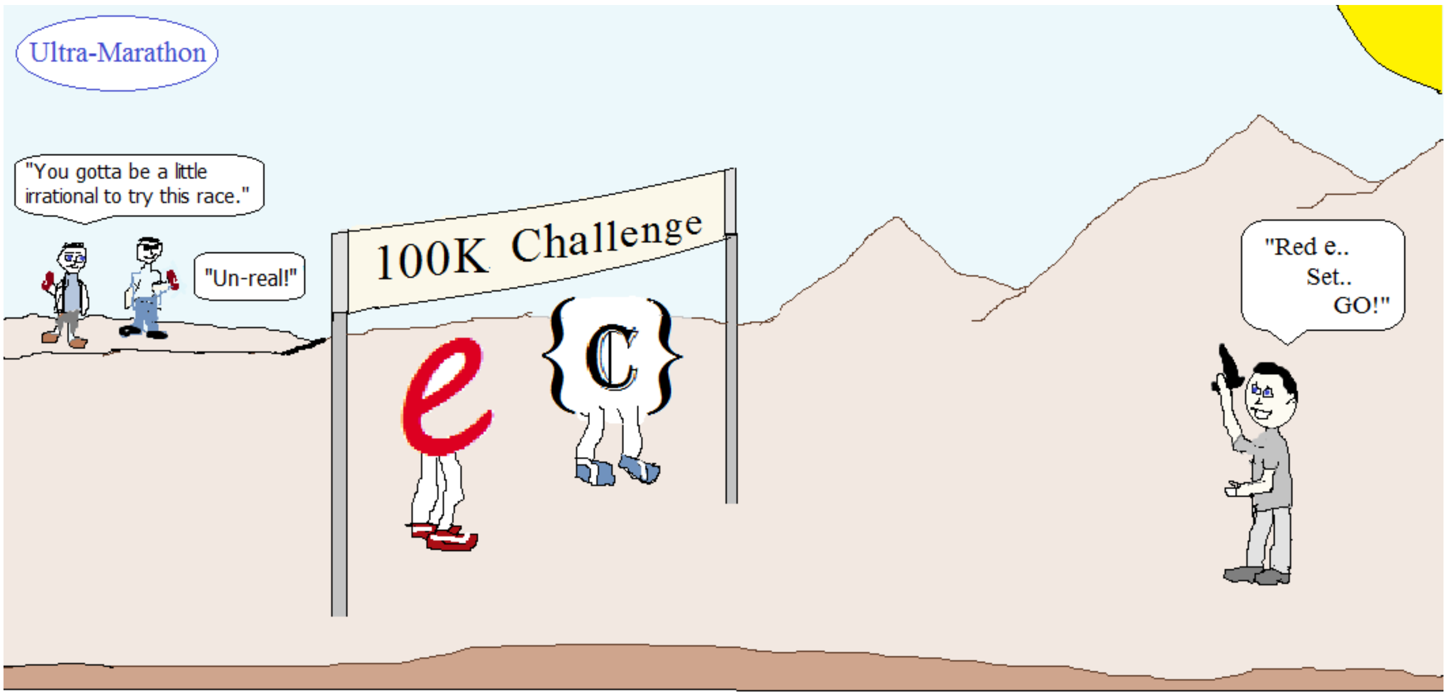
Algebra 2 Preview

Examples and Practice Tests (with Solutions)



Testing the limits of endurance,
these math figures will run on and on...

LanceAF #87 5-24-13
www.mathplane.com

Topics include exponents, graphing systems, factorials, average rate of change, inequalities, inverse functions, and more.

Algebra II Preview

I. Factorials

a) $5! =$

b) $\frac{12!}{10!} =$

c) $\frac{12!}{5!7!} =$

d) $\frac{6!4!}{5!5!} =$

e) $\frac{n!}{(n-3)!} =$

f) $\frac{(n+1)!(n-1)!}{n!(n-2)!} =$

g) How many different ways can the letters A B C D E F G be arranged?

II. Inverse Functions/Equations

Find the inverse:

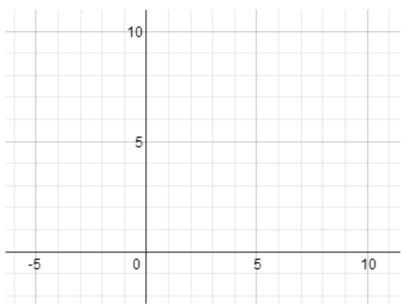
a) $y = 3x + 6$

b) $y = x^2 - 5$

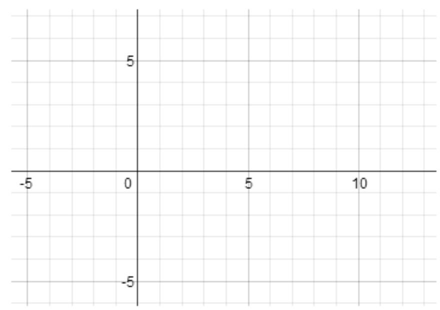
c) $y = -4x^2 + 7$

III. Graphing

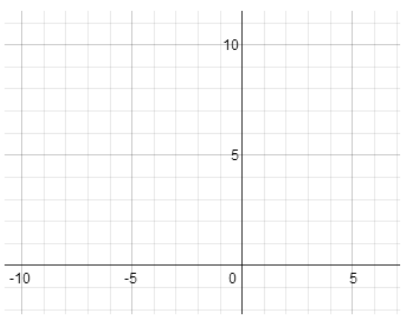
1) $y = |x - 4|$



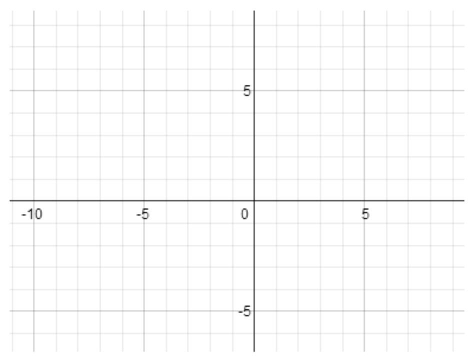
2) $x = y^2$



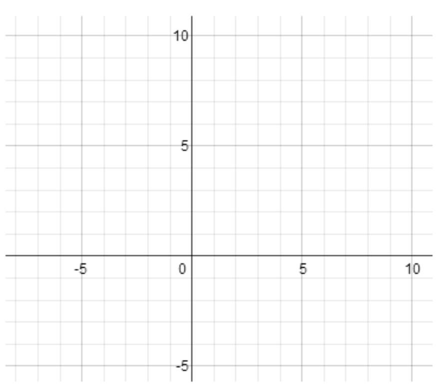
3) $y = 3^x$



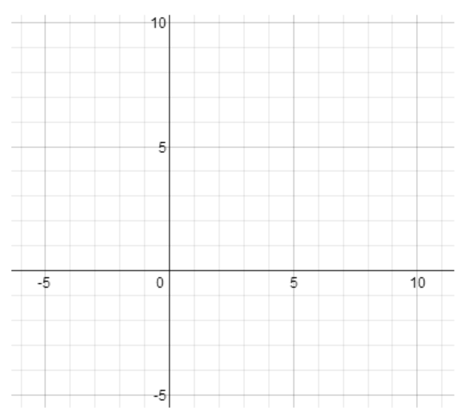
4) $x > -6$
$x + 2y \geq -8$



5) $y \geq x^2 + 4$
$y \leq 2x + 3$



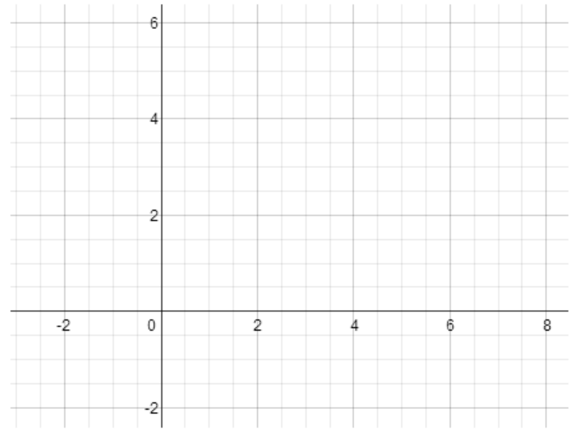
6) $(x - 3)^2 + y = 6$



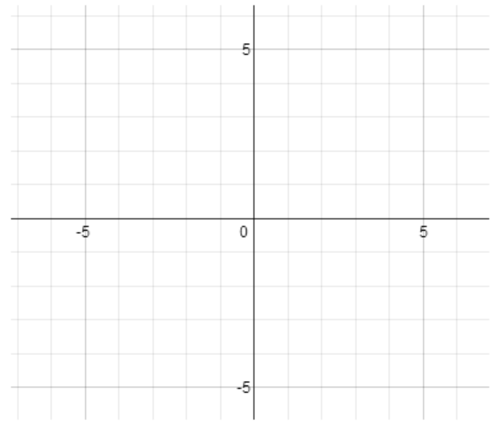
IV. Systems

Solve the following systems.
Graph to confirm your answers.

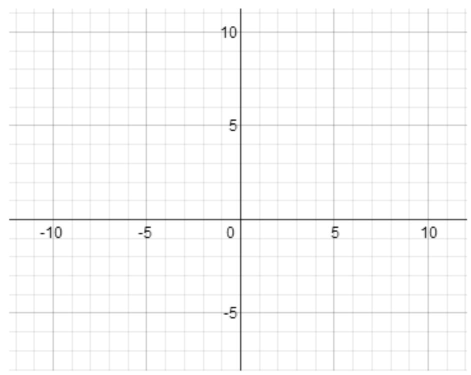
1) $3x + 4y = 25$
$y = x + 1$



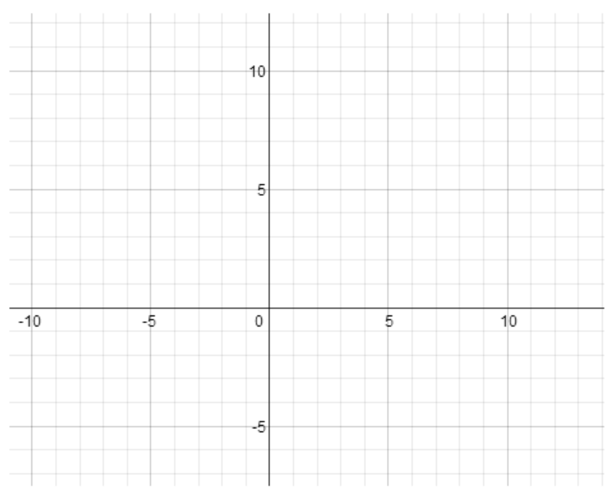
2) $x^2 + y^2 = 16$
$2x - y = 8$



3) $y = 2x^2 + 3$
$y = -6$



4) $y = (x-2)^2$
$y = 3x - 6$



Algebra II Preview

V. Miscellaneous

1) Do the points (3, 4) (8, 18) and (11, 6) form the vertices of a right triangle? Explain why or why not?

2) Solve the following: $x^2 - x - 2 < 0$
Write the answer in *interval notation*.

3) $|2x + 7| \geq 7$ Answer in *set builder notation*.
Use a number line to illustrate the solution set.

Algebra I and II Exponents Exercises

I. Simplify

A) $3^3(3x)^2$

B) $(b^5)^2\, b^3$

C) $(3.1)^5 \cdot (3.1)^{-5}$

D) $(-4x)^2 + 4x^2$

E) $(-2a^2 b)^3 (ab)^3$

F) $(3^6)^2 \cdot (3^{-4})^2$

G) $(10^3)^4 \cdot (4.3 \text{ x } 10^{-9})$

H) $(2xy^2)^4 (-y)^{-3}$

I) $x^{(a-1)} \cdot x^{(1-a)}$

II. Find x

A) $3^{2x} = 9^4$

B) $8^2 = 2^x$

C) $5^5 = 25^x$

III. Reduce

A) $\dfrac{a^{-3} b^2 c^0}{a^{-6} b^5 c^2}$

B) $\dfrac{8d^3 e^4}{4d^0 e^6}$

C) $\dfrac{6f^{-2} g^{-3}}{2^{-1} f^4 g^{-6}}$

Rational Polynomial Equations

Solve for x; check answers (and reveal extraneous solutions!)

1) $\frac{x}{x+2} + \frac{7}{x-5} = \frac{29}{x^2 - 3x - 10}$

2) $\frac{3}{(x-3)} + \frac{4}{(x-4)} = \frac{25}{x^2 - 7x + 12}$

3) $x = \frac{2-x}{x-2}$

4) $\frac{3x+2}{x-1} + \frac{2x+4}{x+2} = 5$

5) $\frac{1}{1-x} = 1 - \frac{x}{x-1}$

6) $\frac{3x}{x-2} + \frac{2x}{x+3} = \frac{30}{(x+3)(x-2)}$

7) $3 - \frac{22}{x+5} = \frac{6x-1}{2x+7}$

Apply the average rate of change formula (with function notation)

$$\frac{f(x+h) - f(x)}{h}$$

Example: $f(x) = 3x + 2$

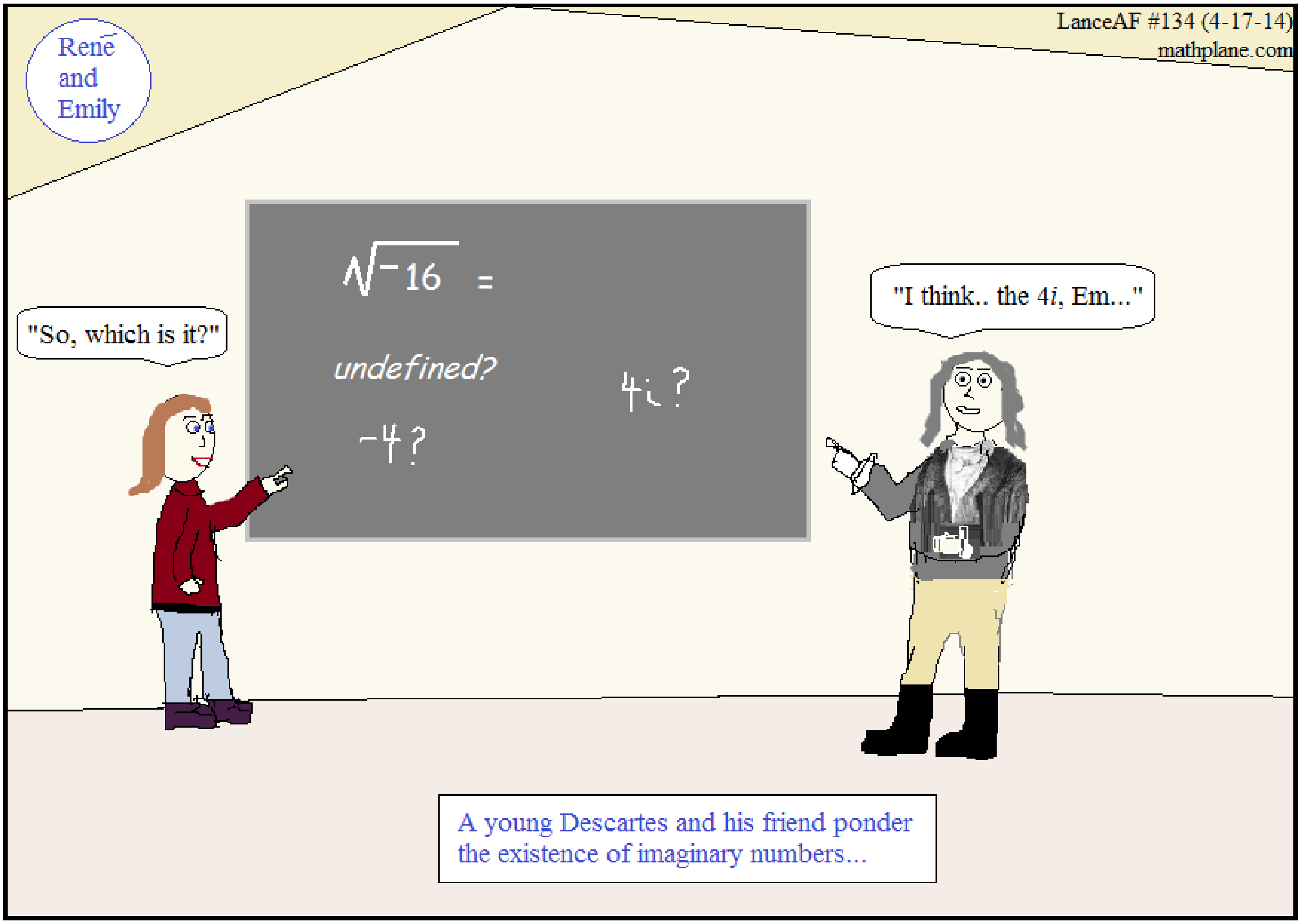
$f(x+h) = 3(x+h) + 2$

$$\frac{3x + 3h + 2 - (3x + 2)}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

$$\frac{f(x+h) - f(x)}{h} = 3$$

1) $f(x) = 6x - 4$ $\frac{f(x+h) - f(x)}{h} =$	2) $f(x) = 3x^2 + 5$ $\frac{f(x+h) - f(x)}{h} =$	3) $f(x) = x^2 + 4x - 1$ $\frac{f(x+h) - f(x)}{h} =$
4) $f(x) = \frac{2}{x+1}$ $\frac{f(x+h) - f(x)}{h} =$	5) $x^3 - 7$ $\frac{f(x+h) - f(x)}{h} =$	6) $f(x) = \sqrt{x} + 1$ $\frac{f(x+h) - f(x)}{h} =$


Rene
and
Emily
LanceAF #134 (4-17-14)
mathplane.com
"So, which is it?"
$\sqrt{-16}$ =
undefined?
-4?
4i?
"I think.. the 4*i*, Em..."
A young Descartes and his friend ponder the existence of imaginary numbers...

Solutions -→

Algebra II Preview

SOLUTIONS

I. Factorials

a) $5! = 120$

$5 \times 4 \times 3 \times 2 \times 1$

b) $\frac{12!}{10!} = 132$

$\frac{12 \times 11 \times 10!}{10!}$

c) $\frac{12!}{5!7!} = \frac{\cancel{12} \cdot 11 \cdot \cancel{10} \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{7!}}$

$\frac{11 \times 9 \times 8}{1} = 792$

d) $\frac{6!4!}{5!5!} = \frac{6 \times 5! \times 4!}{5! \times 5 \times 4!} = \frac{6}{5}$

e) $\frac{n!}{(n-3)!} = n(n-1)(n-2)$

$\frac{n \times (n-1) \times (n-2) \times (n-3)!}{(n-3)!}$

f) $\frac{(n+1)!(n-1)!}{n!(n-2)!} = (n+1)(n-1)$

$\frac{(n+1)\, n! \cdot (n-1)\,(n-2)!}{n! \cdot (n-2)!} = n^2 - 1$

g) How many different ways can the letters A B C D E F G be arranged?

$6! = 720$

(6 in the first slot, 5 choices for the second slot, 4 in the third slot, etc....)

II. Inverse Functions/Equations

Find the inverse:

a) $y = 3x + 6$

$x = 3y + 6$

$3y = x - 6$

$y = \frac{(x-6)}{3}$

b) $y = x^2 - 5$

$x = y^2 - 5$

$y^2 = x + 5$

$y = \sqrt{x+5}$

c) $y = -4x^2 + 7$

$x = -4y^2 + 7$

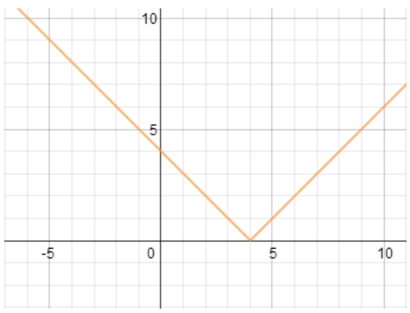
$4y^2 = -x + 7$

$y = \sqrt{\frac{-7+x}{4}}$

III. Graphing

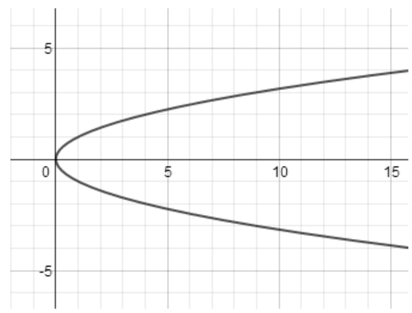
1) $y = |x - 4|$

x	y
2	2
3	1
4	0
5	1
6	2



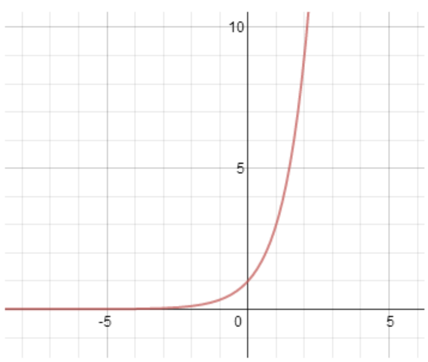
2) $x = y^2$

x	y
4	2
1	1
0	0
1	-1
4	-2



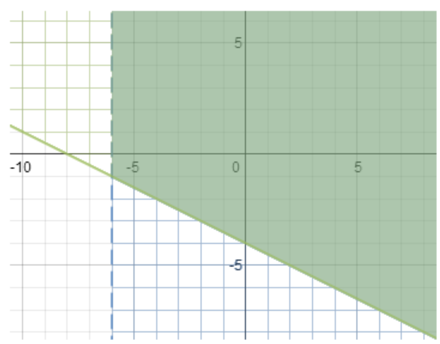
3) $y = 3^x$

x	y
2	9
1	3
0	1
-1	1/3
-2	1/9



4) $x > -6$

$x + 2y \geq -8$



5) $y \geq x^2 + 4$

$y \leq 2x + 3$

$x^2 + 4 = 2x + 3$

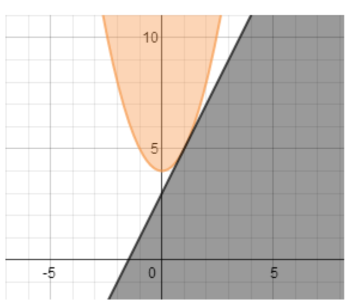
$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

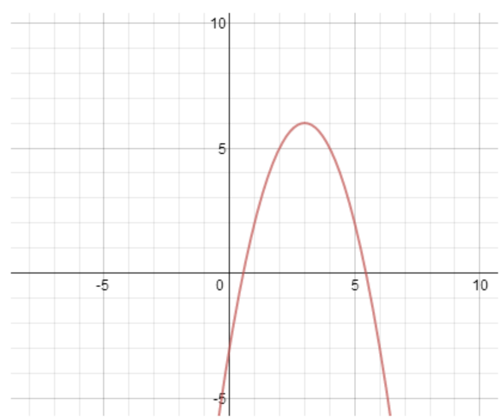
$y = 5$

Only solution is (1, 5)



6) $(x-3)^2 + y = 6$

x	y
-2	-19
-1	-10
0	-3
1	2
2	5
3	6
4	5
5	2
6	-3



IV. Systems

SOLUTIONS

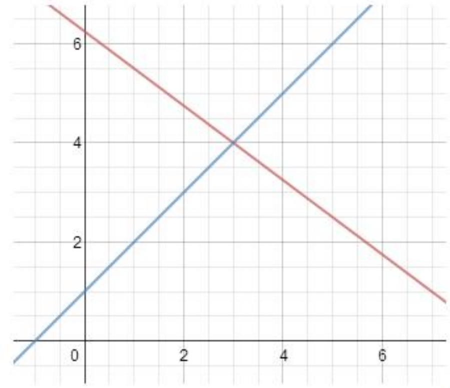
Algebra II Preview

Solve the following systems.
Graph to confirm your answers.

1) $3x + 4y = 25$
$y = x + 1$

(Use Substitution) $3x + 4(x + 1) = 25$
$7x + 4 = 25$
$x = 3$
then, $y = 4$

Check: $3(3) + 4(4) = 25$ ✓
$(4) = (3) + 1$ ✓



2) $x^2 + y^2 = 16$
$2x - y = 8 \longrightarrow y = 2x - 8$

$x^2 + (2x - 8)^2 = 16$

$x^2 + 4x^2 - 32x + 64 = 16$

$5x^2 - 32x + 48 = 0$

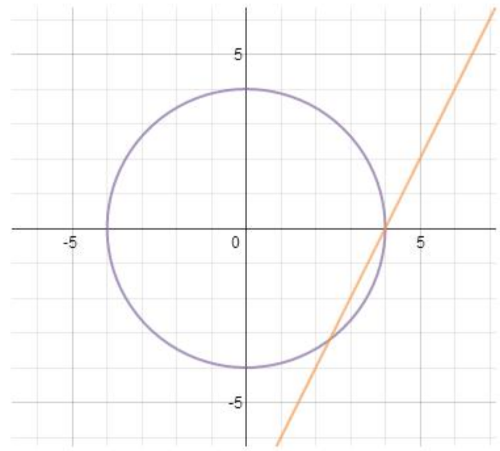
$(5x - 12)(x - 4) = 0$

$\boxed{\begin{array}{l} x = 12/5 \\ y = -16/5 \end{array}}$ $\boxed{\begin{array}{l} x = 4 \\ y = 0 \end{array}}$

Check:

$(4)^2 + (0)^2 = 16$ ✓
$2(4) - (0) = 8$ ✓

$(12/5)^2 + (-16/5)^2 =$
$144/25 + 256/25 =$
16 ✓
$2(12/5) - (-16/5) =$
$40/5 = 8$ ✓



3) $y = 2x^2 + 3$
$y = -6$

(plot these points to help draw the parabola)

x	y
-2	11
-1	5
0	3
1	5
2	11

(set equations equal each other)

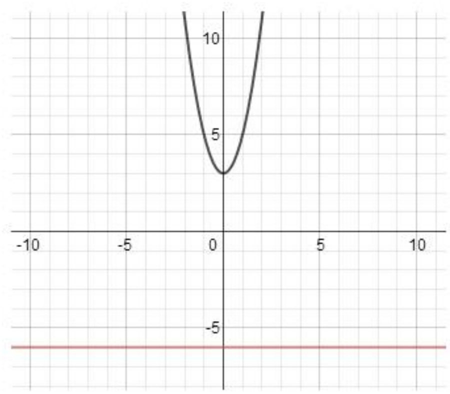
$-6 = 2x^2 + 3$

$-9 = 2x^2$

$-9/2 = x^2$

no real solutions!!

(no "intersections" in the graph)



4) $y = (x-2)^2$
$y = 3x - 6$

(set equations equal to each other to solve)

$(x-2)^2 = 3x - 6$

$x^2 - 4x + 4 = 3x - 6$

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$\boxed{\begin{array}{l} x = 5 \\ y = 9 \end{array}}$ $\boxed{\begin{array}{l} x = 2 \\ y = 0 \end{array}}$

Check:

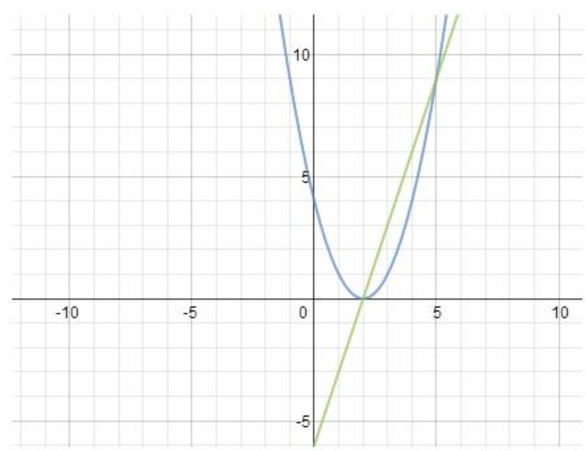
$(9) = ((5) - 2)^2$
$9 = 9$ ✓

$(9) = 3(5) - 6$
$9 = 9$ ✓

$(0) = ((2) - 2)^2$
$0 = 0$ ✓

$(0) = 3(2) - 6$
$0 = 0$ ✓

x	y
5	9
4	4
3	1
2	0
1	1
0	4



V. Miscellaneous

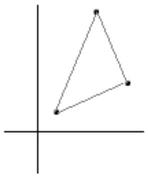
SOLUTIONS

Algebra II Preview

1) Do the points (3, 4) (8, 18) and (11, 6) form the vertices of a right triangle? Explain why or why not?

A right triangle must have one right angle.
And, a right angle is composed of 2 *perpendicular* line segments.

In a coordinate plane, if the slopes of 2 lines are *opposite reciprocals*, then the lines are perpendicular!



slope $= \dfrac{y_1 - y_2}{x_1 - x_2}$

slope of segment (3, 4) to (8, 18): $\dfrac{18-4}{8-3} = \dfrac{14}{5}$

(8, 18) to (11, 6): $\dfrac{6-18}{11-8} = \dfrac{-12}{3} = -4$

(11, 6) to (3, 4): $\dfrac{4-6}{3-11} = \dfrac{-2}{-8} = \dfrac{1}{4}$

Opposite reciprocals! perpendicular sides....

Yes, it is a right triangle...

2) Solve the following: $x^2 - x - 2 < 0$
Write the answer in *interval notation*.

Step 1: Find "critical points" (ignore the inequality)

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

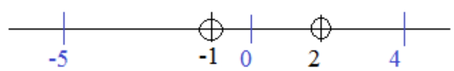
$x = -1, 2$

Step 2: Look at the inequality and "test regions"

Test left region: (try -5) $(-5)^2 - (-5) - 2 < 0$? $28 < 0$? NO

Test middle region: (try 0) $(0)^2 - (0) - 2 < 0$? $-2 < 0$? YES

Test right region: (try 4) $(4)^2 - 4 - 2 < 0$? $18 < 0$? NO



Step 3: Express inequality

Solution: (-1, 2) (interval notation) $-1 < x < 2$

3) $|2x + 7| \geq 7$ Answer in *set builder notation*.
Use a number line to illustrate the solution set.

Step 1: Find "critical points" (ignoring the inequality)

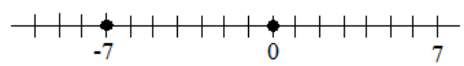
$|2x + 7| = 7$ ("split the absolute value")

$2x + 7 = 7 \rightarrow 2x = 0 \rightarrow x = 0$

$2x + 7 = -7 \rightarrow 2x = -14 \rightarrow x = -7$

Step 2: Look at the inequality (graph: "open or closed circles")

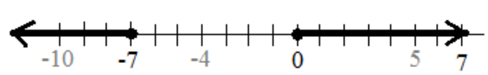
Because the inequality is greater than <u>or equal</u>, it will include 0 and -7



Step 3: Check the regions

Test -10: $|2(-10) + 7| \geq 7$?

$|-13|$ is > 7 YES



Test -4: $|2(-4) + 7| > 7$?

$|-1|$ is not > 7 NO

Solution set: $\{x \mid x \geq 0 \text{ or } x \leq -7\}$
or
$\{x \in \mathbb{R} \mid x \geq 0 \text{ or } x \leq -7\}$

(set builder notation)

Test 5: $|2(5) + 7| > 7$?

$|17|$ is > 7 YES

Algebra I and II Exponents Exercises

Solutions

I. Simplify

A) $3^3(3x)^2$

$27\cdot(3x)(3x) =$

$\boxed{243x^2}$

B) $(b^5)^2\, b^3$

$b^{10}b^3 =$

$\boxed{b^{13}}$

C) $(3.1)^5 \cdot (3.1)^{-5}$

$(3.1)^0 =$

$\boxed{1}$

D) $(-4x)^2 + 4x^2$

$16x^2 + 4x^2 =$

$\boxed{20x^2}$

E) $(-2a^2 b)^3\,(ab)^3$

$-8\cdot a^6\cdot b^3\cdot(a^3 b^3)$

$\boxed{-8a^9 b^6}$

F) $(3^6)^2 \cdot (3^{-4})^2$

$3^{12}3^{-8} = 3^4$

$= \boxed{81}$

G) $(10^3)^4 \cdot (4.3 \text{ x } 10^{-9})$

$10^{12}\cdot(4.3)\cdot(10^{-9}) =$

$\boxed{4.3 \text{ x } 10^3 \text{ or } 4300}$

H) $(2xy^2)^4\,(-y)^{-3}$

$2^4x^4y^8 \cdot \dfrac{1}{(-y)^3}$

$\dfrac{16x^4y^8}{-1\cdot y^3} = \boxed{-16x^4y^5}$

I) $x^{(a-1)} \cdot x^{(1-a)}$

$x^{(a-1)+(1-a)} =$

$x^0 = \boxed{1}$

II. Find x

A) $3^{2x} = 9^4$

"find common base" (3)

$3^{2x} = (3^2)^4$

$3^{2x} = 3^8$

then, "drop base and solve"

$2x = 8 \quad \boxed{x = 4}$

B) $8^2 = 2^x$

use common base 2

$(2^3)^2 = 2^x$

$2^6 = 2^x$

$\boxed{x = 6}$

C) $5^5 = 25^x$

$5^5 = (5^2)^x$

$5^5 = 5^{2x}$

$5 = 2x$

$\boxed{x = 5/2 \text{ or } 2.5}$

III. Reduce

A) $\dfrac{a^{-3}b^2c^0}{a^{-6}b^5c^2}$

"collect each variable"

$\dfrac{a^{-3}}{a^{-6}} = \dfrac{a^6}{a^3} = a^3$

$\dfrac{b^2}{b^5} = \dfrac{1}{b^3}$ and $\dfrac{1}{c^2}$

$\boxed{\dfrac{a^3}{b^3c^2}}$

B) $\dfrac{8d^3e^4}{4d^0e^6}$

$\dfrac{8}{4} = 2$

$\dfrac{d^3}{1} = d^3$

$\dfrac{e^4}{e^6} = \dfrac{1}{e^2}$

$\boxed{\dfrac{2d^3}{e^2}}$

C) $\dfrac{6f^{-2}g^{-3}}{2^{-1}f^4g^{-6}}$

$\dfrac{6}{2^{-1}} = 6 \text{ x } 2 = 12$

$\dfrac{f^{-2}}{f^4} = \dfrac{1}{f^6}$

$\dfrac{g^{-3}}{g^{-6}} = g^3$

$\boxed{\dfrac{12g^3}{f^6}}$

Rational Polynomial Equations

SOLUTIONS

Solve for x; check answers (and reveal extraneous solutions!)

1) $\dfrac{x}{x+2} + \dfrac{7}{x-5} = \dfrac{29}{x^2 - 3x - 10}$

$\dfrac{x}{(x+2)} + \dfrac{7}{(x-5)} = \dfrac{29}{(x+2)(x-5)}$

$\dfrac{x(x-5)}{(x+2)(x-5)} + \dfrac{7(x+2)}{(x-5)(x+2)} = \dfrac{29}{(x+2)(x-5)}$

$x^2 - 5x + 7x + 14 = 29$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$\boxed{x = 3, -5}$

if x = 3, then

$\dfrac{3}{5} + \dfrac{7}{-2} = \dfrac{29}{-10}$

$\dfrac{-6}{-10} + \dfrac{35}{-10} = \dfrac{29}{-10}$ ✓

if x = -5, then

$\dfrac{-5}{-3} + \dfrac{7}{-10} = \dfrac{29}{30}$

$\dfrac{50}{30} + \dfrac{-21}{30} = \dfrac{29}{30}$ ✓

2) $\dfrac{3}{(x-3)} + \dfrac{4}{(x-4)} = \dfrac{25}{x^2 - 7x + 12}$

$\dfrac{3(x-4)}{(x-3)(x-4)} + \dfrac{4(x-3)}{(x-4)(x-3)} = \dfrac{25}{(x-3)(x-4)}$

$3x - 12 + 4x - 12 = 25$

$7x - 24 = 25$

$\boxed{x = 7}$

if x = 7, then

$\dfrac{3}{4} + \dfrac{4}{3} = \dfrac{25}{12}$

$\dfrac{9}{12} + \dfrac{16}{12} = \dfrac{25}{12}$ ✓

3) $\dfrac{x}{1} = \dfrac{2-x}{x-2}$

(cross multiply)

$x(x-2) = 1(2-x)$

$x^2 - 2x - 2 + x = 0$

$(x-2)(x+1) = 0$

$\boxed{x = \cancel{2}, -1}$

2 is an extraneous solution

if x = 2, then

$2 = \dfrac{0}{0}$ ✗

if x = -1, then

$-1 = \dfrac{3}{-3}$ ✓

4) $\dfrac{3x+2}{x-1} + \dfrac{2x+4}{x+2} = 5$

$\dfrac{3x+2}{x-1} + \dfrac{2\cancel{(x+2)}}{\cancel{(x+2)}} = 5$

$\dfrac{3x+2}{x-1} = 3$

$3x + 2 = 3(x-1)$

$3x + 2 = 3x - 3$

$\boxed{\text{NO SOLUTION}}$

5) $\dfrac{1}{1-x} = 1 - \dfrac{x}{x-1}$

$\dfrac{-1(1)}{-1(1-x)} = \dfrac{x-1}{x-1} - \dfrac{x}{x-1}$

$\dfrac{-1}{(x-1)} = \dfrac{x-1-x}{x-1}$

$\dfrac{-1}{(x-1)} = \dfrac{-1}{x-1}$

$\boxed{x = \text{all real numbers EXCEPT } 1}$

if x = 5, then $\dfrac{1}{-4} = 1 - \dfrac{5}{4}$ ✓

6) $\dfrac{3x}{x-2} + \dfrac{2x}{x+3} = \dfrac{30}{(x+3)(x-2)}$

$\dfrac{3x(x+3)}{(x-2)(x+3)} + \dfrac{2x(x-2)}{(x+3)(x-2)} = \dfrac{30}{(x+3)(x-2)}$

$3x^2 + 9x + 2x^2 - 4x = 30$

$5x^2 + 5x - 30 = 0$

$5(x^2 + x - 6) = 0$

$5(x+3)(x-2) = 0$

$\boxed{x = \cancel{-3}, \cancel{2}}$

Extraneous -- no solutions!

if x = -3, then

$\dfrac{-9}{-5} + \dfrac{-6}{0} = \dfrac{30}{0}$ ✗

if x = 2, then

$\dfrac{6}{0} + \dfrac{4}{5} = \dfrac{30}{0}$ ✗

7) $3 - \dfrac{22}{x+5} = \dfrac{6x-1}{2x+7}$

combine left side

$\dfrac{3x-7}{x+5} = \dfrac{6x-1}{2x+7}$

cross multiply

$6x^2 + 21x - 14x - 49 = 6x^2 + 30x - x - 5$

$-44 = 22x$

$\boxed{x = -2}$

if x = -2, then

$3 - \dfrac{22}{3} = \dfrac{-13}{3}$ ✓

Apply the average rate of change formula (with function notation)

$$\frac{f(x+h) - f(x)}{h}$$

Example: $f(x) = 3x + 2$

$$f(x+h) = 3(x+h) + 2$$

$$\frac{3x + 3h + 2 - (3x+2)}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

$$\frac{f(x+h) - f(x)}{h} = 3$$

1) $f(x) = 6x - 4$

$$f(x+h) = 6(x+h) - 4$$

then, $$\frac{6x + 6h - 4 - (6x - 4)}{h}$$

$$\frac{6h}{h} = 6$$

$$\frac{f(x+h) - f(x)}{h} = 6$$

2) $f(x) = 3x^2 + 5$

$$f(x+h) = 3(x+h)^2 + 5$$

$$= 3x^2 + 6xh + 3h^2 + 5$$

then, $$\frac{3x^2 + 6xh + 3h^2 + 5 - (3x^2 + 5)}{h}$$

$$\frac{6xh + 3h^2}{h} = \frac{h(6x + 3h)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 6x + 3h$$

3) $f(x) = x^2 + 4x - 1$

$$f(x) = (x+h)^2 + 4(x+h) - 1$$

$$= x^2 + 2xh + h^2 + 4x + 4h - 1$$

then, $$\frac{x^2 + 2xh + h^2 + 4x + 4h - 1 + (x^2 + 4x - 1)}{h}$$

$$\frac{2xh + h^2 + 4h}{h} = \frac{h(2x + h + 4)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 2x + h + 4$$

4) $f(x) = \frac{2}{x+1}$

$$f(x+h) = \frac{2}{(x+h)+1}$$

then, $$\frac{\frac{2}{(x+h)+1} - \frac{2}{x+1}}{h}$$

$$\frac{\frac{2(x+1) - 2(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$\frac{2x + 2 - 2x - 2h - 2}{(x+h+1)(x+1)} \cdot \frac{1}{h} =$$

$$\frac{-2h}{h(x+h+1)(x+1)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-2}{(x+1)(x+h+1)}$$

5) $x^3 - 7$

$$f(x+h) = (x+h)^3 - 7$$

$$= (x+h)(x^2 + 2xh + h^2) - 7$$

$$= x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3 - 7$$

then, $$\frac{x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3 - 7 - (x^3 - 7)}{h}$$

$$\frac{3x^2h + 3xh^2 + h^3}{h}$$

$$\frac{h(3x^2 + 3xh + h^2)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2$$

6) $f(x) = \sqrt{x+1}$

$$f(x+h) = \sqrt{(x+h)+1}$$

then, $$\frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h} \cdot \frac{(\sqrt{(x+h)+1} + \sqrt{x+1})}{(\sqrt{(x+h)+1} + \sqrt{x+1})}$$

(using conjugate of numerator)

$$\frac{(x+h)+1 - (x+1)}{h\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}$$

$$\frac{h}{h\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{\sqrt{(x+h)+1} + \sqrt{x+1}}$$

A couple of examples…

Solving linear systems: 2 equations, 3 unknowns

Example: $x + 2y - z = 6$

$2x + 7y + z = 10$

Step 1: Use elimination/combination method to find y (eliminate x; solve for y in terms of z)

(mult. by -2) $x + 2y - z = 6$

$$\begin{array}{r} -2x - 4y + 2z = -12 \\ 2x + 7y + z = 10 \\ \hline 3y + 3z = -2 \end{array}$$

$3y = -2 - 3z$

$$\boxed{y = -2/3 - z}$$

Step 2: Use elimination/combination method to find x (eliminate y; solve for x in terms of z)

$x + 2y - z = 6$

(mult. by -7) $-7x - 14y + 7z = -42$

(mult. by 2) $2x + 7y + z = 10$

$4x + 14y + 2z = 20$

$$\begin{array}{r} -7x - 14y + 7z = -42 \\ 4x + 14y + 2z = 20 \\ \hline -3x + 9z = -22 \end{array}$$

$-3x = -22 - 9z$

$$\boxed{x = 22/3 + 3z}$$

Step 3: Check your results

Suppose $z = 0$:

(22/3, -2/3, 0) is one solution

$x = 22/3 + 3(0) = 22/3$

$y = -2/3 - (0) = -2/3$

$x + 2y - z = 6$ $\quad (22/3) + 2(-2/3) - (0) = 18/3 = 6$ ✓

$2x + 7y + z = 10$ $\quad 2(22/3) + 7(-2/3) + (0) = 30/3 = 10$ ✓

Suppose $z = 2$:

(40/3, -8/3, 2) is another solution

$x = 22/3 + 3(2) = 40/3$

$y = -2/3 - (2) = -8/3$

$x + 2y - z = 6$ $\quad (40/3) + 2(-8/3) - (2) = 18/3 = 6$ ✓

$2x + 7y + z = 10$ $\quad 2(40/3) + 7(-8/3) + (2) = 30/3 = 10$ ✓

Step 4: Express a general solution (in terms of z)

$(x, y, z) = (22/3 + 3z, -2/3 - z, z)$

$\boxed{\text{Solving linear systems: 2 equations, 3 unknowns}}$

Example: Solve the following system in terms of y. Then, provide three specific solutions.

$$2x + 5y - z = 8$$
$$x - 2y + 3z = 6$$

Step 1: Use substitution method to find z (eliminate x; solve for z in terms of y)

$$2x + 5y - z = 8$$
$$x = 2y - 3z + 6$$

$$2(2y - 3z + 6) + 5y - z = 8$$
$$4y - 6z + 12 + 5y - z = 8$$
$$-7z = -9y - 4$$
$$z = 4/7 + 9/7y$$

$$\boxed{z = 4/7 + 9/7y}$$

Step 2: Use elimination/combination method to find x (eliminate z; solve for x in terms of y)

(mult. by 3) $2x + 5y - z = 8$

$$6x + 15y - 3z = 24$$
$$x - 2y + 3z = 6$$
$$7x + 13y = 30$$
$$7x = 30 - 13y$$

$$\boxed{x = 30/7 - 13/7y}$$

Step 3: Check answers and find 3 points

system: $2x + 5y - z = 8$
$x - 2y + 3z = 6$

solution:

$x = 30/7 - (13/7)y$
$y = y$
$z = 4/7 + (9/7)y$

General Solution
(in terms of y)

$$\left(\frac{30}{7} - \frac{13y}{7},\ y,\ \frac{4}{7} + \frac{9y}{7}\right)$$

Note: The solution is a 3-dimensional line;
As y increases by 1,
x decreases by 13/7 and z increases by 9/7

Let y = 0: $x = 30/7 - (13/7)(0) = 30/7$
$z = 4/7 + (9/7)(0) = 4/7$

$2(30/7) + 5(0) - (4/7) = 56/7 = 8$ ✓
$(30/7) - 2(0) + 3(4/7) = 42/7 = 6$ ✓

$\boxed{(30/7, 0, 4/7)}$

Let y = 1: $x = 30/7 - (13/7)(1) = 17/7$
$z = 4/7 + (9/7)(1) = 13/7$

$2(17/7) + 5(1) - (13/7) = 8$ ✓
$(17/7) - 2(1) + 3(13/7) = 6$ ✓

$\boxed{(17/7, 1, 13/7)}$

Let y = 2 $x = 30/7 - (13/7)(2) = 4/7$
$z = 4/7 + (9/7)(2) = 22/7$

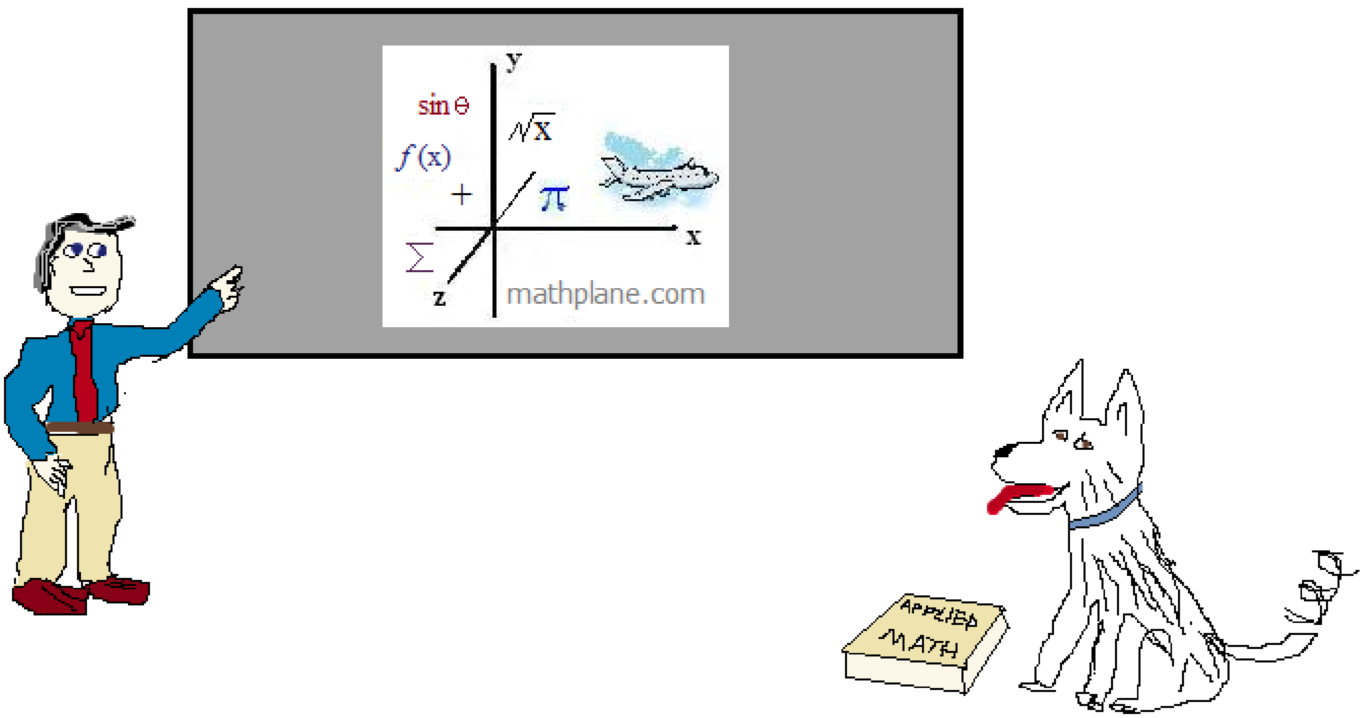
$2(4/7) + 5(2) - (22/7) = 8$ ✓
$(4/7) - 2(2) + 3(22/7) = 6$ ✓

$\boxed{(4/7, 2, 22/7)}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy



Also, at Facebook, Google+, TeachersPayTeachers, TES,and Pinterest

And, Mathplane *Express* for mobile at mathplane.ORG

One more question:

$$\frac{x+6}{x-1} \geq 0$$

Can you solve and graph (on number line)?

Solve and graph on a number line. (Express your answer in interval notation)

$$\frac{x+6}{x-1} \geq 0$$

Step 1: Identify the critical points

(numerator)

$$\frac{x+6}{x-1} = 0$$

$x + 6 = 0$

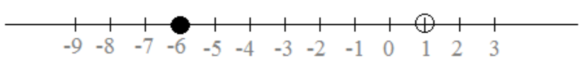
$x = -6$

(denominator)

$$\frac{x+6}{x-1} = 0$$

at $x = 1$,
the equation is undefined..

Step 2: "open circle" or "closed circle"; then, test regions



Since the inequality is > or equal, the critical points will be "closed" circles.. HOWEVER, since the equation is undefined at 1, there will be an "open" circle at 1.

Test regions

$x < -6$: try -9 --- $\frac{(-9)+6}{(-9)-1} \geq 0$

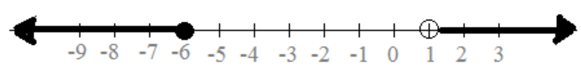
$\frac{-3}{-10} \geq 0$ YES

$-6 < x < 1$: try 0 --- $\frac{(0)+6}{(0)-1} \geq 0$

$\frac{6}{-1} \geq 0$ NO

$x > 1$: try 5 --- $\frac{(5)+6}{(5)-1} \geq 0$

$\frac{11}{4} \geq 0$ YES



Interval Notation:

$(-\infty, -6] \cup (1, \infty)$

closed bracket   open parentheses