagram A, what are the measures of angles D, E, and F?

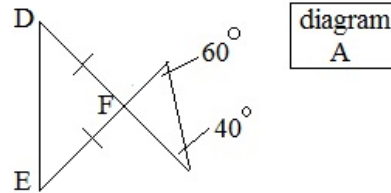   D
   F
   60°
   40°
   E

   diagram
   A

*5) 'Trick question': If 2 sides of an isosceles triangle are 6 and 10 inches, what is the
                        length of the 3rd side?

**6) Challenge question: If 2 sides of a triangle are 7 and 12 inches, what is
                          the length of the 3rd side?

1) Given $\triangle ABC \cong \triangle DEF$

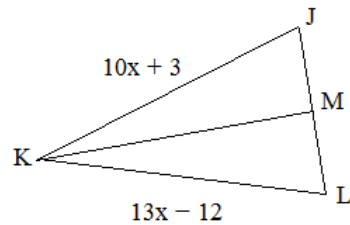In triangle ABC, which side is the smallest?

$\overline{AC} = x^2$
$\overline{DE} = 8x - 11$
$\overline{FD} = 2x + 3$
$\overline{FE} = 4x + 6$

2) Given $\overline{KM}$ is a perpendicular bisector of $\overline{JL}$; $\overline{JL} = 5x - 5$
What is the length of $\overline{JM}$?

10x + 3

J

M

K

L

13x − 12

3) Always, Sometimes, or Never?
Two triangles are congruent if
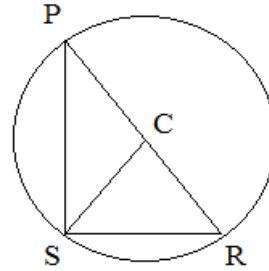2 sides and 1 angle are congruent to corresponding parts of another.

4) If the perimeter of an equilateral triangle is $6y + 18$ and one side is $4y - 14$,
what is the perimeter?

5) In $\triangle$ ABC,

if $\overline{AC} > \overline{BC} > \overline{AB}$, list the 3 angles in order of size (from largest to smallest)

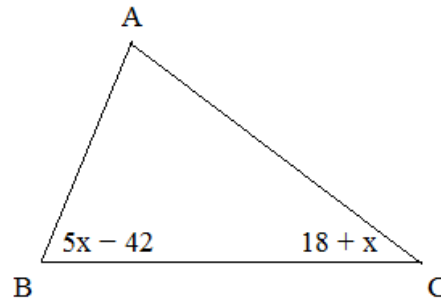6) In Circle C, $PS \perp SR$

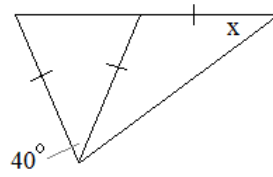$$\angle P = 38^{\circ}$$

Find   a) $\angle$ PSC

b) $\angle$ R

7) Given: $\overline{AC} > \overline{AB}$

What are the restrictions of x?
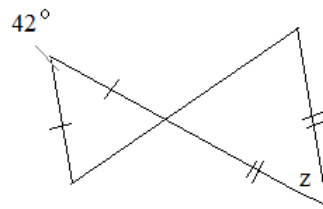
5x − 42   18 + x

B   C

A

8) Find x
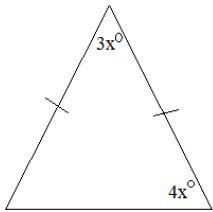
$40°$

x

9) Find y

$68°$

y

10) Find z

$42°$
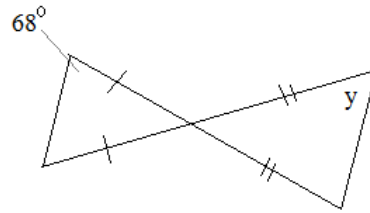
z

11) The vertices of a triangle are $(2, -6)$ $(5, -2)$ $(7, -6)$

Is this triangle scalene, isosceles, or equilateral?

12)

3x°

4x°

What is x?

13)

45

y

50

x

43

14)

a

b

c

d

e
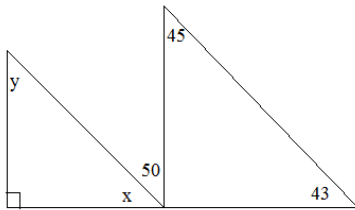
a) 180

b) 240

c) 270

d) 360

e) 450

a + b + c + d + e = ?

15) The measure of one angle in a right triangle is 5 times the measure of another.
    What are the angles?

16) If the perimeter of this isoceles triangle is less than 45,
    which side is the base?

10

x + 7

2x - 8

17) Given: $\angle$ B = 3x + 10

$\angle$ C = 46 − 3y

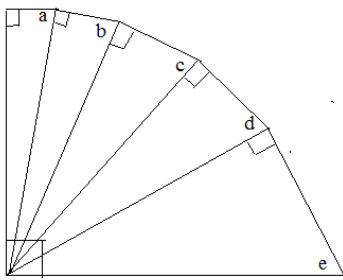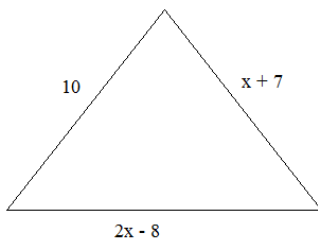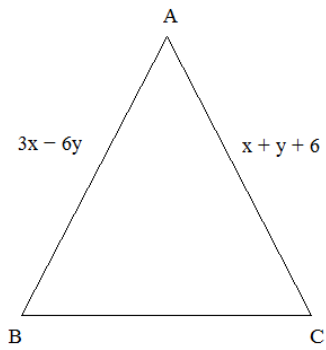$\triangle$ ABC is an isosceles triangle with base $\overline{BC}$

What is the measure of angle A?

A

3x − 6y   x + y + 6

B   C

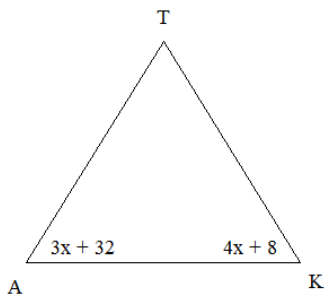18) $\triangle$ DEF is an isosceles triangle with base $\overline{DE}$

Determine the measure of the angles and the measure of the sides...

F

5x + 1   −4y

20x + 3°

D   E   −12y + 15°

19) $\overline{TA} = \overline{AK}$

Find the measures of all the angles.

T

3x + 32   4x + 8

A   K
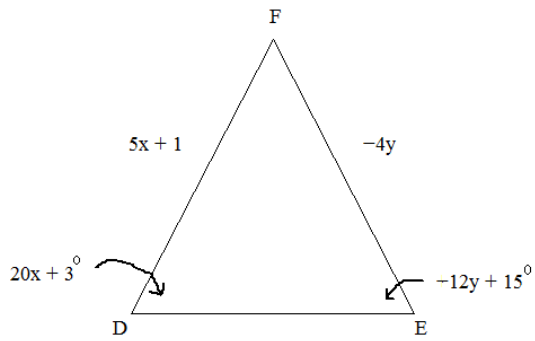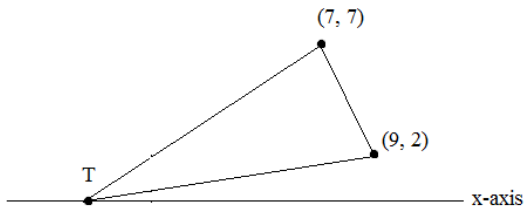
20) Are the points (1, 0)   (7, 3)   (-1, 4) the vertices of a right triangle?

Justify using the distance formula:

Justify using slope:

21) Assume $(-3, 0)$ and $(5, 0)$ are vertices of an isosceles right triangle.
    Can you identify the 3rd vertex? (there are 6 possibilities)

22) If $\triangle$ RHT is a right triangle, and T is on the x-axis, what is T?   (note: figure not drawn to scale!)

```
                    (7, 7)
                      ●
                     /|
                    / |
                   /  |
                  /   ● (9, 2)
              T  /   /
          ●─────────────────────────── x-axis
```
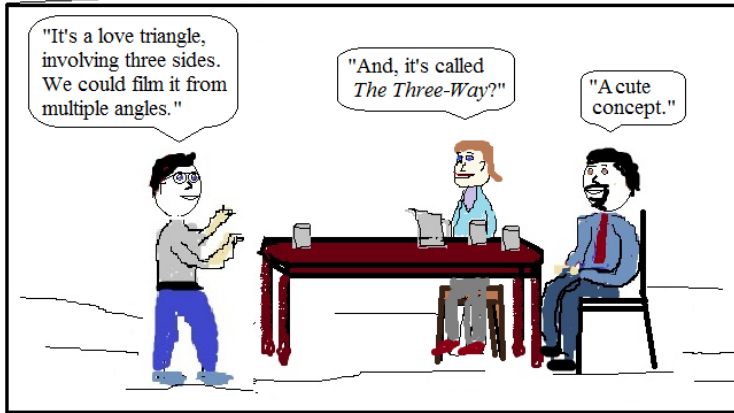
23) In $\triangle$ XYZ,

$\overline{XY} = 5$

$\overline{XZ} = 12$ list angles from largest to smallest....

$\overline{YZ} = 6$
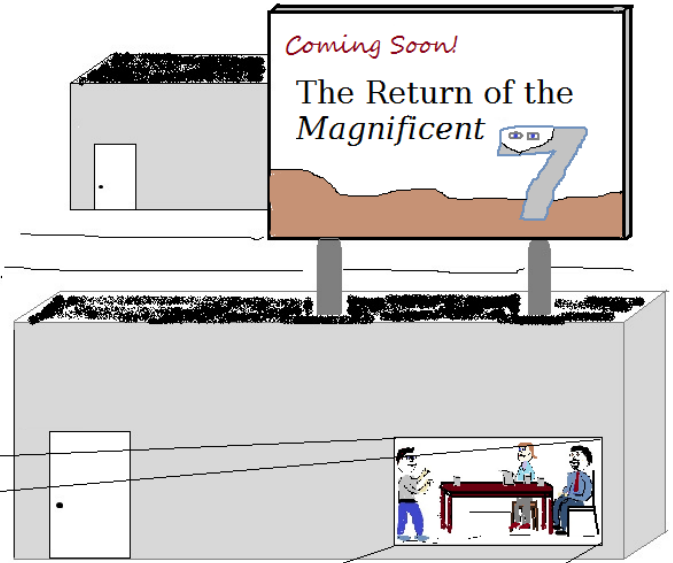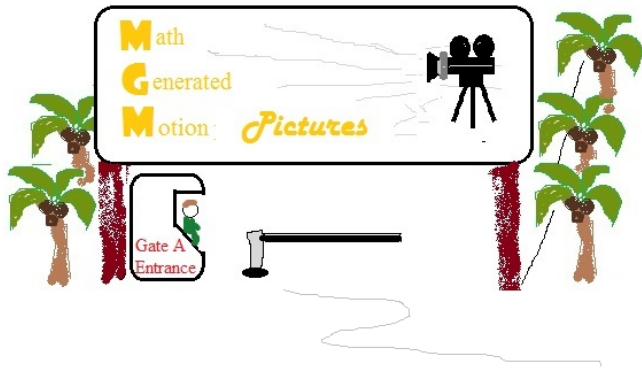
The Math Guy pitches his geometry idea....

_____

CHALLENGE QUESTION:
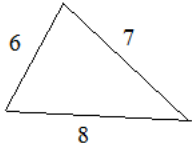
Find angle d:
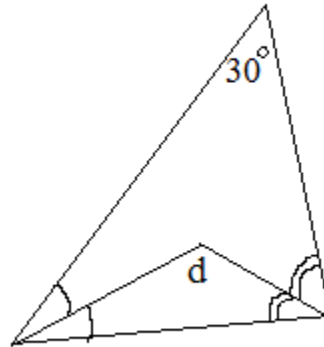
SOLUTIONS

Sides: Equilateral, Isosceles, Scalene

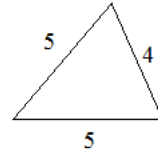Angles: Right, Acute, Obtuse

I. Identify the following:

sides: scalene

angles: acute

sides: isosceles

angles: acute

△ ABC where   m∠A = 25°
m∠B = 35°

sides: scalene

angles: obtuse
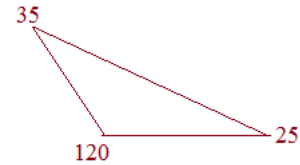
II. Classify the following triangles:

"30-60-90 triangle":

sides: scalene

angles: right
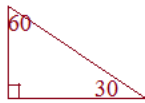
"45-45-90 triangle":

sides: isosceles

angles: right

"60-60-60 equiangular triangle":

sides: equilateral

angles: acute

III. Always, Sometimes, or Never?

1) An equilateral triangle is obtuse.   NEVER   all angles are 60°   (acute)

2) A right triangle is isosceles.   SOMETIMES   (if it is a 45-45-90, then it is right)

3) The sum of the interior angles of an obtuse triangle is 180°.

ALWAYS   Sum of interior angles of ALL triangles is 180°

Triangle Characteristics: Applications Quiz	SOLUTIONS

1) If the perimeter of an equilateral triangle is 18 feet, then what are the lengths of each side?

Since it is an equilateral triangle, all sides are the same.

$A = B = C$

$A + B + C = 18'$

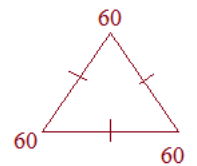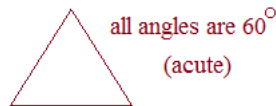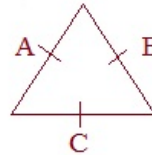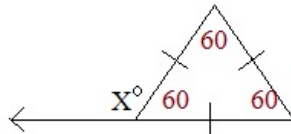| 6 feet |
|---|

A △ B
C

2) What is the measure of exterior angle X?

Since the triangle has 3 equal sides, it must be equilateral. Therefore, all angles are 60°

60
$X°$ 60   60
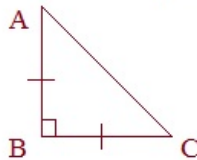
(supplementary angles)
$X + 60 = 180$

$X = 120°$

3) If △ABC is isosceles and right, what are the measures of each angle?

A
B   C

(Isosceles) $\overline{AB} = \overline{BC}$

(Right) $\angle B = 90°$

$\angle A + \angle B + \angle C = 180$

$\angle A = \angle C = 45°$

4) In diagram A, what are the measures of angles D, E, and F?

Sum of angles in a △ is 180..
And, vertical angles are congruent.
Therefore, F = 80

△DEF is isosceles because $\overline{DF} = \overline{EF}$

so, $\angle D = \angle E = 50°$
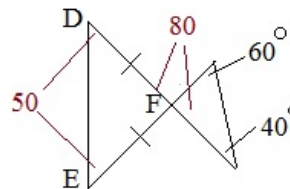
D   80   60°
50   F
40°
E

diagram A

*5) 'Trick question': If 2 sides of an isosceles triangle are 6 and 10 inches, what is the length of the 3rd side?

10   10
6

6   6
10

⇒
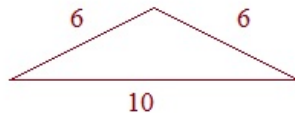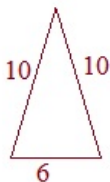
3rd side: 6 inches OR 10 inches

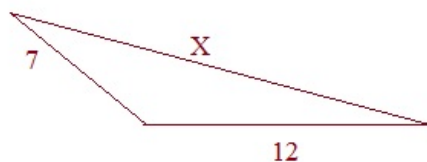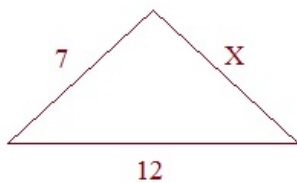****6) Challenge question: If 2 sides of a triangle are 7 and 12 inches, what is the length of the 3rd side?**

If largest side is 12 inches, then X must be larger than 5

If X is the largest side, then X cannot be larger than 19
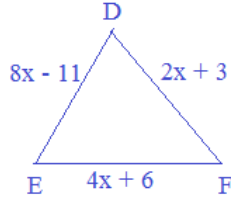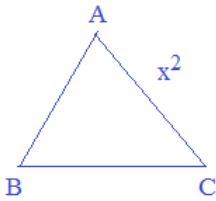
The length of the third side: $5 < X < 19$ inches

7   X
12

7   X
12

1) Given $\triangle ABC \cong \triangle DEF$
   In triangle ABC, which side is the smallest?

$\overline{AC} = x^2$
$\overline{DE} = 8x - 11$
$\overline{FD} = 2x + 3$
$\overline{FE} = 4x + 6$

Corresponding Parts Congruent Triangles Congruent

$AC = DF$
$x^2 = 2x + 3$
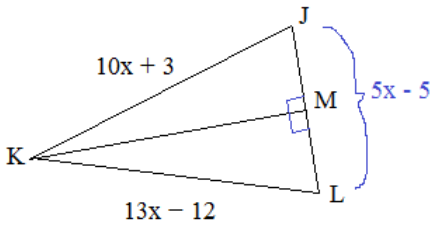$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = -1, 3$ If x = -1, then DE = 8(-1) - 11 = -19
             Side cannot be negative, so x ≠ -1

A
x^2

D
8x - 11   2x + 3

B   C   E   4x + 6   F

Since x = 3, DF = AC = 9
EF = BC = 18
DE = AB = 13

2) Given $\overline{KM}$ is a perpendicular bisector of $\overline{JL}$; $JL = 5x - 5$
   What is the length of $\overline{JM}$?

Quick proof: $KM \cong KM$ (reflexive property)
             $JM \cong ML$ (def. bisector)
             JMK and LMK are right angles
                   (def. of perpendicular)
             $\triangle JMK \cong \triangle LMK$ (Side-Angle-Side or HL)
Therefore, $\overline{JK} \cong \overline{KL}$ CPCTC

10x + 3
J
M   5x - 5
K
13x − 12   L

$10x + 3 = 13x - 12$
$15 = 3x$ If x = 5, then $\overline{JL} = 20$ and $\overline{JM} = 10$
$x = 5$

3) Always, Sometimes, or Never?
   Two triangles are congruent if
   2 sides and 1 angle are congruent to corresponding parts of another.

   SOMETIMES...   If the included angles are congruent, then the triangles must be congruent.
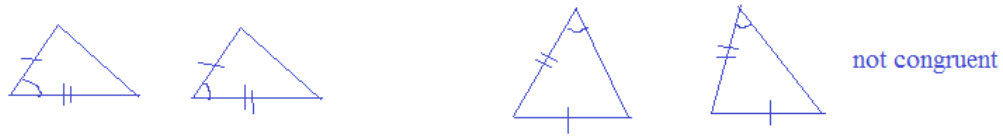
   not congruent

4) If the perimeter of an equilateral triangle is 6y + 18 and one side is 4y − 14,
   what is the perimeter?
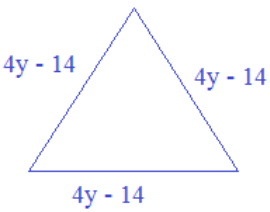
   If one side is 4y - 14, then all 3 sides are 4y - 14

   Therefore, the perimeter is 3(4y - 14)

   $6y + 18 = 3(4y - 14)$

   $6y + 18 = 12y - 42$

   $60 = 6y$
   $y = 10$

4y - 14   4y - 14

4y - 14

Since y = 10,
each side is 26
and the perimeter
is 78

5) In △ ABC,

if $\overline{AC} > \overline{BC} > \overline{AB}$, list the 3 angles in order of size (from largest to smallest)

Draw a diagram and assign values:

A

5   10

B   8   C
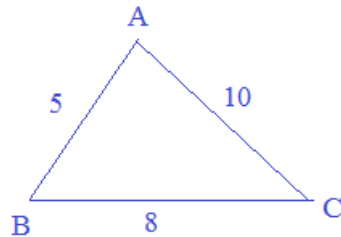
Since B is opposite the largest side, it is the largest angle...
And, since C is opposite the smallest side, it is the smallest angle...
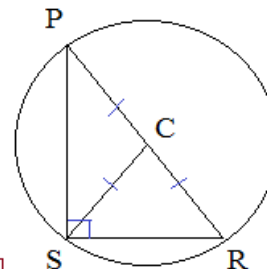
B (largest), A (middle), C (smallest)

6) In Circle C, PS ⊥ SR

$\angle P = 38^{o}$

Find   a) ∠ PSC

       b) ∠ R

If ∠P = 38 degrees
then,   ∠PSC = 38 degrees

If ∠PSC is 38 degrees,
then   ∠CSR = 52 degrees

Therefore,   ∠R = 52 degrees

P

C

S   R

**all radii are congruent

(angles-sides theorem if sides are congruent, then opposite angles are congruent)

7) Given: $\overline{AC} > \overline{AB}$

What are the restrictions of x?

If AC > AB, then   ∠B > ∠ C

5x - 42 > 18 + x

4x > 60

x > 15

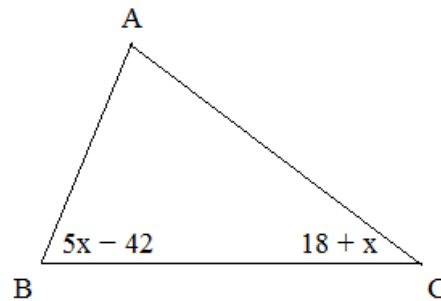Since the sum of interior angles of triangle is 180,   B + C < 180

5x - 42 + 18 + x < 180

6x - 24 < 180

6x < 204

x < 34

A

5x − 42   18 + x

B   C

15 < x < 34

8) Find x

Sum of angles must
be 180 degrees
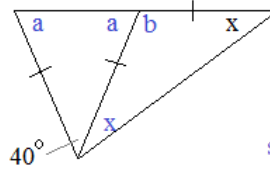
$a + a + 40 = 180$
$2a = 140$
$a = 70$

Since $a = 70$, $b = 110$ (supplementary)

If $b = 110$, then $x + x = 70...$ therefore, $x = 35$

Find y

a a b x

x

40°

Triangle Properties Questions

SOLUTIONS

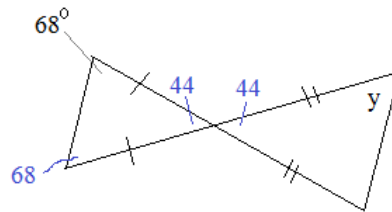since sides are congruent, the opposite
angles are congruent...

9) Find y

"sides-angles theorem", so
other angle is 68...
therefore, 3rd angle is 44 degrees

(vertical angles, so other
angle is 44 degrees)

$y + y + 44 = 180$
$2y = 136$
$y = 68$

68°

44 44 y

68

note: the 2 isosceles triangles
are similar
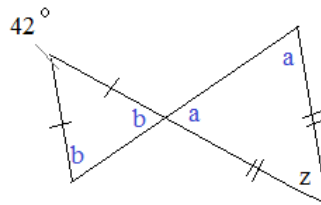
10) Find z

$b + b + 42 = 180$
$2b = 138$
$b = 69$

$a = b$ (vertical angles)

$a = 69$
then, $z = 42$

42°

a

b a

b z

note: the 2 triangles are similar
(isosceles)

11) The vertices of a triangle are $(2, -6)$ $(5, -2)$ $(7, -6)$
Is this triangle scalene, isosceles, or equilateral?

To determine sides, use the distance formula:

$(2, -6)$ to $(5, -2)$ $\quad d = \sqrt{(5-2)^2 + (-2-(-6))^2}$

$= \sqrt{9 + 16} = 5$
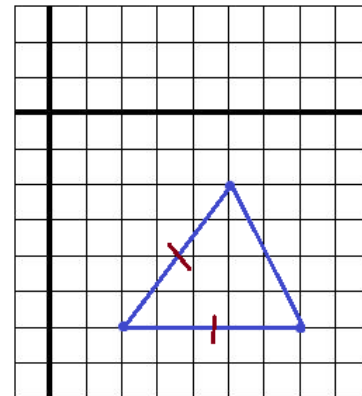
$(5, -2)$ to $(7, -6)$ $\quad d = \sqrt{(7-5)^2 + (-6-(-2))^2}$
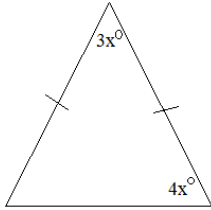
$= \sqrt{4 + 16} = 2\sqrt{5}$

$(7, -6)$ to $(2, -6)$ $\quad d = 5$
(horizontal line segment)

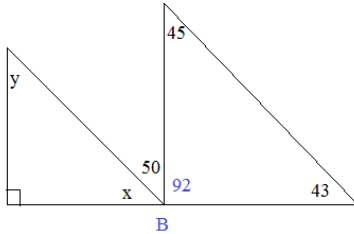Since 2 sides are the same length,
the triangle is isosceles

mathplane.com

12)

$3x°$

$4x°$

What is x?

Since triangle is isosceles, the other angle is 4x.

$3x + 4x + 4x = 180$

$11x = 180$

13)

45

y

50

x   92   43

B

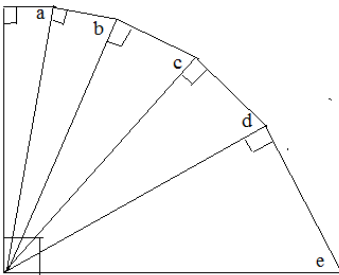Since interior angles add up to 180:

$45 + 43 + B = 180$

$B = 92$

Angle along line must add up to 180:

$92 + 50 + x = 180$ $\boxed{x = 38}$

then, $\boxed{y = 52}$

14)

a
b
c
d
e

$a + b + c + d + e = ?$

5 triangles, so the sum of the interior angles must be 5 x 180 = 900

We know there are 5 (small) 90 degree angles..
And, the sum of the lower angles is one (large) 90 degree angle..
So, we know the 6 angles add up to 540...
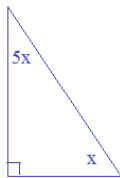
therefore, the remaining angles are 360...

a) 180

b) 240

c) 270

d) 360

e) 450

15) The measure of one angle in a right triangle is 5 times the measure of another.
     What are the angles?

$5x$
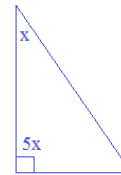
x

$5x + x + 90 = 180$

$6x = 90$

$x = 15$

$\boxed{15\text{-}75\text{-}90}$

OR

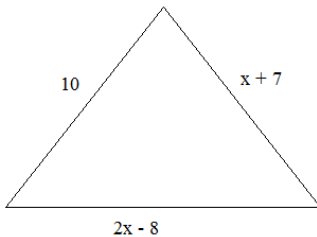since $5x = 90$,
$x = 18$

$18 + ? + 90 = 180$

$? = 72$

$\boxed{18\text{-}72\text{-}90}$

x

$5x$

16) If the perimeter of this isoceles triangle is less than 45,
     which side is the base?

10   x + 7

2x - 8

If 10 is the base, then
$x + 7 = 2x - 8$
$x = 15$
and, the perimeter is $10 + 22 + 22 = 54$
($54 > 45$, so 10 is not the base)

$\boxed{x + 7 \text{ is the base}}$

If x + 7 is the base, then
$10 = 2x - 8$
$x = 9$
and, the perimeter is $10 + 16 + 10 = 36$

If 2x - 8 is the base, then
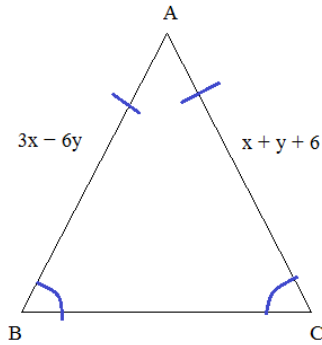$10 = x + 7$
$x = 3$
and, the perimeter is $10 + 10 + (-2)$
***a side cannot have a length *negative* 2

mathplane.com

17) Given: $\angle\,B = 3x + 10$

             $\angle\,C = 46 - 3y$

    $\triangle$ABC is an isosceles triangle with base $\overline{BC}$

    What is the measure of angle A?

Solutions

congruent sides:

$3x - 6y = x + y + 6$

$2x - 7y = 6$

congruent angles:

$3x + 10 = 46 - 3y$

$3x + 3y = 36$

$x + y = 12$

solve system:

$2x - 7y = 6$

$x + y = 12$
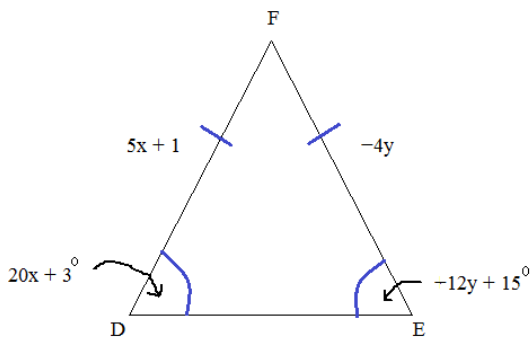
$\longrightarrow -2x - 2y = -24$

$-9y = -18$

$y = 2$

then, $x = 10$

If $x = 10$ and $y = 2$, then $\overline{AB} = \overline{AC} = 18$ ✓

$\angle\,B = \angle\,C = 40^{0}$ therefore, angle A $= 100^{0}$

18) $\triangle$ DEF is an isosceles triangle with base $\overline{DE}$

    Determine the measure of the angles and the measure of the sides...

$5x + 1 = -4y$

$5x + 4y = -1$

$20x + 3 = -12y + 15$
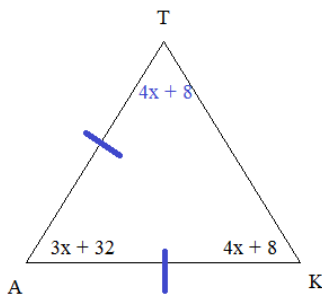
$20x + 12y = 12$

$5x + 3y = 3$

$y = -4$

and

$x = 3$

angles are 63, 63, and 54

and the sides are 16 and 16...

NOTE: The base side requires trignometry to determine its length....

19) $\overline{TA} = \overline{AK}$

    Find the measures of all the angles.

Since TA = AK, angle T and angle K are congruent
(because if congruent angles, then congruent sides)

angle T + angle K + angle A = 180 degrees

$4x + 8 + 4x + 8 + 3x + 32 = 180$

$11x = 132$

$x = 12$

Angle T = 56

Angle K = 56

Angle A = 68

20) Are the points (1, 0) (7, 3) (-1, 4) the vertices of a right triangle?

SOLUTIONS

Justify using the distance formula:

Find distance between points then, apply Pythagorean Theorem

(1, 0) to (7, 3) $\sqrt{45}$ a

(1, 0) to (-1, 4) $\sqrt{20}$ b

(7, 3) to (-1, 4) $\sqrt{65}$ c

Pythagorean Theorem

$a^2 + b^2 = c^2$

Justify using slope:

find slope between points..
If any have opposite reciprocal - i.e. perpendicular then, there is a right angle

YES, it's a right triangle

slope of b: -2
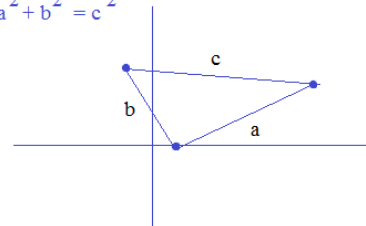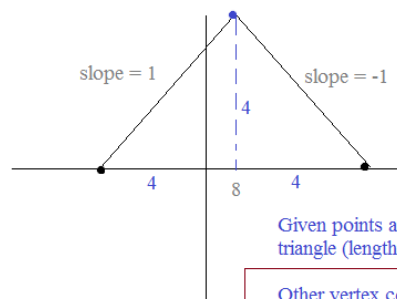slope of a: 1/2

21) Assume (−3, 0) and (5, 0) are vertices of an isosceles right triangle. Can you identify the 3rd vertex? (there are 6 possibilities)

8

8
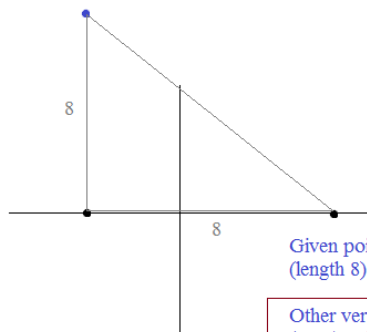
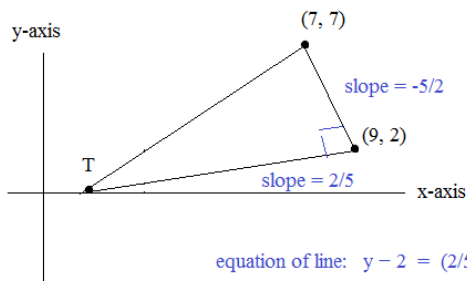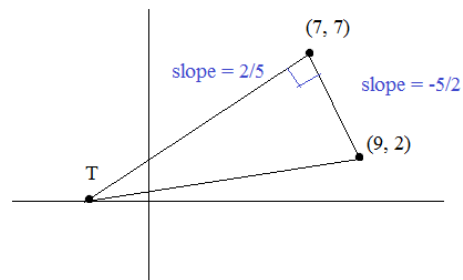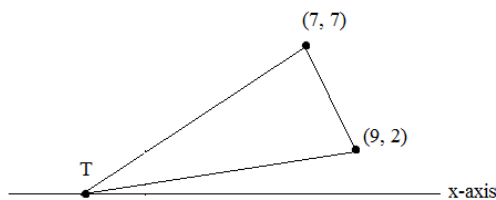Given points are leg of right triangle... (length 8)

Other vertex could be
(-3, 8)   (-3, -8)   (5, 8)   (5, -8)

slope = 1   4   slope = -1

4   8   4

Given points are hypotenuse of right triangle (length 8)

Other vertex could be
(1, 4)   (1, -4)

22) If △ RHT is a right triangle, and T is on the x-axis, what is T? (note: figure not drawn to scale!)

(7, 7)

(9, 2)

T

x-axis

(7, 7)

slope = 2/5   slope = -5/2

(9, 2)

T

y-axis

(7, 7)

slope = -5/2

(9, 2)

T

slope = 2/5

x-axis

equation of line: $y - 2 = (2/5)(x - 9)$

$-2 = (2/5)x - 18/5$

$8/5 = (2/5)x$

$x = 4$ (4, 0)

Equation of line: slope = 2/5 point: (7, 7)

$y - 7 = (2/5)(x - 7)$

x-intercept: (?, 0)

$0 - 7 = (2/5)(x - 7)$

$-35/2 = (x - 7)$
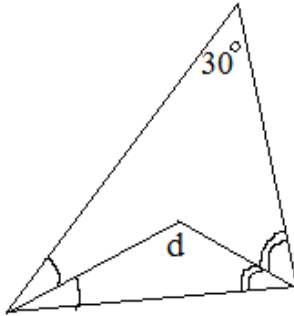
$x = -21/2 = -10.5$ (-10.5, 0)

23) In △XYZ,

$\overline{XY} = 5$

$\overline{XZ} = 12$

$\overline{YZ} = 6$

list angles from largest to smallest....

NONE, because the triangle does not exist!!

5   6

12

Find angle d:

(big triangle)

$2x + 2y + 30° = 180°$

$2x + 2y = 150°$

$x + y = 75°$
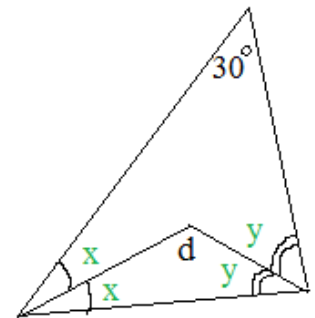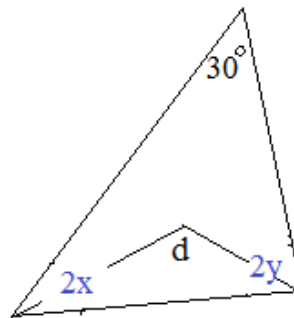
(small triangle)

$x + y + d = 180°$

$75° + d = 180°$

$d = 105°$
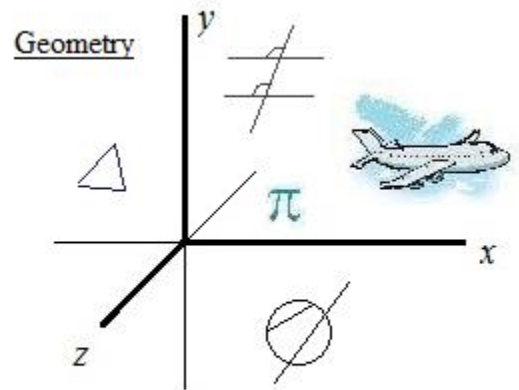
ANSWER

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know...

Cheers...

Geometry

6 more triangle questions:

1) What is the name of a triangle where all sides have different lengths?

2) A triangle has sides of length 8 and 13. What are the possible lengths of the 3rd side?

3) Two sides of an isosceles triangle are 5 and 7 feet. What is the perimeter of the triangle?

4) DEF is a right triangle. If angle E is 37 degrees, what are the measures of D and F?

5) Where do the altitudes of a right triangle intersect?

6) If the 3 altitudes of triangle ABC intersect outside the triangle, what type of triangle is ABC?

Answers to **6 more triangle questions**:

1) *Scalene* Triangle

2) Length of 3rd side (S):

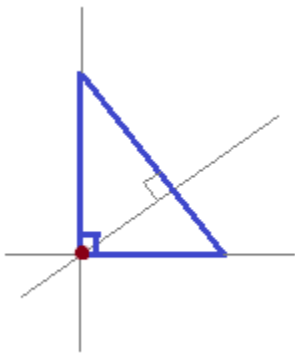$5 < S < 21$ (i.e. any length between 5 and 21)

3) a) $5 + 5 + 7 = 17$ OR,

  b) $7 + 7 + 5 = 19$

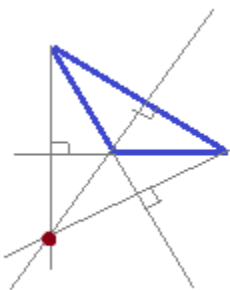4) $D = 53$ $F = 90$ OR,

  $F = 53$ $D = 90$

5)   At the right angle vertex

(3 altitudes of right triangle)

6) ABC is *Obtuse*

(3 altitudes of obtuse triangle)