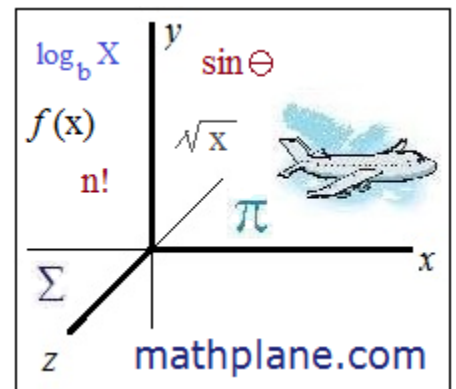


% Percentages %

Notes, Examples, and Quiz (w/Solutions)



Finding the percentage of one number out of another:

Set up the ratios. Solve.

Example: What percentage of 30 is 12? Or, "12 is what percentage of 30?"

Set up the ratios: $\frac{X}{100} = \frac{12}{30}$ Part/Portion
Whole

Solve for X: $\frac{X}{100} = \frac{2}{5}$ Reduce the fraction

$5X = 2(100)$ Cross multiply

$X = 40$ 40%

Alternate method: Convert the ratio into a decimal. Then, convert the decimal into a percentage.
 (i.e. move 2 decimal places and add % symbol)

Example: 23 is what percentage of 58? Or, "what percent of 58 is 23?"

Convert the ratio into a decimal: $\frac{23}{58}$ is approximately .39655

Convert into a percentage: $.39655 \xrightarrow{\text{2 decimal places}} 39.655\%$

Quick check:
 20 out of 50 is 40%
 23 out of 58 is close to 40%

X% of what number is another number:

Set up ratios. Solve.

Example: 30% of what number is 42?

Set up ratios: $\frac{30}{100} = \frac{42}{X}$ Part that's taken
Whole

Solve: $\frac{3}{10} = \frac{42}{X}$

$3X = 420$ X = 140

Shortcut: Set up decimal equation. Solve.

Example: 28% of what number is 20?

$.28X = 20$

$X = \frac{20}{.28} \approx 71.43$ (i.e. 28% of 71.43 is approx. 20...)

Percentages: Notes & Examples

Steps to determine percentage increase/decrease:

Percentage Increase/Decrease

- 1) Find Change
- 2) Establish "starting point"
- 3) Solve
- 4) Check for "reasonableness"

Example: What is the percentage increase from 20 to 25?

Find change: the increase is 5

"starting point": the starting point is 20

solve: what percentage is 5 out of 20? $\frac{5}{20} = .25$ therefore, 25%

Example: What is the percentage decrease from 25 to 20?

Find change: the decrease is 5 (i.e. -5)

"starting point" (basis): the starting point is 25

solve: what percentage is 5 out of 25? $\frac{5}{25} = .20$ therefore, 20%

check: 10% of 25 is 2.5... So, 20% of 25 is 5...

***NOTE: In the above examples, the numbers are 20 and 25... And, the change is 5...
But, the percentages are different! (*percentage change depends on the starting point!*)

Example: After a 30% increase, the total is 100. What was the original amount?

Suppose you simply decreased 30% from 100. The result is 70. But, that is NOT correct!
IF you added 30% to 70, the answer is not 100... It is 91...

SOLUTION: original amount + increase = final amount

$$X + (30\% \text{ of } X) = 100$$

$$X + .30X = 100$$

$$1.3X = 100$$

$$X = 76.9 \text{ (approximately)}$$

check:

30% of 76.9 is approx.
23.1

$76.9 + 23.1 = 100$ ✓

Example: A \$34 shirt is on sale for 20% off. What is the price of the shirt?

original amount + increase/decrease = final amount

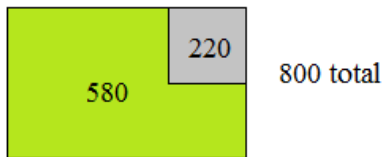
$$\$34 + (-.20)(\$34) = \text{final price}$$

$$\$34 - \$6.8 = \text{\$27.20}$$

Word problems:

Like most word problems, a solid strategy is to draw a picture and/or label variables. Then, construct the formula and solve.

Example: Joe's entire backyard is 800 square feet. If the garden is 220 square feet, what percentage of Joe's backyard is the garden?



$$\text{percentage} = \frac{X \text{ (percent)}}{100} = \frac{220}{800}$$

"portion" (garden)
whole (backyard)

$$\frac{X}{100} = \frac{11}{40}$$

$$40X = 1100 \quad X = 27.5$$

27.5%

("check for reasonableness": 25% or 1/4 of 800 is 200... So, 220 out of 800 should be a little bit more than 25%)

Example: Sam enjoyed the dinner and service at his favorite restaurant. The final bill was \$48. If he left an 18% tip, how much did he spend at the restaurant?

$$\text{Total cost} = \text{dinner bill} + \text{tip}$$

$$.18 \times \$48 = \$8.64$$

$$\text{Cost} = \$48 + (18\% \text{ of } \$48)$$

$$\text{or, } \frac{18}{100} = \frac{X}{48} \quad X = 8.64$$

$$\$48 + \$8.64 = \boxed{\$56.64}$$

Example: After a 35% discount, the cost of a shirt \$19.50. What is the original price of the shirt (without the discount)?

$$\text{Original price} - \text{discount} = \text{Cost of shirt}$$

35% of X

$$X - (35\% \text{ of } X) = \$19.50$$

$$\frac{35}{100} = \frac{\text{discount}}{X}$$

$$X - .35X = \$19.50$$

$$35X = 100(\text{discount})$$

$$.65X = \$19.50$$

$$\text{discount} = \frac{35X}{100} = .35X$$

$$\boxed{X = \$30}$$

"75% of this class passed the test --
-- which is 50% more than last
year's class...."

24 students
A - 5
B - 8
C -

A Sense of Percents

Okay, 75% of 24
is 18.. And,---

"25% last year!
What a bunch of idiots!!"

And, there's a 100%
chance *he'll* be in
next year's class...

Pre-Algebra

Questions: Assuming each class has 24 students, how
many passed last year? what percentage passed?
(Answers will come in time!)

LAF #150 (8-7-14)
mathplane.com

Practice Test (And, Solutions) ->

Percentages Quiz

I. Find X in each expression:

1) 35% of 200 is X

2) 4% of 20 is X

3) 22% of X is 11

4) 55% of X is 100

5) X% of 48 is 6

6) X% of 6 is 48

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6

2) 90 to 108

3) 30 to 70

4) 41 to 38

5) 26.50 to 21.50

6) 9 to 0

Find the result:

7) cut 13% from 200

8) decrease 88 by 25%

9) increase 34 by 20%

10) increase 25 by 500%

III. Word problems/applications

- 1) The price of the lunch special is \$9. If sales tax is 7%, what is the total cost?
- 2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?
- 3) The price of gas has gone up from \$2.75 to \$4.35. What is the percentage increase?
- 4) If you borrow \$500 at an annual interest rate of 6%, how much will you owe after 1 year?
- 5) After a 20% discount, the price of a men's suit is \$185. What was the *original* price of the suit?

***Challenge: Last year, an investment lost 30% of its value.
What *percentage* increase is necessary this year to recover the lost value?

IV: two-step questions

- 1) You earned 64 points on a math quiz. Your grade was 80%.

How many points were needed to earn a 90% ?

- 2) A bowl contains red, blue, and white candies.
58 out of 102 candies were blue.
What percentage were not blue?

- 3) A store sells candy for 50 cents each..
Sammy buys as much candy as he can afford.
When he gets to the counter, he learns that there is 8% sales tax.
He returns 2 pieces of candy, and then pays for the remaining candy (including 8% tax)....

How much candy did he buy?

What did it cost?

Hidden Message

Solve/Answer questions below.
Translate numbers to letters.

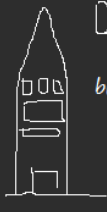
Clue: "It may be 4%"

% %

% Letter Key %									
1	2	3	4	5	6	7	8	9	0
A	C	E	I	M	N	O	R	S	T

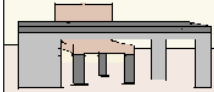
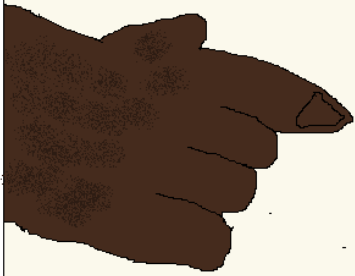
- 1) 40% of 5 → _____
- 2) A bag contains 20 colored marbles (red, blue, or green).
If 35% are blue, 6 are green, how many are red? → _____
- 3) 20% of math students will get an 'A'.
If a class has 14 girls and 11 boys,
how many students will earn 'A's'? → _____
- 4) An 80% free throw shooter attempts 25 shots.
How many shots does he expect to miss? → _____
- 5) 2 out of 50: % → _____
- 6) 18% of 217 3 .06 → _____
- 7) 150% of 6 → _____
- 8) A matinee cost \$5.40. If the show ordinarily cost 9 dollars,
what discount (%) did you receive? 0% → _____
- 9) Dinner cost \$30 plus sales tax. If tax is 10% and you leave
\$40, what tip did you leave your waiter? → _____
- 10) Inside the instructions box above, what percentage of
the letters are 'a' ? % → _____
- 11) .8% of 1000 → _____
- 12) A shirt retails for \$28. If you get a 25% discount,
how much does the shirt cost you? \$2 → _____
- 13) 30% of produced cars are white. If 1698 cars are white,
what is the total number manufactured? 566 → _____
- 14) A square has an area of 100 sq. feet.
If you reduce the length of each side by 40%, what is the
area of the new square? 6 ft² → _____

Ratios / Proportions



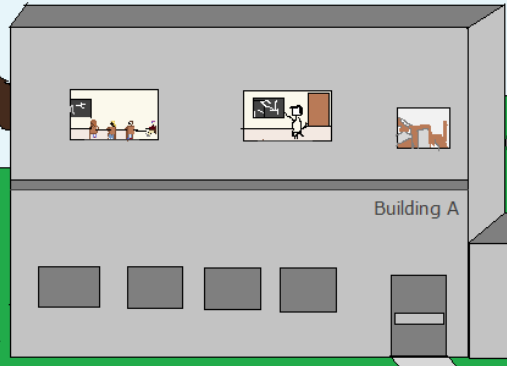
building : model
 $2000' : 20''$

plane : paw

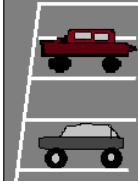


"His class rocks!"

"I think the teacher likes the blonde girl."



"... this is maintenance again at Empire State High School.... yeah, we're gonna need another 3 desks..."



Although he had an enormous impact on the class, Mr. Kong wasn't rehired to teach math...

Solutions-→

Percentages Quiz

SOLUTIONS

I. Find X in each expression:

1) 35% of 200 is X $\frac{35}{100} = \frac{X}{200}$
 $.35 \times 200 = 70$ OR $100X = 35(200)$
 $X = 70$

2) 4% of 20 is X note: 40% of 20 is 8
then, 4% of 20 is .8
 $.04 \times 20 = .8$

3) 22% of X is 11
 $\frac{22}{100} = \frac{11}{X}$ ← part
← whole
 $22X = 1100$
 $X = 50$

4) 55% of X is 100
 $\frac{55}{100} = \frac{100}{X}$ $55X = 10000$
 $X \approx 181.82$

5) X% of 48 is 6 "6 out of 48 is what %?"
 $\frac{X}{100} = \frac{6}{48}$
 $\frac{X}{100} = \frac{1}{8}$ $X = 12.5$

6) X% of 6 is 48
 $\frac{X}{100} = \frac{48}{6}$
 $X = 800$

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6 increase amount: 2 starting amount: 4 $\frac{2}{4}$ out of 4 is 50%

2) 90 to 108 increase: 18 starting basis: 90
 $\frac{18}{90} = .20$ 20%

3) 30 to 70 increase: 40 starting amount: 30 $\frac{40}{30} = 1.33$
 133.33%

4) 41 to 38 decrease: 3 starting point: 41 $\frac{3}{41} \approx .0732$
 $\text{approx. } 7.32\%$

5) 26.50 to 21.50 decrease: 5 start: 26.50 $\frac{5}{26.5} \approx .189$
 $\text{approx. } 18.9\%$

6) 9 to 0 taking all away $\rightarrow 100\%$
 decrease: 9 starting basis: 9
 $\frac{9}{9} = 1$ (move 2 decimal places)

Find the result:

7) cut 13% from 200
 $.13 \times 200 = 26$ $200 - 26 = 174$

8) decrease 88 by 25%
 25% of 88 is 22... $88 - 22 = 66$ $\frac{25}{100} = \frac{22}{88}$

9) increase 34 by 20%
 $.20 \times 34 = 6.8$ then, $34 + 6.8 = 40.8$

10) increase 25 by 500%
 increase 25 by 100% is +25
 so, increase 25 by 500% is $5 \times 25 = 125$
 $25 + 125 = 150$

- 1) The price of the lunch special is \$9. If sales tax is 7%, what is the total cost?

$$\begin{aligned} \text{Cost} &= \text{lunch price} + \text{sales tax} \\ &= \$9 + (.07)(\$9) = \boxed{\$9.63} \end{aligned}$$

- 2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?

$$\begin{aligned} \text{total population: } 357 + 395 &= 752 & \frac{X}{100} &= \frac{357}{752} \begin{array}{l} \text{"portion"} \\ \text{"whole"} \end{array} & 752X &= 100(357) \\ & & & & & \boxed{X = 47.47} \end{aligned}$$

- 3) The price of gas has gone up from \$2.75 to \$4.35. What is the percentage increase?

$$\begin{aligned} \text{The increase is } \$1.60 & & \frac{1.6}{2.75} & \text{ is approx. } .582 \\ \text{The "starting point" (basis) is } \$2.75 & & & \end{aligned}$$

price has gone up about 58.2%

- 4) If you borrow \$500 at an annual interest rate of 6%, how much will you owe after 1 year?

$$\begin{aligned} \text{After 1 year, you will owe } \$500 + \text{interest} &= \\ \$500 + .06(\$500) &= \boxed{\$530} \end{aligned}$$

- 5) After a 20% discount, the price of a men's suit is \$185. What was the *original* price of the suit?

Notice, we are looking for the *original* price.
So, we need to construct the appropriate formula and variables....

$$\begin{aligned} \text{Original price} - \text{discount} &= \text{final price} \\ X - X(.20) &= \$185 \end{aligned}$$

$$\begin{aligned} \text{Quick check: } 231.25 & & .80X &= \$185 \\ 20\% \text{ of } 231.25 \text{ is } 46.25 & & & \\ 231.25 - 46.25 = 185 & \checkmark & & \boxed{X = \$231.25} \end{aligned}$$

***Challenge: Last year, an investment lost 30% of its value.
What *percentage* increase is necessary this year to recover the lost value?

Suppose the investment were \$100...

$$\begin{aligned} \text{After 1 year, the investment would be worth } \$70.. \\ 100 - .30(100) &= 70 \end{aligned}$$

So, what *percentage* is necessary to increase 70 to 100?

increase is 30; starting basis is 70

$$\frac{30}{70} \cong .4286 \quad \boxed{\text{approximately } 42.86\%}$$

SOLUTIONS

IV: two-step questions

1) You earned 64 points on a math quiz. Your grade was 80%.

How many points were needed to earn a 90% ?

Step 1: Find the total points $\frac{64}{X} = \frac{80}{100}$ $X = 80$ The test had a total of 80 points....

Step 2: Use total points find 90% $\frac{Y}{80} = \frac{90}{100}$ $Y = 72$ You would need to get 72 points to earn a 90%

2) A bowl contains red, blue, and white candies. 58 out of 102 candies were blue. What percentage were not blue?

Step 1: find the percentage that were blue $\frac{58}{102} \frac{\text{blue}}{\text{total}} = .57 \text{ approx. } 57\%$

Step 2: Subtract % of blue from 100%. the rest are not blue... $1 - \frac{58}{102} = \frac{44}{102}$ 43% (approximately)

3) A store sells candy for 50 cents each.. Sammy buys as much candy as he can afford. When he gets to the counter, he learns that there is 8% sales tax. He returns 2 pieces of candy, and then pays for the remaining candy (including 8% tax)....

How much candy did he buy?

What did it cost?

Let C = # of candy pieces

Money in Sammy's pocket = .50C (without sales tax)

Money in Sammy's pocket = [(C - 2) x .50](1.08)

Set amounts equal to each other:



.50C = [(C - 2) x .50](1.08)

.50C = .54C - 1.08

-.04C = -1.08

C = 27

27 pieces x .50 = 13.50

25 pieces x .50 = 12.50 + .08(12.50) = 13.50

He bought 25 pieces and spent 13.50....

Hidden Message

Clue: "It may be 4%"

Solve/Answer questions below.
Translate numbers to letters.

% %

Solutions

% Letter Key %

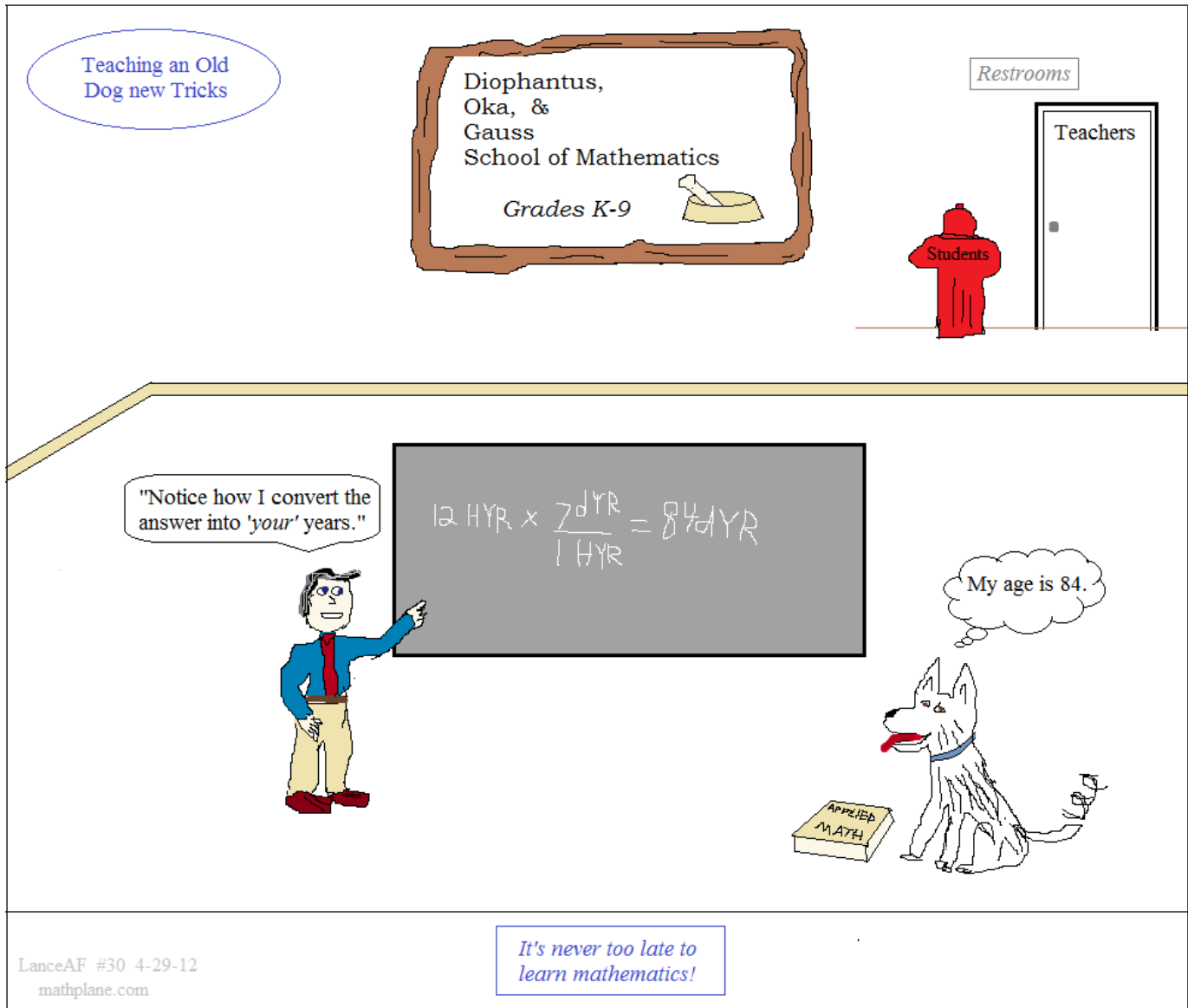
1	2	3	4	5	6	7	8	9	0
A	C	E	I	M	N	O	R	S	T

- 1) 40% of 5 $.40 \times 5 = 2$ → C
- 2) A bag contains 20 colored marbles (red, blue, or green). 20 total: 7 blue (.35 x 20)
If 35% are blue, 6 are green, how many are red? 6 green
7 red → O
- 3) 20% of math students will get an 'A'. $.20 (14 + 11) = 5$
If a class has 14 girls and 11 boys, how many students will earn 'A's?' → M
- 4) An 80% free throw shooter attempts 25 shots. An 80% shooter expects to
How many shots does he expect to miss? miss 20%... $25 \times .20 = 5$ → M
- 5) 2 out of 50: % 2 out of 50 is equivalent
to 4 out of 100 ---> 4% $\frac{2}{50} = .04$ → I
- 6) 18% of 217 $.18 \times 217 = 39.06$ $.06$ → S
- 7) 150% of 6 100% of 6 is 6... 50% of 6 is 3 $150\% = 9$ $1.50 \times 6 = 9$
original discount matinee
 $9.00 - (9.00x) = 5.40$
 $-(9.00x) = -3.60$
 $x = .40$ (40%) → S
- 8) A matinee cost \$5.40. If the show ordinarily cost 9 dollars,
what discount (%) did you receive? 0% → I
- 9) Dinner cost \$30 plus sales tax. If tax is 10% and you leave \$40, what tip did you leave your waiter? total bill: 30 dinner If you leave \$40,
+ 3 tax then the tip is \$7.
33 dollars → O
- 10) Inside the instructions box above, what percentage of the letters are 'a' ? there are 50 total letters 3 out of 50
and 3 'a's... (see above) is 6% % → N
- 11) .8% of 1000 $.008 \times 1000 = 8$ 8% of 100 is 8
so, .8% of 1000 is 8 → R
- 12) A shirt retails for \$28. If you get a 25% discount, how much does the shirt cost you? 25% of 28 = 7
therefore, shirt costs $28 - 7 = 21$ \$2 → A
- 13) 30% of produced cars are white. If 1698 cars are white, what is the total number manufactured? $\frac{30}{100} = \frac{1698 \text{ white}}{x \text{ total}}$ cross multiply
and solve to
get 5660 0 → T
- 14) A square has an area of 100 sq. feet. $\frac{100}{\text{sq ft}}$ $\frac{10}{10}$ $\frac{6}{\text{sq ft}}$ $\frac{6}{6}$ 6 ft^2 → E
If you reduce the length of each side by 40%, what is the area of the new square? $10 - (.40 \times 10) = 6$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know!

Cheers



Find more comics and math resources at the Mathplane site, TeachersPayTeachers, and TES

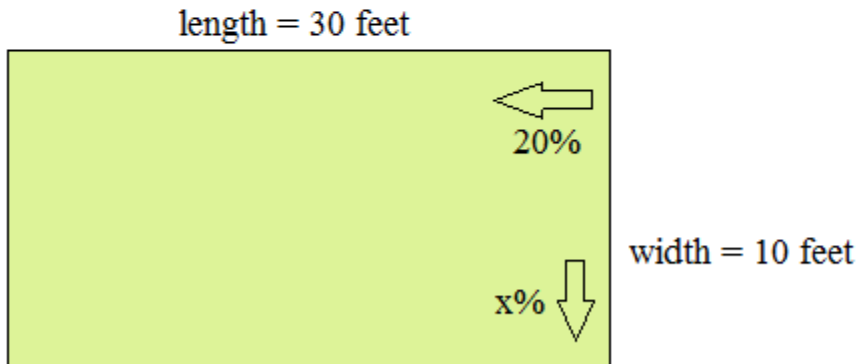
Also, Mathplane *Express* for mobile at Mathplane.ORG

TWO MORE QUESTIONS....

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width

- a) to maintain the same area?
- b) to maintain the same perimeter?

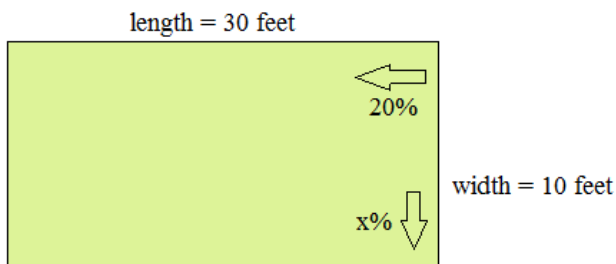


ANSWERS on next page....

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width

- a) to maintain the same area?
- b) to maintain the same perimeter?



ANSWERS

a) Area....

Original area is 300 square feet...

New length is $30' - (.20 \times 30') = 24'$

$24' \times (\text{new width}) = 300 \text{ square feet}$

new width = 12.5'

10 feet ---> 12.5 feet (increase of 2.5 feet)

percentage increase: $\frac{2.5 \text{ feet}}{10 \text{ feet}} = 25\%$

b) Perimeter....

Original perimeter is 80 feet....

New length is 24'...

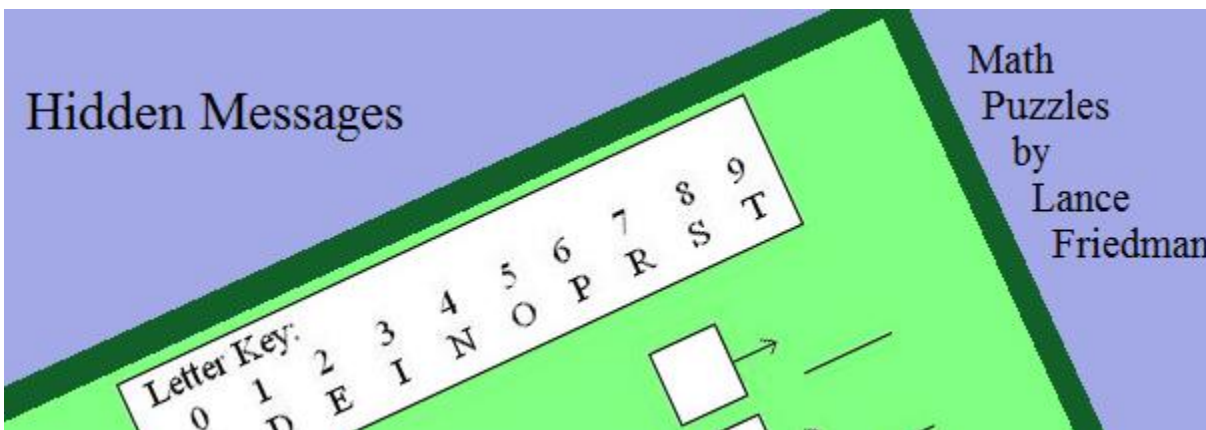
$(2 \times 24') + (2 \times \text{new width}) = 80 \text{ feet}$

$2 \times \text{new width} = 32 \text{ feet}$

new width = 16 feet

10 feet ---> 16 feet (increase of 6 feet)

percentage increase: $\frac{6 \text{ feet}}{10 \text{ feet}} = 60\%$



Available at mathplane.com